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Equivalent resistance and noise of cascaded mesoscopic cavities

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SUMMARY

Since the time of the early studies on mesoscopic devices, it was recognized that, due mainly to the ballistic nature of transport, their resistances and the other electrical quantities could not be combined following the traditional rules of circuit theory. Here, we focus specifically on the conductance and noise resulting from a chain of cascaded mesoscopic cavities and discuss an existing semiclassical model for conductance and shot noise, recasting it into an equivalent circuit description. Numerical simulations show that such a model fails if the cavities are identical and is only partially valid if cavities are unequal. We discuss possible explanations for such a somewhat surprising behaviour. Copyright © 2007 John Wiley & Sons, Ltd.

INTRODUCTION

As soon as fabrication of high-mobility mesoscopic devices became possible, as a result of the definition of quasi-1D channels on GaAs/AlGaAs heterostructures with etching and electrostatic depletion, the question was raised whether basic principles of circuit theory could be applied to a combination of such devices. It was soon realized, both experimentally [1] and theoretically [2], that this in general is not true. For example, the resistance of two constrictions in series is close to that of the narrower constriction, rather than to the sum of the resistances of the two constrictions. This result is intuitively apparent, if we realize that in such nanostructures resistance is not the consequence of a drift–diffusion process as in macroscopic conductors, but is rather the consequence of scattering off the geometric features of the device, and, specifically, of the limited number of transverse modes that can propagate through the structure. This provides a simple and intuitive picture explaining why the resistance of the narrower constriction dominates: it is the

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narrower constriction that does determine the maximum number of transverse modes that can propagate and thus, through the Landauer–Büttiker formula, the conductance.

Such a behaviour, however, undergoes significant changes when the width of the region between the constrictions is large compared to the constrictions themselves. In such a case this region starts behaving as a quasi-reservoir, i.e. in a way similar to a classical contact, with a well-defined chemical potential. The reason for the quasi-reservoir behaviour is mainly in the diffraction at the apertures defining the regions, which are much narrower than the regions themselves: this leads to a quantum chaotic behaviour [3], filling all the states at the same energy with the same occupancy. Having quasi-reservoirs between the constrictions makes each constriction behave like a classical resistor and therefore the resulting resistance is the sum of those of the single constrictions.

The picture, however, is not always so simple: indeed, our numerical simulations have shown that a remarkable exception exists, i.e. the case of identical cavities. If we cascade multiple identical cavities, the total resistance will be much less than the sum of the resistances of the contacts defining the cavities. In order to observe this effect, the cavities need to be exactly identical: even slight differences in the length or in the positions of the constrictions seem to destroy it.

A related anomaly is observed in the shot-noise behaviour of cascaded cavities: while a single cavity with symmetric apertures is supposed to yield a suppression of shot noise down to $\frac{1}{4}$ [4, 5] of the full shot noise expected from Schottky's theorem [6], cascaded cavities were predicted [7] to yield a larger Fano factor (which is the ratio of the shot noise power spectral density to the full shot power spectral density $2eI$ from Schottky's theorem, where e is the electron charge and I the average current). In particular, the semiclassical model of Reference [7] predicts a Fano factor of $\frac{1}{3}$ when the number of cascaded cavities tends to infinity, as in the case of a one-dimensional conductor with an infinite number of barriers [8].

We shall present such a semiclassical model in terms of a circuit equivalent and then we shall discuss the discrepancies that we have observed in our numerical results, in particular the case of identical cavities *versus* that of cavities differing in length or for some other feature.

Understanding the extent to which the behaviour of circuits based on nanoscale devices can be represented in terms of traditional circuit equivalents provides a measure of the applicability of SPICE-like models to the analysis of their behaviour. It is well known, for example, that for single-electron transistors a SPICE model is meaningful only if the capacitances at the nodes of the device are much larger than those inside the device, otherwise a description more complex than the $I-V$ relationships at the terminals is required [9].

MODEL

We are interested in the analysis of resistance and noise in mesoscopic cavities defined in the high-mobility 2-dimensional electron gas (2DEG) obtained by modulation doping in a semiconductor heterostructure. They can be fabricated with several different techniques, such as etching [10] or electrostatic depletion operated by negatively biased metal gates at the semiconductor surface [7].

In particular, using a series of negatively biased split gates, it is possible to define a series of mesoscopic cavities, i.e. a series of wide regions of the 2DEG connected with the outside only through narrow input and output constrictions.

