Channel noise modelling of nanoMOSFETs in a partially ballistic transport regime

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G. Mugnaini, G. Iannaccone, *Channel noise modelling of nanoMOSFETs in a partially ballistic transport regime*, Journal of Computational Electronics, **5**, 2, pp.91-95 (2006).

J Comput Electron (2006) 5:91–95
DOI 10.1007/s10825-006-8825-2

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Abstract In this paper, we present a novel compact model for channel noise in FETs where the effects of far-from-equilibrium transport are considered in a fundamental way. The intermediate range between the drift-diffusion and the ballistic transport regimes is covered through an analytical treatment based on Büttiker virtual probes approach to inelastic scattering. The channel noise is interpreted as due to the contribution of thermal noise at the source end and of shot noise associated with local ballistic transport at the drain end. The model can be improved through the inclusion of Fermi and Coulomb correlations, that provide a significant suppression of shot noise.

Keywords Noise · MOSFETs · nanoMOSFETs · Compact models

1. Introduction

The noise in submicrometer MOSFETs is still a controversial issue in the area of mixed-signal modeling. Experiments [1, 2] have shown that in short FETs the channel noise can be larger than that predicted by the conventional model based on thermal noise valid for long channel devices. The most accepted approach adopted to reproduce such excess noise is based on the introduction of far-from-equilibrium transport through an approximate energy transport model, which therefore models noise as a Johnson-Nyquist noise with a temperature gradient given by the electric field. We believe that such approach can not be a fully realistic description, because it is not compatible with ballistic transport, indeed describing a drift-diffusion regime subject to overshoot due to electron heating, that does not hold for ultra-short channel devices.

In this work we present a different treatment for channel noise valid in the transition from drift-diffusion to ballistic transport, inspired by the Büttiker model of virtual probes [3]. On the basis of the Büttiker probes approach we build a physically based model valid in any operating regime and for any ratio of the channel length L to the mean free path λ, thus describing noise in the intermediate regime between drift-diffusion and ballistic transport in MOSFETs.

2. Noise in ballistic MOSFETs

Following [4, 5], the vertical electrostatics on the peak of the electrostatic potential in a non-degenerate ballistic MOSFET with generic architecture and subject to quantum confinement is:

$$\begin{aligned} &2C_g(V_g - \phi_m + \chi - \phi_c) \\ &= Q_b + \frac{qN_c}{2}\left[\exp\left(\frac{\phi_c - V_s}{\phi_t}\right) + \exp\left(-\frac{\phi_c - V_d}{\phi_t}\right)\right] \end{aligned} \qquad (1)$$

where C_g is the effective gate capacitance, ϕ_c is the electrostatic potential in the channel defined as in [6], V_d, V_s are the drain and source Fermi potentials, N_c is the effective state density in all the conduction sub-bands under nondegenerate statistics and Q_b is the depletion charge in the body. On

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the top of the peak, the two opposite fluxes can be written:

$$I^+ = \frac{qN_c v_{\text{th}}}{2}\exp\left(\frac{\phi_c - V_s}{\phi_t}\right),$$

$$I^- = \frac{qN_c v_{\text{th}}}{2}\exp\left(\frac{\phi_c - V_d}{\phi_t}\right),$$

where v_{th} is the averaged unidirectional thermal velocity on the peak of the channel, that can be calculated as in [4]. We will assume Poissonian injection from the drain and source in a MOSFET. From the classical model for shot noise:

$$S_I(f) = 2q(I^+ + I^-), \tag{2}$$

We find:

$$S_I(f) = q^2 N_c v_{\text{th}}\left(e^{\frac{\phi_c - V_s}{\phi_t}} + e^{\frac{\phi_c - V_d}{\phi_t}}\right) \tag{3}$$

With some algebraic manipulations, we find:

$$S_I(f) = 2qQ_m v_{\text{th}}, \tag{4}$$

where:

$$Q_m = Q_n \mathcal{W}\left[\frac{1}{2}e^{\frac{V_g - V_s - V_T}{\phi_t}} + \frac{1}{2}e^{\frac{V_g - V_d - V_T}{\phi_t}}\right] \tag{5}$$

is the density of mobile charge at the peak of the electrostatic potential, and $\mathcal{W}$ is the Lambert $\mathcal{W}$-function [4]. It is clear that this expression is the same in [7] in the case of non-degenerate conductor.

