Approaches to the tunneling time based on the Larmor clock and particle absorption as particular cases of the stay-time method

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The Larmor clock and the particle absorption approaches to the tunneling time are reexamined and extended to cover two- and three-dimensional systems and multichannel scattering. We show that both of them can be viewed as particular cases of the recent stay-time method. Furthermore, the barrier traversal time proposed by Büttiker on the basis of the Larmor clock is found to correspond to the root-mean-square value of the actual tunneling time, rather than to the mean tunneling time.

The question of determining the characteristic times in the motion of a particle in a classically forbidden region is still receiving considerable attention. Several articles have appeared recently which address the whole subject and compare the various approaches to the tunneling time problem.[1,2] One recent work[3] proposes an approach based on Feynman path integrals, and allows us to obtain a real-valued mean time spent by the particle in a given region (which is called "stay time") and also the standard deviation of its distribution. Moreover, this approach explains the results of many other studies addressing the subject from different points of view.

In this Brief Report we consider two well known approaches to the tunneling time problem, basically consisting of applying a perturbation to the region of interest. We refer to the method based on the Larmor precession,[4–7] where the perturbation is a uniform magnetic field, and to the one based on the absorption of particles in the barrier,[8–10] where the perturbation is a uniform pure imaginary potential, which acts on the wave function as an optical "absorber." The equivalence of these approaches has been demonstrated by Muga, Brouard, and Sala[10] using the projection operator technique.

We show that these approaches can be easily viewed as particular cases of the stay-time method.[3] In order to do this, we reformulate both the approaches considered in such a way that their validity is extended to multichannel scattering and to two- and three-dimensional systems. Moreover, this procedure sheds new light on the time proposed by Büttiker on the basis of the Larmor clock.[6]

It has been shown[3] that the time spent by a particle in a given region of space Ω can be obtained by adding at time t_0 a uniform perturbative potential V to the region being considered, in such a way that the Hamiltonian $\hat{H}_V(V,t_0;\mathbf{r},t)$ of the system is

$$\hat{H}_V(V,t_0;\mathbf{r},t) = \hat{H}_0(\mathbf{r},t) + Vu(t-t_0)\Theta_\Omega(\mathbf{r}), \tag{1}$$

where $\hat{H}_0$ is the Hamiltonian in the absence of perturbation, $\Theta_\Omega(\mathbf{r})$ equals 1 if $\mathbf{r}\in\Omega$ and 0 otherwise, and $u(t-t_0)$ is the Heaviside function. The particle is described by a wave function $\Psi_V(V,t_0;\mathbf{r},t)$ which is the solution of the Schrödinger equation:

$$\hat{H}_V(V,t_0;\mathbf{r},t)\Psi_V(V,t_0;\mathbf{r},t) = i\hbar\frac{\partial}{\partial t}\Psi_V(V,t_0;\mathbf{r},t), \tag{2}$$

$\hbar$ being the reduced Planck's constant. We call $\Psi_0(\mathbf{r},t)$ the wave function of the unperturbed system, i.e., $\Psi_0(\mathbf{r}_1,t_1) = \Psi_V(0,t_0;\mathbf{r}_1,t_1)$.

By exploiting the properties of Feynman path integrals we have proposed that the mean time spent in Ω from time t_0 to t_1 by a particle which is found in $\mathbf{r}_1$ at t_1 is

$$\overline{\tau}(t_0;\mathbf{r}_1,t_1) \equiv \mathrm{Re}\left\{\frac{i\hbar}{\Psi_0}\frac{\partial\Psi_V}{\partial V}\right\}\Bigg|_{V=0}. \tag{3}$$

We have called $\overline{\tau}(t_0;\mathbf{r}_1,t_1)$ mean "stay time."

This method also allows us to obtain the mean-square stay time $\overline{\tau^2}(t_0;\mathbf{r}_1,t_1)$ and the standard deviation $\sigma_\tau(t_0;\mathbf{r}_1,t_1)$ of the stay time, in the forms, respectively,

$$\overline{\tau^2}(t_0;\mathbf{r}_1,t_1) = \left|\frac{i\hbar}{\Psi_0}\frac{\partial\Psi_V}{\partial V}\right|^2\Bigg|_{V=0} \tag{4}$$

$$\sigma_\tau(t_0;\mathbf{r}_1,t_1) = \left|\mathrm{Im}\left\{\frac{i\hbar}{\Psi_0}\frac{\partial\Psi_V}{\partial V}\right\}\Bigg|_{V=0}\right|, \tag{5}$$

where, in (3)–(5), $\Psi_V(V,t_0;\mathbf{r},t)$ is valued in $\mathbf{r}_1$ at t_1.