Previous calculations [3, 11] have shown that the shape of the cavity and the detailed behaviour of the potential are not essential in determining resistance additivity and the mentioned shot noise suppression. The really relevant parameters are the widths of the cavities and of the constrictions.
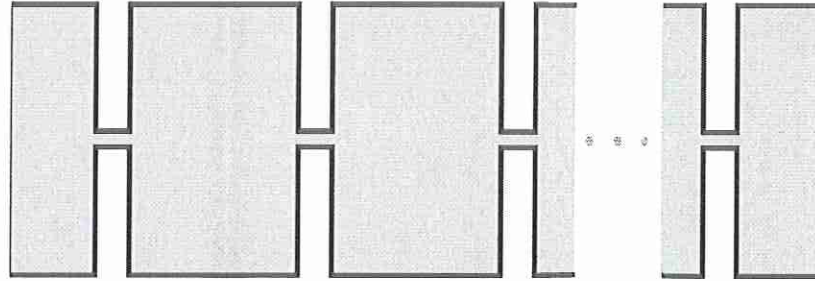
Figure 1. Series of rectangular cavities.

For this reason, here we consider model cavities defined by hard walls and with a rectangular shape (Figure 1), resulting in a significant reduction of the computational complexity. The conductance and the shot noise power spectral density are computed from the transmission matrix of the structure, which is in turn determined with the recursive Green's function method.

The structure is subdivided into transverse sections (the first and the last of which are semi-infinite to include open boundary conditions), chosen in such a way that within each of them there is no significant variation of the transverse confinement potential along the longitudinal direction (since we are considering rectangular cavities, there is indeed no variation at all within a section). Using a representation in the transverse eigenmode space for the transverse direction and in real space for the longitudinal direction, it is possible to analytically obtain the diagonal Green's function matrix of each transverse section, which is initially considered as isolated from its neighbours and 'closed' with Dirichlet boundary conditions at its ends. Then we can compose Green's function matrices of neighbouring sections by introducing a proper perturbation potential $\hat{V}$ which connects the two sections; in particular the Green's function matrix $\hat{G}$ of the perturbed structure (the one in which the sections are coupled) can be obtained from the Green's function matrix $\hat{G}^0$ of the unperturbed structure (the one with the two sections decoupled) using the Dyson equation

$$\hat{G} = \hat{G}^0 + \hat{G}^0 \hat{V} \hat{G} \tag{1}$$

From this implicit (with respect to $\hat{G}$) equation, it is possible to derive explicit relationships useful for the recursive composition of the Green's function matrices of adjacent sections [12]. Starting from one end of the structure and recursively composing Green's function matrices of adjacent sections, we finally obtain the Green's function matrix (and from this the transmission matrix t) of the overall structure.

From the transmission matrix t, the conductance can be computed using the Landauer–Büttiker formula

$$G = \frac{2e^2}{h} \sum_{n,m} |t_{nm}|^2 = \frac{2e^2}{h} \sum_n w_n \tag{2}$$

while the shot noise power spectral density can be evaluated following Büttiker [13] as

$$S_I = 4\frac{e^3}{h}|V|\sum_n w_n(1 - w_n) \tag{3}$$

(the w_n's are the eigenvalues of the matrix $tt^\dagger$, V is the externally applied voltage, and h is Planck's constant). Dividing the shot noise power spectral density by the full shot noise power spectral density $S_{I_{fs}} = 2eI = 2eGV$, we have that the Fano factor γ is given by

$$\gamma = \frac{\sum_n w_n(1 - w_n)}{\sum_n w_n} \tag{4}$$

As a result of the large number of modes propagating in the wide regions (of the order of a few hundreds), fast fluctuations originating from quantum interference are observed as a function of energy. They become particularly significant when constrictions that allow only a few modes to propagate are considered, since their amplitude, of the order of a conductance quantum, becomes comparable to the total conductance. For this reason, we average over a range of energies around the Fermi level, about 9 meV in our simulations, considering 100 samples. In particular, in the case of the Fano factor, the average has been performed separately for the numerator and the denominator of the corresponding expression, as in the case of a real measurement, in which the noise power spectral density is measured (and averaged) separately from the current. In order to make shot noise observable over thermal noise, the measurement has to be performed with an applied bias V much greater than kT/e (where k is the Boltzmann constant and T is the temperature); therefore, the energy interval over which the transition from 0 to 1 of the Fermi function takes place is much less than the difference between the chemical potentials of the leads (eV). Thus the average can be performed as a constant average, over the whole energy range.