It is useful to remark that, when V_{ds} is small enough, the device operates in the linear region, then we expect that near equilibrium transport occurs, and therefore shot noise reduces to thermal noise. Indeed, from the expression for the current:

$$I_{ds} = Q_n v_{\text{th}} \mathcal{W}\left[\frac{e^{\frac{V_g - V_s - V_T}{\phi_t}} + e^{\frac{V_g - V_d - V_T}{\phi_t}}}{2}\right]\tanh\left(\frac{V_{ds}}{2\phi_t}\right). \tag{6}$$

we find that the conductance in the linear region is:

$$g_m = \frac{Q_m v_{\text{th}}}{2\phi_t}, \tag{7}$$

then we have:

$$S_I(f) = 2qQ_m v_{\text{th}} = 4g_m kT \tag{8}$$

as we expect in thermal equilibrium.

3. Noise in MOSFETs subject to intermediate transport regime

The equilibrium Johnson-Nyquist noise formulation is not valid when the MOSFET operates in far-from-equilibrium transport as in the case of ballistic and quasi-ballistic transport, where the number of inelastic scatterers is small.

Starting from the Büttiker approach to incoherent transport, we can interpret a generic MOSFET as a long enough chain of elementary ballistic MOSFETs with channel length equal to the mean free path λ. In such interpretation, when the number of internal contacts, i.e. the scatterers, is large, we expect that the ballistic chain behaves as a near thermal equilibrium MOSFET, that is a drift-diffusion MOSFET, whereas if the number of internal contacts is small, the behavior is quasi-ballistic, and in the limit $N = L/\lambda = 1$ the device is fully ballistic.

Adopting this interpretation, we can follow the considerations in [4], where now the elementary ballistic MOSFETs are considered with their noise generator.

It is interesting to observe that the model of ballistic chain resembles the description proposed in [8].

The aboveseen model can be simplified if we observe that, since the first $N - 1$ MOSFETs work near their linear region as suggested in Fig. 2, we can assimilate them to a single equivalent noisy drift-diffusion MOSFET, as represented in Fig. 1, with the associated thermal equilibrium noise source.

Therefore the fully ballistic chain can be approximated with the series of a drift-diffusion transistor of length $(N - 1)\lambda = L - \lambda$ and a ballistic transistor.

The elementary noise generator is given by (8): then, following the impedance field approach, we have that the total thermal noise along the drift-diffusion section of the channel is given by

$$S_I(f) = \frac{4kT\int_{V_s}^{V_d} g_m^2\, dV_{Fn}}{(L-\lambda)^2 I_{ds}} \tag{9}$$

and assuming nondegenerate statistics and the simplified vertical electrostatics exposed in [4], we can find an expression

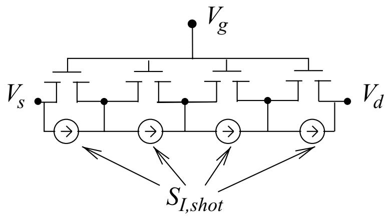


Fig. 1 Noise macromodel of a generic MOSFET in terms of a suitable chain of ballistic MOSFETs. The noise generators S_I are described by the shot noise expression (3). The internal contacts act as scatterers for the carrier fluxes of ballistic subchannels. The internal Fermi levels are given by the current continuity along the channel