The stay-time approach is shown to be self-consistent and effective in deriving the results of studies addressing the tunneling time from different points of view. Except for the approach based on Bohm's causal interpretation of quantum mechanics pioneered and recently reviewed by Leavens *et al.*,[11,12] virtually all the other approaches can be derived by the means of the stay-time method.[3] The Büttiker-Landauer method of the oscillating barrier[13] will be addressed elsewere. Here we want to show that the stay-time approach is a useful tool for obtaining the results of the methods based on the Larmor clock and on particle absorption.

PARTICLE ABSORPTION METHOD

This method has the undoubted virtue of being very intuitive. It is well known that a state with a finite lifetime can be described by adding a pure imaginary component to the energy of the state. If we apply a uniform pure imaginary potential $-i\Gamma/2$ to the whole space at time t_0, the Schrödinger equation becomes

$$\left[\hat{H}_0 - i\frac{\Gamma}{2}u(t-t_0)\right]\Psi' = i\hbar\frac{\partial\Psi'}{\partial t}. \tag{6}$$

It can be easily verified that the solution of (6) is $\Psi' = \Psi_0 \exp\{-(\Gamma/2)\hbar(t-t_0)u(t-t_0)\}$, where, as we wrote above, Ψ_0 is the solution of the unperturbed equation [Eq. (6) with $\Gamma = 0$]. Therefore, after integrating the probability density on the whole space, we write

$$\int |\Psi'(\mathbf{r},t)|^2 d\mathbf{r} = \exp\left[-\frac{\Gamma}{\hbar}(t-t_0)u(t-t_0)\right] \times \int |\Psi_0(\mathbf{r},t)|^2 d\mathbf{r}, \tag{7}$$

i.e., the probability of finding the particle decays exponentially from the moment the absorbing potential is applied with a characteristic time constant $\tau' = \hbar/\Gamma$, which is the mean lifetime of the state.

The basic idea[8–10] for obtaining the time spent in Ω is to apply the absorbing potential $-i\Gamma/2$ at time t_0 only to region Ω, and to state that, at least to first order in Γ, the probability density in $\mathbf{r}_1$ at t_1 has the form

$$|\Psi_\Gamma(\Gamma,t_0;\mathbf{r}_1,t_1)|^2 = |\Psi_0(\mathbf{r}_1,t_1)|^2 \times \exp[-(\Gamma/\hbar)\tau_{\rm abs}(t_0;\mathbf{r}_1,t_1)], \tag{8}$$

where $\tau_{\rm abs}(t_0;\mathbf{r}_1,t_1)$ is assumed to be the mean time spent in Ω, in the time interval (t_0,t_1), by the particle found in $\mathbf{r}_1$ at t_1, and $\Psi_\Gamma(\Gamma,t_0;\mathbf{r},t)$ solves the equation

$$\left[\hat{H}_0 - i\frac{\Gamma}{2}u(t-t_0)\Theta_\Omega(\mathbf{r})\right]\Psi_\Gamma = i\hbar\frac{\partial\Psi_\Gamma}{\partial t}. \tag{9}$$

From (8) we can draw a definition of $\tau_{\rm abs}(t_0;\mathbf{r}_1,t_1)$ as

$$\tau_{\rm abs}(t_0;\mathbf{r}_1,t_1) = -\frac{\hbar}{|\Psi_0|^2}\frac{\partial|\Psi_\Gamma|^2}{\partial\Gamma}\bigg|_{\Gamma=0} = \mathrm{Re}\left\{-\frac{2\hbar}{\Psi_0}\frac{\partial\Psi_\Gamma}{\partial\Gamma}\right\}\bigg|_{\Gamma=0}, \tag{10}$$

where $\Psi_0(\mathbf{r},t)$ and $\Psi_\Gamma(\Gamma,t_0;\mathbf{r},t)$ are valued in $\mathbf{r}_1$ at t_1.