RESULTS

As already mentioned in the Introduction, if the width of the constrictions defining the cavities is of the same order of magnitude as that of the cavities themselves, the resistances of cascaded constrictions do not add.

For example, we have performed a numerical simulation of an 80 nm wide and 50 nm long rectangular cavity, delimited by 40 nm wide constrictions, each one characterized by a normalized resistance (with respect to the resistance quantum $h/(2e^2)$) equal to 1. We find for the complete cavity a normalized resistance of 1.115, which is almost the same as the resistance of a single constriction. Analogously, the series of two such cavities (containing three constrictions) has a total resistance of 1.27 if the constrictions are aligned and of 1.245 if the constrictions are vertically shifted (in the specific case the second constriction is 10nm under the first and the third constriction is 10 nm above the first).

For constrictions that are much narrower than the cavities, such that the cavities act (as previously discussed) as quasi-reservoirs, we can apply a semiclassical model developed by Oberholzer et al. [7]. Here we recast this model into the form of an equivalent circuit, which can be treated more intuitively.

Let us consider a series of N cavities (delimited by $N + 1$ constrictions) with a voltage V applied to the outer leads. If we assume that all propagating modes in each cavity (for example, the ith) at a given energy E are occupied with the same probability f_{i+1} (for the reasons presented in the Introduction), the cavity can be considered a reservoir, except for two issues: f_{i+1} at a given energy depends on the balance of the fluxes entering and exiting the cavity at that energy, and, when a voltage is applied to the external leads, the cavity is not in thermal equilibrium. This
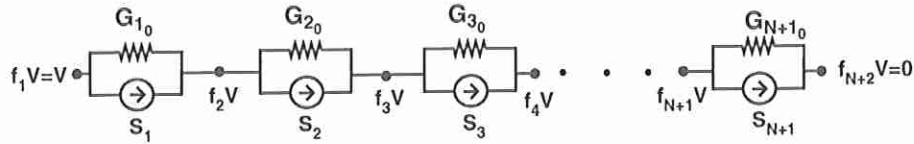
Figure 2. Equivalent circuit of a series of cavities.

latter aspect is apparent from the fact that, even at zero temperature, f_{i+1} can assume any value between 0 and 1, depending on transport conditions.

Under the above assumption, each cavity can be viewed just as two constrictions in series, connected through the very special node represented by the cavity itself. For simplicity, let us assume zero temperature operation; the occupation factor in the left lead of the series is equal to 1, while the occupation factor in the right lead of the series is equal to 0. We can devise an equivalent circuit (Figure 2) representing each constriction with a resistor and a current noise source: the series of cavities is therefore modelled with a series of noisy resistors, whose resistances add up. If f_i and f_{i+1} are the occupation factors of the regions, respectively, to the left and to the right of the ith constriction, the current through the generic ith constriction is given by

$$I_i = V G_i = V \frac{2e^2}{h} \sum_n w_{in}(f_i - f_{i+1}) = G_{i_0}(f_i - f_{i+1})V \tag{5}$$

(the w_{in}'s are the eigenvalues of $t_i t_i^\dagger$, where t_i is the transmission matrix of the ith constriction, while G_{i_0} is its zero temperature conductance) while, following Büttiker [14], the current noise source of the ith constriction has a value given by

$$S_{I_i} = \frac{4e^3 V}{h} \sum_n [w_{in} f_i(1 - f_i) + w_{in} f_{i+1}(1 - f_{i+1}) + (1 - w_{in})w_{in}(f_i - f_{i+1})^2] \tag{6}$$

Assuming that the zero temperature conductances G_{i_0} of all the constrictions are identical and equal to G_0 and enforcing the identity of all the currents I_i through the constrictions, we have that

$$f_i = \frac{N + 2 - i}{N + 1} \tag{7}$$

Using the equivalent electric circuit, we can find the power spectral density of the noise current for the entire structure, which is given by

$$S_I = \frac{1}{(N + 1)^2} \sum_{i=1}^{N+1} S_{I_i} \tag{8}$$

If the transmission eigenvalues w_{in} for the constrictions are either 0 or 1, we find that the Fano factor is

$$\gamma = \frac{S_I}{2eI} = \frac{1}{3}\left(1 - \frac{1}{(N + 1)^2}\right) \tag{9}$$