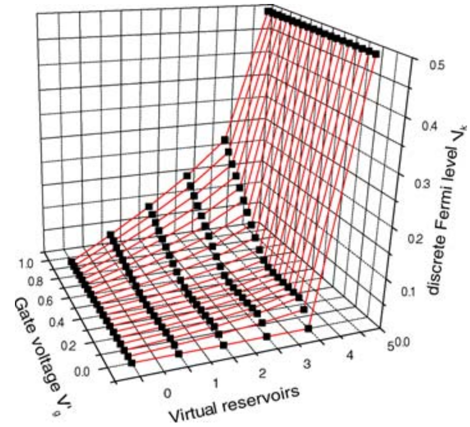


Fig. 2 Behavior of the discrete Fermi levels at the internal contacts along the ballistic chain. The voltage drops mainly on the last ballistic MOSFET of the chain

similar to that proposed in [9]:

$$S_I(f) = 4kT\frac{\mu}{(L-\lambda)} \times \frac{\frac{2}{3}\left(Q_{ms}^2 + Q_{ms}Q_{md} + Q_{md}^2\right) + Q_n\left(Q_{ms} + Q_{md}\right)}{Q_{ms} + 2Q_n + Q_{md}} \tag{10}$$

where the low-field mobility is related to the mean free path λ [4]:

$$\mu = \frac{v_{\text{th}}\lambda}{2\phi_t} \tag{11}$$

This expression is valid from weak to strong inversion. In saturation and strong inversion we have:

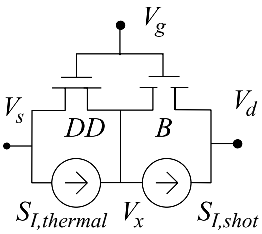
$$S_I(f) = \frac{8}{3}kTg_m \tag{12}$$

while a saturated MOSFET in weak inversion:

$$S_I(f) = 2kTg_m \tag{13}$$

Therefore the complete noise model is represented in Fig.3.

Fig. 3 Noise macromodel of a generic MOSFET. The noise generator $S_{I,\text{thermal}}$ is given by (10), $S_{I,\text{shot}}$ is given by (3)



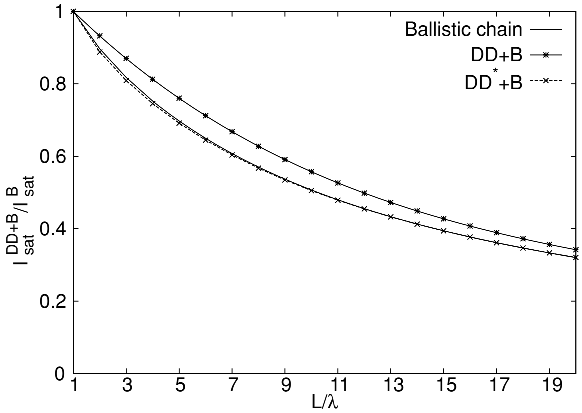


Fig. 4 Ratio $B = I_{sat}^{DD+B}/I_{sat}^{B}$ vs. L/λ, in the case of $DD + B$ model, improved $DD^* + B$ model [4] and the complete ballistic chain

It is remarkable that if we use the proposed macromodel in order to describe a conventional MOSFET, as discussed in [4], where the saturation velocity effect is present, the last ballistic MOSFET accounts for the saturation of carrier velocity, and gives a further contribution to the noise in the channel. The fact that the saturated part of the channel contributes to total noise was not recognised in [10], where the authors state that the saturated part of the channel near the drain do not give noise because it is insensitive to the drain voltage variations. The effects of far-from-equilibrium transport can be appreciated in Fig. 5, where the noise in the ballistic chain, the DD+B macromodel and the drift-diffusion (thermal) noise model are compared. As we can see, the effects of far-from-equilibrium transport are more evident the smaller the normalised L is, both in noise behavior as in Figs. 4 and 6, and in static current as represented in Fig. 4.

This noise model, as the current model in [4], can be improved if we consider the effective mobility degradation in the first $N - 1$ ballistic MOSFETs.