Comparison of (9) with (1) and (2) straightforwardly yields (details are given in the Appendix)

$$\frac{\partial\Psi_\Gamma}{\partial\Gamma}\bigg|_{\Gamma=0} = -\frac{i}{2}\frac{\partial\Psi_V}{\partial V}\bigg|_{V=0}. \tag{11}$$

This result, after substitution in (10), allows us to obtain

$$\tau_{\rm abs}(t_0;\mathbf{r}_1,t_1) = \mathrm{Re}\left\{\frac{i\hbar}{\Psi_0}\frac{\partial\Psi_V}{\partial V}\right\}\bigg|_{V=0} = \overline{\tau}(t_0;\mathbf{r}_1,t_1), \tag{12}$$

i.e., the time obtained by superimposing an imaginary absorbing potential on the region Ω coincides with the mean stay time in Ω given by (3).

LARMOR CLOCK METHOD

The Larmor clock method was originally proposed by Baz'[4] and Rybachenko[5] and has been successively reexamined by several authors.[6,7,14–16] We formulate this method in a slightly different way, in order to extend its range of applicability and put in evidence its relations with the stay-time approach.

In this case the perturbative element is a uniform magnetic field $\mathbf{B}$ applied to the region Ω at time t_0. Let its direction be the z axis ($\mathbf{B} = [0,0,B\Theta_\Omega(\mathbf{r})u(t-t_0)]$).

It is worth noting that not all system configurations are feasible. The point is that the lines of flux of the magnetic field must be closed lines. This implies that while one-dimensional and two-dimensional regions are always feasible (the lines of flux can pass through the considered plane far enough from the system not to affect wave function evolution), the situation is different for three-dimensional regions. In this case the region Ω has to be chosen in such a way that the lines of flux of the magnetic field pass, out of Ω, only through regions in which the wave function is almost vanishing, for instance, regions where the potential is almost infinite.

In the following calculations we shall assume that the magnetic field is confined only to region Ω and that, if it is present also in other regions, this fact does not affect wave function evolution, because the wave function in these regions is practically zero.

The Hamiltonian to be dealt with is

$$\hat{H}_B = \left[[-i\hbar\nabla - q\mathbf{A}(\mathbf{r},t)]^2/2m + V_0(\mathbf{r},t)\right] \times I - (\hbar\omega_L/2)\sigma_z\Theta_\Omega(\mathbf{r})u(t-t_0), \tag{13}$$

where σ_x, σ_y, and σ_z are the Pauli spin matrices, I is the unit 2×2 matrix, m and q are the mass and the charge of the particle, respectively, $\omega_L = qB/m$ is the Larmor precession frequency, $V_0(\mathbf{r},t)$ is the scalar potential, and $\mathbf{A}$ is the vector potential such as $\mathbf{B}(\mathbf{r},t) = \nabla\wedge\mathbf{A}(\mathbf{r},t)$. The wave function is a two-component vector

$$\mathbf{\Psi}_B(B,t_0;\mathbf{r},t) = \begin{pmatrix}\Psi_B^+(B,t_0;\mathbf{r},t)\\ \Psi_B^-(B,t_0;\mathbf{r},t)\end{pmatrix}. \tag{14}$$

Let the spinors of the unperturbed wave function $\mathbf{\Psi}_0$ be labeled Ψ_0^+ and Ψ_0^- and let $\mathbf{\Psi}_0$ be polarized before t_0 in a direction perpendicular to the magnetic field, say, the x direction, implying $\Psi_0^+ = \Psi_0^-$ for $t \le t_0$. If there is no magnetic field the electron conserves its spin so that $\Psi_0^+ = \Psi_0^-$ for any t. We are not interested in normalized wave function so let us assume $\Psi_0^\pm = \Psi_0$.

A particle found in $\mathbf{r}_1$ at time t_1 has spin expectation values in the three directions

$$\langle S_y(t_0;\mathbf{r}_1,t_1)\rangle = \frac{\hbar}{2}\frac{\mathbf{\Psi}_B^*\sigma_y\mathbf{\Psi}_B}{\mathbf{\Psi}_B^*\mathbf{\Psi}_B} = \frac{\hbar\,\mathrm{Im}\{\Psi_B^{-*}\Psi_B^+\}}{\Psi_B^{+*}\Psi_B^+ + \Psi_B^{-*}\Psi_B^-}, \tag{15}$$