which tends to $\frac{1}{3}$ as the number of cavities is increased (analogously to what happens in the case of diffusive conductors).
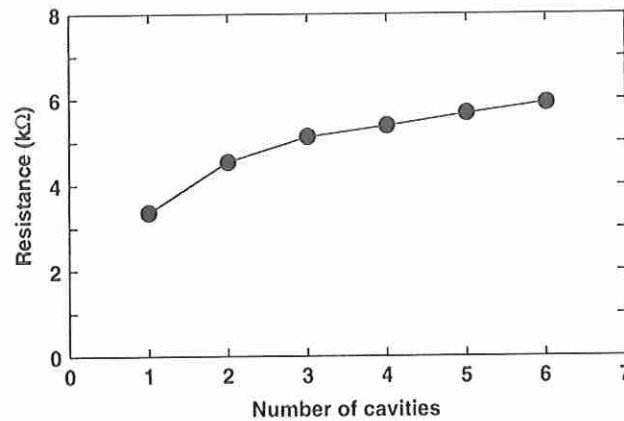
Figure 3. Resistance of a series of exactly identical cavities as a function of
the number of cascaded cavities.

In this model each cavity acts as a quasi-reservoir, with all the states at the same energy having the same occupancy. This is possible only if there is strong mode mixing that allows electrons entering the cavity to explore all the possible transverse modes, as a result of reflections off the entrance and exit walls. This is simply due to the presence of rapid variations in the transverse potential in correspondence of the constrictions. Therefore, a classically regular shape, such as a rectangle, produces the very same effect, from the point of view of interest to us, as a classically chaotic (i.e. nonintegrable) shape, such as a stadium.

Oberholzer *et al.* [7] have performed some experiments on a series of real cavities realized in a heterostructure GaAs/AlGaAs, which seem to confirm the results of the analytical model, although limited to the observation of only two cascaded cavities.

Numerical simulations, however, provide a somewhat different picture, as will be described in the following. We have considered cavities 5 μm long and 8 μm wide, with constrictions that are quite narrow with respect to the cavity width: only 200 nm. Material parameters are those for gallium arsenide with an effective mass $m^* = 0.067 m_0$ (where m_0 is the free electron mass). Calculations have been performed around a Fermi level of 9.02 meV within a range of ±0.02 meV. For such a Fermi energy, there are a total of about 300 modes propagating in the wider regions, and an appropriate number of evanescent modes must be included: we have increased the number of transverse modes considered in the calculation until no significant variation was observed in the results; between 400 and 600 transverse modes were sufficient for all calculations.

We have started with a series of exactly identical cavities, for which we have noticed that the resulting overall resistance is not at all the sum of the resistances of the cascaded constrictions (at the considered Fermi energy the resistance of each constriction is about $\frac{1}{8}$ of a resistance quantum $h/(2e^2)$, i.e. 1613 Ω): it is just slightly larger than the resistance of a single cavity (which in turns corresponds to the sum of the resistances of the constrictions defining it), as can be seen in Figure 3, where we have reported the value of the total resistance as a function of the number of cascaded cavities. If for identical cavities the additivity of the constriction resistances fails, also the previously described circuit model for noise has to fail. Indeed, the computed Fano factor, instead of increasing towards $\frac{1}{3}$ as the number of cascaded cavities is increased, does not vary
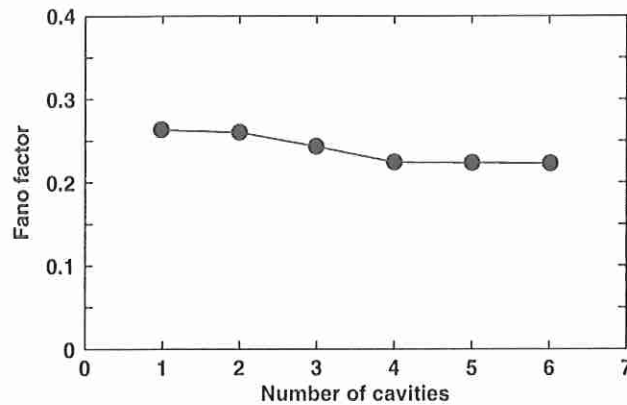
Figure 4. Fano factor of a series of exactly identical cavities as a function of
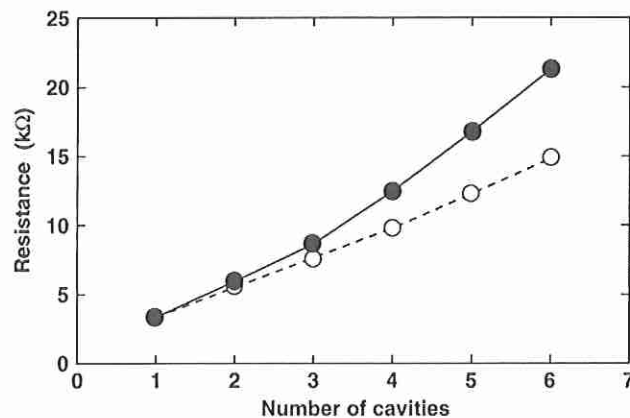the number of cascaded cavities.