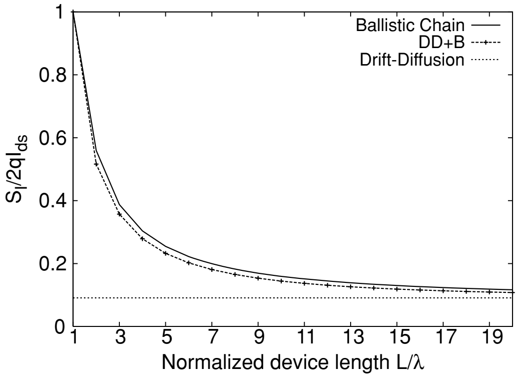


Fig. 5 Comparison of normalised power spectral densities $S_I/2IQ_{ds}$ *vs.* the ratio L/λ for the cases of ballistic chain with shot noise generators, DD+B with thermal noise in the DD section and shot noise in the ballistic transistor and DD transistor with totally thermal noise

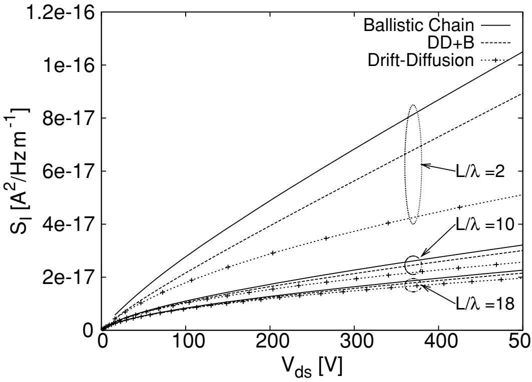


Fig. 6 Comparison of noise power spectral density as a function of V_{ds} for the cases of ballistic chain with shot noise generators, DD+B with thermal noise in the drift-diffusion section and shot noise in the last ballistic transistor and drift-diffusion transistor with totally thermal noise and for different normalised lengths

An important aspect is that the proposed noise macromodel does not suffer from the problem of evaluating the parameter $N(V_{gs}, V_{ds})$ in model [8], that links the total noise with the total current of the ballistic chain. The authors recognise that theoretical details of the calculation of such parameter are arduous, and they assume it as a suitable constant.

Therefore, the proposed model gives an alternative treatment and an insight of noise in a ballistic chain and then in an arbitrary MOSFET.

4. Effect of suppression of shot noise in the ballistic section

It is clear that if the hypothesis of Poissonian process does not hold, a suppression of shot noise in the ballistic section will emerge. Such a suppression has been predicted theoretically as a consequence of Pauli exclusion principle under strongly degenerate conditions and of long-range Coulomb interaction among carriers, that acts mainly through a space distribution of electrons more regular than a Poissonian statistics, thus reducing the current noise.

A complete analytical treatment of such two aspects in the model is difficult [7], therefore we will introduce a simplified electrostatics on the top of the barrier in the ballistic section, as in Fig. 7, and we will consider only the first subband populated.

Therefore by means of results in [11], we have that the fluctuations of the peak of the barrier in first subband δE_c is:

$$\delta E_c = \frac{2\int_0^\infty N_{2D}\,[\delta f_s + \delta f_d]\,dE}{C_c + C_{Q+} + C_{Q-}} \tag{14}$$

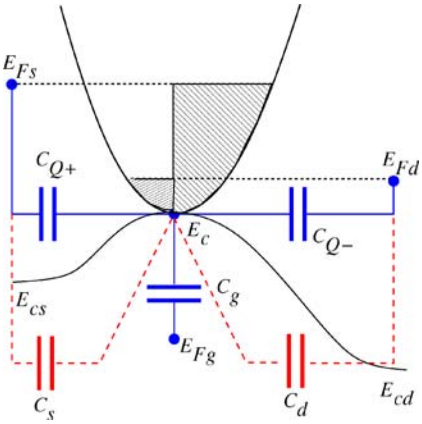


Fig. 7 Modified electrostatics for the ballistic section. The capacitances C_s and C_d represent the geometrical effects of local source and drain on the peak of the barrier