$$\langle S_z(t_0;\mathbf{r}_1,t_1)\rangle = \frac{\hbar}{2}\frac{\mathbf{\Psi}_B^*\sigma_z\mathbf{\Psi}_B}{\mathbf{\Psi}_B^*\mathbf{\Psi}_B} = \frac{\hbar}{2}\frac{\Psi_B^{+*}\Psi_B^+ - \Psi_B^{-*}\Psi_B^-}{\Psi_B^{+*}\Psi_B^+ + \Psi_B^{-*}\Psi_B^-}, \tag{16}$$

$$\langle S_x(t_0;\mathbf{r}_1,t_1)\rangle = \left(\hbar^2/4 - \langle S_y(t_0;\mathbf{r}_1,t_1)\rangle^2 - \langle S_z(t_0;\mathbf{r}_1,t_1)\rangle^2\right)^{1/2}, \tag{17}$$

where $\mathbf{\Psi}_B$ is valued in $\mathbf{r}_1$ at t_1. There are two concurring effects in the change of spin orientation. One is Larmor precession, according to which the spin rotates

on a plane perpendicular to the magnetic field direction (the xy plane in our case) with constant frequency ω_L. The other is the Zeeman effect, according to which electrons with spin along the magnetic field direction acquire energy $\hbar\omega_L/2$ and particles with spin in the opposite direction lose the same energy resulting in different behavior with respect to a given potential configuration, and in varying spin expectation values along the z axis. In particular, in the case of tunneling of a potential barrier, particles polarized in the direction of magnetic field have a greater probability of traversing the barrier, so that the z-axis component of the spin expectation value of the tunneled particles increases.

According to the well known methods based on the Larmor clock,[6] one can define three characteristic times,

$$\tau_y(t_0;\mathbf{r}_1,t_1) = -\frac{2}{\hbar}\left.\frac{\partial\langle S_y(t_0;\mathbf{r}_1,t_1)\rangle}{\partial\omega_L}\right|_{B=0}, \tag{18}$$

$$\tau_z(t_0;\mathbf{r}_1,t_1) = \frac{2}{\hbar}\left.\frac{\partial\langle S_z(t_0;\mathbf{r}_1,t_1)\rangle}{\partial\omega_L}\right|_{B=0}, \tag{19}$$

$$\tau_x(t_0;\mathbf{r}_1,t_1) = \left[\tau_y(t_0;\mathbf{r}_1,t_1)^2 + \tau_z(t_0;\mathbf{r}_1,t_1)^2\right]^{1/2}. \tag{20}$$

Now, let us add to the region Ω at time t_0 a constant potential V also. Therefore, the Hamiltonian is

$$\hat{H}_{BV} = \hat{H}_B + V\Theta_\Omega(\mathbf{r})u(t-t_0), \tag{21}$$

and the corresponding wave function is $\mathbf{\Psi}_{BV}(B,V,t_0;\mathbf{r},t)$. Let us note that if $V=0$ $\mathbf{\Psi}_{BV}$ reduces to the $\mathbf{\Psi}_B$ of (14) and if $B=0$ each component of $\mathbf{\Psi}_{BV}$ reduces to Ψ_V solution of (2).

The Hamiltonian $\hat{H}_{BV}$ of (21) is a 2×2 diagonal matrix and can be written as

$$\hat{H}_{BV}(B,V,t_0;\mathbf{r},t) = \begin{bmatrix} \hat{H}^+_{BV}(B,V,t_0;\mathbf{r},t) & 0 \\ 0 & \hat{H}^-_{BV}(B,V,t_0;\mathbf{r},t) \end{bmatrix}, \tag{22}$$

where

$$\hat{H}^\pm_{BV}(B,V,t_0;\mathbf{r},t) = \frac{[-i\hbar\nabla - q\mathbf{A}(\mathbf{r},t)]^2}{2m} + V_0(\mathbf{r},t) + \left(V \mp \frac{\hbar\omega_L}{2}\right)\Theta_\Omega(\mathbf{r})u(t-t_0). \tag{23}$$

Let us point out that

$$\hat{H}^-_{BV}(B,V,t_0;\mathbf{r},t) = \hat{H}^+_{BV}[B,V'(B),t_0;\mathbf{r},t], \tag{24}$$

where $V'(B) = V + \hbar\omega_L$, i.e., the Hamiltonian for the spinor Ψ^-_{BV} with a perturbative potential V is equal to the Hamiltonian for Ψ^+_{BV} with a perturbative potential $V'(B) = V + \hbar\omega_L$. Given that at time t_0 we have $\Psi^+_B = \Psi^-_B = \Psi_0$ we can write

$$\Psi^-_{BV}(B,V,t_0;\mathbf{r},t) = \Psi^+_{BV}(B,V'(B),t_0;\mathbf{r},t). \tag{25}$$