Figure 5. Resistance, as a function of the number of cascaded cavities, of a series of cavities differing for
the cavity length (solid dots) or in the vertical position of the delimiting constrictions (empty dots).

significantly and even exhibits a slight decrease. This can be clearly noticed in Figure 4, where
we report our results for the Fano factor as a function of the number of cavities.

These surprising results have induced us to analyse the behaviour of cascaded cavities if very
small differences are introduced between them. In particular, we have considered cavities differing
in their length, the vertical positions of the constrictions, the value of the bottom potential inside
them. In all such cases, we have noticed that the introduction of differences between the cascaded
structures leads to a partial recovery of the results from the semiclassical model.

In Figures 5 and 6 we show the results obtained in terms of the normalized resistance and of the
Fano factor for a series of cavities differing for the cavity length and for the vertical position of
the delimiting constrictions (all 200 nm wide), as a function of the number of considered cavities.
In Figure 5 we report the results for the resistance of cavities that have a length of 5.05, 4.95, 5.1,
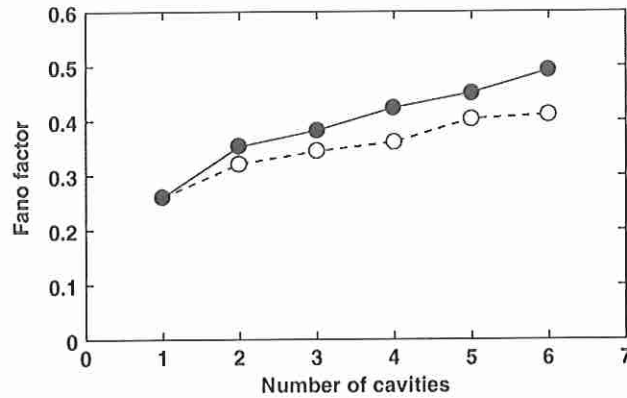
Figure 6. Fano factor, as a function of the number of cascaded cavities, of a series of cavities differing in the cavity length (solid dots) or in the vertical position of the delimiting constrictions (empty dots).
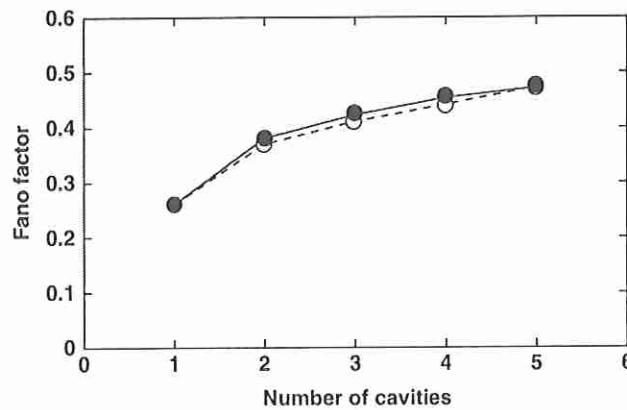

Figure 7. Fano factor as a function of the number of cavities for a series of cavities differing in the value of the potential at the bottom of each cavity (solid dots); the correspondent results for an analogous series of cavities in which the shifts in the bottom potential are an order of magnitude smaller (empty dots).

4.9, 5.15, 4.85 µm (solid dots) or the exit constriction vertically shifted, with respect to the centre, by 1200, −1200, 1800, −1800, 2400, −2400 nm (empty dots). It is apparent that in this case the total resistance actually increases linearly or more than linearly with the number of the cavities and is even larger than the sum of the resistances of each constriction. The behaviour of the Fano factor, reported for the same structures in Figure 6, is however still somewhat different from that predicted from the circuit model, reaching values clearly larger than $\frac{1}{3}$.