where N_{2D} is the density of states of the first subband, f_s, f_d are the occupation factors at source and drain, respectively, the quantum capacitances C_{Q+}, C_{Q-} are given by:

$$C_{Q+} = -2q^2 \frac{1}{C_{Q+}} \int_0^\infty N_{2D} \frac{\partial f_s}{\partial E_c} dE, \tag{15}$$

$$C_{Q+} = -2q^2 \frac{1}{C_{Q+}} \int_0^\infty N_{2D} \frac{\partial f_s}{\partial E_c} dE \tag{16}$$

and the term $C_c = C_g + C_s + C_d$ is the total geometrical capacitance, containing the vertical term C_g and the horizontal terms C_s, C_d The current fluctuations depends on the fluctuations of the occupation factors and of the channel barrier. We introduce the factors

$$\gamma_s(E) = 1 - \frac{v_{\text{th},s} C_{Q+} - v_{\text{th},d} C_{Q-}}{C_c + C_{Q+} + C_{Q-}} \frac{1}{v_y}, \tag{17}$$

and

$$\gamma_d(E) = 1 + \frac{v_{\text{th},s} C_{Q+} - v_{\text{th},d} C_{Q-}}{C_c + C_{Q+} + C_{Q-}} \frac{1}{v_y}, \tag{18}$$

where v_y is the longitudinal velocity and the averaged velocities $v_{th,s}$, $v_{th,d}$ are defined as:

$$v_{\text{th},s} = -2q^2 \frac{1}{C_{Q+}} \int_0^\infty v_y N_{2D} \frac{\partial f_s}{\partial E_c} dE \tag{19}$$

and

$$v_{\text{th},d} = -2q^2 \frac{1}{C_{Q-}} \int_0^\infty v_y N_{2D} \frac{\partial f_d}{\partial E_c} dE, \tag{20}$$

then we can write:

$$\delta I = 2q \int_0^\infty v_y N_{2D} \left[\gamma_s \delta f_s - \gamma_d \delta f_d\right] dE \tag{21}$$

Since $\overline{\delta f_s^2} = f_s(1 - f_s)$ and $\overline{\delta f_d^2} = f_d(1 - f_d)$ the spectral power density of the current is:

$$S_I = 4q^2 \int_0^\infty v_y N_{2D} \left[\gamma_s^2 f_s(1 - f_s) - \gamma_s^2 f_d(1 - f_d)\right] dE \tag{22}$$

The expression describes both effects of Pauli and Coulomb correlations on the shot noise. Because of the first $N - 1$ transistors work near linear region and then their noise is approximately thermal noise, we have that such expression shows its effects only on the last ballistic MOSFET of the chain. Therefore the complete noise model is given by Eq. (9) for the drift-diffusion section and the (22) for the ballistic section.

5. Conclusions

The excess noise in short channel MOSFET is introduced as a far-from-equilibrium noise, locally present along the structure. This inherent noise process is not apparent if the MOSFET channel is long enough, masked by the local equilibrium that gives the conventional thermal noise, because of the large number of inelastic scattering events.

It has been verified that if the number of inelastic scatterers (the internal contacts) is of the order of tens or smaller, the far-from-equilibrium behavior is apparent. A simplified model for noise in an arbitrary MOSFET is presented, showing that the first $N - 1$ shot noise generators can be aggregated in an equivalent thermal generator. The issue of shot noise suppression by Fermi and Coulomb correlations is considered, obtaining a description of noise in an arbitrary MOSFET given by a thermal noise generator in series with a partially suppressed shot noise generator.

In conclusion, the proposed model gives a fully physical description of the transition between the thermal noise regime and the shot noise regime with a simple analytical treatment suitable for the circuit-level simulators.

Acknowledgments Support from MIUR-PRIN Programme (contract 2004097150) and from the European Commission through the Network of Excellence SINANO (contract 506844) is gratefully acknowledged.

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