It is shown in the Appendix that from (25) we obtain

$$\left.\frac{\partial[\Psi^-_{BV} - \Psi^+_{BV})}{\partial\omega_L}\right|_{\substack{B=0\\V=0}} = \hbar\left.\frac{\partial\Psi^+_{BV}}{\partial V}\right|_{\substack{B=0\\V=0}}, \tag{26}$$

or, more simply,

$$\left.\frac{\partial[\Psi^-_B - \Psi^+_B]}{\partial\omega_L}\right|_{B=0} = \hbar\left.\frac{\partial\Psi_V}{\partial V}\right|_{V=0}. \tag{27}$$

Substitution of (27) in (3), (4), and (15)–(20) yields

$$\tau_y(t_0;\mathbf{r}_1,t_1) = \overline{\tau}(t_0;\mathbf{r}_1,t_1), \tag{28}$$

$$|\tau_z(t_0;\mathbf{r}_1,t_1)| = \sigma_\tau(t_0;\mathbf{r}_1,t_1), \tag{29}$$

$$\tau_x(t_0;\mathbf{r}_1,t_1) = [\overline{\tau^2}(t_0;\mathbf{r}_1,t_1)]^{1/2}. \tag{30}$$

The time τ_y was originally proposed by Baz'[4] and Rybachenko[5] as the time spent by the particle in the region Ω. The underlying assumption is that spin precession in the classically forbidden region is still proportional, at least to first order, to the Larmor frequency ω_L. Büttiker[6] afterwards proposed that the whole spin rotation had to be considered (precession on the xy plane and Zeeman rotation) and that it was linear with respect to time with the same constant ω_L, at least to first order. This idea leads one to consider τ_x as the actual time spent in the forbidden region.

From (28) and (30) it can be seen that the mean stay time in Ω is equal to the Larmor time τ_y, and the time τ_x proposed by Büttiker is the root-mean-square (rms) stay time. To first order in ω_L, we have

$$\begin{aligned}\langle S_y(t_0;\mathbf{r}_1,t_1)\rangle &= -(\hbar/2)\omega_L\tau_y(t_0;\mathbf{r}_1,t_1)\\ &= -(\hbar/2)\omega_L\overline{\tau}(t_0;\mathbf{r}_1,t_1),\end{aligned} \tag{31}$$

and, to second order in ω_L, we have

$$\begin{aligned}\langle S_x(t_0;\mathbf{r}_1,t_1)\rangle &= (\hbar/2)\left[1-\omega_L^2[\tau_x(\mathbf{r}_1,t_1)]^2/2\right]\\ &= (\hbar/2)[1-\omega_L^2\overline{\tau^2}(t_0;\mathbf{r}_1,t_1)/2].\end{aligned} \tag{32}$$

It is interesting to note that τ_x appears squared in (32); the mean variation of the spin expectation value in the x direction is proportional to τ_x^2; therefore, it seems highly plausible to give τ_x the meaning of rms time spent in Ω.

DISCUSSION

We have shown that the methods for defining and obtaining the tunneling time based on particle absorption and on the Larmor clock can be seen as particular cases of the stay time method.

Since the times $\tau_{\rm abs}$ and τ_y are equal to the mean stay time, they satisfy, in the case of the tunneling time, the same consistency requirements, i.e., the additivity over different regions and over different scattering channels, the consistency with the dwell time defined on a different basis by other authors,[17,18] and with the time of passage through a surface.[3] They also suffer from the same problems, i.e., possible superluminal velocities and nonzero reflection times in regions on the far side of a barrier. These questions have already been pointed out in the

case of the Larmor clock approach by Leavens *et al.*[16] and have been addressed in the case of the stay time[3] and there is no need to propose the same considerations again.

We want just to point out that the approaches reviewed in this paper, extend their field of validity to two- and three-dimensional problems and to multichannel scattering. Moreover, the unification of different approaches helps to make order in the great number of proposals concerning the tunneling time problem.

We have found that the time proposed by Büttiker on the basis of Larmor clock is actually equal to the rms stay time. The effect of this statement is twofold: on one hand it attributes to τ_x and τ_y a physical meaning, in the sense that they are actual characteristic times of the tunneling process; on the other hand it sheds a different light on the time proposed by Büttiker.