It is interesting to observe that these results are obtained even for very small differences between the cavities, such as the variation in the length of one of the cavities by a few per cent, and that they seem to have very little dependence on the magnitude of the differences. As an example, we report in Figure 7 the Fano factor as a function of the number of cavities for two series of variations in the values of the potential at the bottom of the cavity, differing by an order of magnitude. The

results represented with solid dots are for the second cavity with a bottom potential, with respect to that of the first cavity, 0.4 meV higher, the third 0.4 meV lower, the fourth 0.8 meV higher, the fifth 0.8 meV lower, the sixth 0.6 meV higher, while those indicated with empty dots are for a series of shifts for the bottom potential of 0.04, −0.04, 0.08, −0.08, and 0.06 meV.

Analogous results have been obtained repeating for stadium-shaped cavities some of the simulations performed on rectangular cavities, in such a way to confirm the validity of our considerations independently of the shape of the structure.

The fact that very small variations in the potential details can lead to a crossover between two quite different regimes could hint at the presence of a resonance effect in the case of exactly identical cavities. Starting from this hypothesis, we have performed simulations considering two cascaded cavities of different length, varying the bottom potential of one of the cavities in such a way as to restore the resonance effect hypothesized for identical cavities, as it should be possible, at least if the resonance involved only one energy level. We have, however, been unable to find a condition restoring the behaviour observed for identical cavities.

A possible alternative explanation is instead based on the consideration that the wave functions in each cavity can be decomposed into a transverse component (transverse modes) and a longitudinal component (longitudinal modes): as long as the constrictions are narrow, the longitudinal modes in each cavity can be seen as quasi-bound states with nodes in correspondence with the ends of the cavity. Therefore, if the cavities are exactly identical, the propagating longitudinal modes (propagating because there actually are openings) of each cavity have nodes also in correspondence with all the other constrictions. Thus, the presence of the other constrictions does not have a large effect on conductance and noise (a similar effect could be also the explanation for the strong dependence of the conductance of a cavity containing a transverse potential barrier on the longitudinal position of the barrier, which has been observed recently [15]). The concurrent presence of a beaming effect, i.e. of a contribution to transmission from direct trajectories between the constrictions, could instead be the reason of the dependence of such a phenomenon on the alignment of all the constrictions.

The fact that increasing the number of unequal cascaded cavities we do not obtain for the Fano factor the value $\frac{1}{3}$ predicted from the equivalent circuit for an infinite number of cavities, but, rather, a larger value, could instead be explained by the fact that the results of the described theoretical model, which has a semiclassical basis, are not totally applicable to the considered situations, in which the number of propagating modes inside the constrictions is very small. Such a phenomenon could be analogous to what happens in diffusive mesoscopic conductors, where the Fano factor becomes greater than the $\frac{1}{3}$ value in the case of a limited number of propagating modes [16].

CONCLUSION

We have analysed the conductance and noise behaviour of a nanoelectronic circuit consisting of cascaded mesoscopic cavities. While in general resistance additivity of cascaded structures is violated in mesoscopic circuits, due to the nonlocal nature of the resistance of ballistic conductors, a form of partial resistance additivity is recovered in the case of cascaded constrictions separated by wide regions, behaving as quasi-reservoirs. This justifies an equivalent circuit model that can be used to predict also the shot noise power spectral density of a series of cavities. Such an additivity, however, breaks down if the cascaded structures are identical, as demonstrated by our

numerical simulations. In the case of identical cavities the Fano factor is about the same as for a single cavity, while for unequal cavities, for which resistance additivity approximately holds, it does increase above the value $\frac{1}{3}$ predicted by the circuit model. We have proposed possible explanations for the observed discrepancies: the disappearance of resistance additivity for identical cavities may be the result of the nodes for the longitudinal modes located in correspondence to the intermediate constrictions, while the presence of a beaming effect between constrictions may also play a role. The increase of the Fano factor for nonequal cavities above the $\frac{1}{3}$ value predicted by the circuit model can be explained with the fact that the small number of modes propagating through the constrictions may lead too far from the classical limit. Further work is, however, needed to completely clarify the origin of these effects.

From a practical point of view, the case of nonequal cavities is more interesting, because, considering fabrication tolerances and the presence of random distributions of impurities, it is impossible to obtain two identical cavities. In general, a better understanding of these effects is important to gain an understanding of how far circuit models can be pushed for the analysis of interconnected nanoscale devices.

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