One of the unphysical properties of the time proposed by Büttiker is that transmission and reflection times are both greater than the dwell time. This is impossible from a classical point of view and is usually attributed, in a quantum mechanical framework, to some interference effects. From the point of view of the stay time the reason is simply that Büttiker times for transmission and reflection are rms times, so it is not strange that they are greater than the dwell time, which is a mean time.

The time Büttiker finds in the case of Larmor clock is equal to the time obtained by the means of the oscillating barrier approach.[13] Also in that case, we have shown that what is defined is the square of the characteristic interaction time,[3,19] so it is not surprising to us that the resulting time is, again, a mean square time.

Finally, τ_z is shown to be equal to the standard deviation of the stay time. It is also equal to the imaginary part of the complex time obtained by Sokolovski and Baskin.[20] Schulman and Ziolkowski have shown[21] that the major contribution to the propagator through a potential barrier comes from a path corresponding to a time spent under the barrier which is pure imaginary and equal to τ_z. Since the real and imaginary parts of the complex pole of a scattering matrix define the location and width of the resonance, respectively, these authors argue that the imaginary time is a measure of the effective spread in significant tunneling times. Also, these considerations support and strengthen our interpretation of $|\tau_z|$ as the standard deviation σ_τ.

APPENDIX

In order to derive (11), let us perform the derivative with respect to V of the Schrödinger equation (2) and evaluate it for $V = 0$; we obtain

$$\left[\hat{H}_0(\mathbf{r},t) - i\hbar\frac{\partial}{\partial t}\right]\frac{\partial\Psi_V}{\partial V}\bigg|_{V=0} = -\Theta_\Omega(\mathbf{r})u(t-t_0)\Psi_0; \tag{A1}$$

then let us differentiate (9) with respect to Γ and evaluate it for $\Gamma = 0$, i.e.,

$$\left[\hat{H}_0(\mathbf{r},t) - i\hbar\frac{\partial}{\partial t}\right]\frac{2}{i}\frac{\partial\Psi_\Gamma}{\partial\Gamma}\bigg|_{\Gamma=0} = \Theta_\Omega(\mathbf{r})u(t-t_0)\Psi_0. \tag{A2}$$

Adding (A2) and (A1) yields

$$\left[\hat{H}_0(\mathbf{r},t) - i\hbar\frac{\partial}{\partial t}\right]\left(\frac{\partial\Psi_V}{\partial V}\bigg|_{V=0} + \frac{2}{i}\frac{\partial\Psi_\Gamma}{\partial\Gamma}\bigg|_{\Gamma=0}\right) = 0. \tag{A3}$$

It follows directly that

$$\frac{\partial\Psi_V}{\partial V}\bigg|_{V=0} = -\frac{2}{i}\frac{\partial\Psi_\Gamma}{\partial\Gamma}\bigg|_{\Gamma=0} + \tilde{\Psi}, \tag{A4}$$

where $\tilde{\Psi}$ is any solution of the unperturbed Schrödinger equation, and can be uniquely determined by the initial conditions.

For $t < t_0$ the derivatives in (A4) are null, because both the pertubations are not yet introduced; therefore, $\tilde{\Psi} = 0$. As a solution of the unperturbed Schrödinger equation, $\tilde{\Psi}$ conserves the norm, implying that it remains null for $t > t_0$ also.

Now, in order to derive (26), let us point out that the second term of (25) depends on B also through $V'(B) = V + \hbar\omega_L$, because ω_L is proportional to B. Therefore, by performing on (25) the total derivative with respect to B, we obtain

$$\frac{\partial\Psi^-_{BV}}{\partial B} = \frac{\partial\Psi^+_{BV}}{\partial B} + \frac{\partial\Psi^+_{BV}}{\partial V'}\frac{\partial V'(B)}{\partial B}, \tag{A5}$$

where $\partial\Psi^+_{BV}/\partial V' = \partial\Psi^+_{BV}/\partial V$ and $\partial V'(B)/\partial\omega_L = \hbar$. If we remember that ω_L is proportional to B we can write (A5) as

$$\frac{\partial\Psi^-_{BV}}{\partial\omega_L} = \frac{\partial\Psi^+_{BV}}{\partial\omega_L} + \hbar\frac{\partial\Psi^+_{BV}}{\partial V}. \tag{A6}$$

For $B = 0$ and $V = 0$ it follows that $V'(B) = 0$ and we finally obtain (26).

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