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We show that shot noise in a resonant-tunneling diode biased in the negative differential resistance regions of the I-V characteristic is enhanced with respect to "full" shot noise. We provide experimental results showing a Fano factor of up to 6.6, and show that it is a dramatic effect caused by electron-electron interaction through the Coulomb force, enhanced by the particular shape of the density of states in the well. We also present numerical results from the proposed theory, which are in agreement with the experiment, demonstrating that the model accounts for physics relevant to the phenomenon. [S0031-9007(97)05143-0]

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Deviations from the purely poissonian shot noise (the so-called "full" shot noise) in mesoscopic devices and resonant tunneling structures have been the subject of growing interest in the last decade [1–11]. The main reason is that noise is a very sensitive probe of electron-electron interaction [12], because of both the Pauli principle and the Coulomb force, and provides information but obtainable from dc and ac characterization; furthermore, noise depends strongly on the details of device structure, so that the capability of modeling it in nanoscale devices implies and requires a deep understanding of the collective transport mechanisms of electrons.

Almost all published theoretical and experimental studies have focused on the suppression of shot noise due to negative correlation between current pulses caused by single electrons traversing the device. Such correlation may be introduced by Pauli exclusion, which limits the density of electrons in phase space, and/or by Coulomb repulsion, depending on the details of the structure and on the dominant transport mechanism [6–8], and make the pulse distribution subpoissonian, leading to suppressed shot noise.

In particular, for the case of resonant tunneling structures, several theoretical and experimental studies have appeared in the literature [2–11], assessing that the power spectral density of the noise current S in such devices may be suppressed down to half the "full" shot noise value $S_{\mathrm{full}} = 2qI$, i.e., that associated with a purely poissonian process.

In this Letter, we propose a theoretical model and show experimental evidence of the opposite behavior, that is, of enhanced shot noise with respect to S_{full}, which is to be expected in resonant tunneling structures biased in the negative differential resistance region of the I-V characteristic. An attempt to model such phenomenon has been presented in Ref. [13].

We shall show that in such a condition Coulomb interaction and the shape of the density of states in the well introduce positive correlation between consecutive current pulses, leading to a superpoissonian pulse distribution, which implies a superpoissonian shot noise.

First, we shall show an intuitive physical picture of the phenomenon, then we shall express it in terms of a model for transport and noise in generic resonant tunneling structures presented elsewhere [8,14]. Furthermore, we shall show the experimental results, exhibiting a noise power spectral density almost 6.6 times greater than S_{full}, and compare it with the results provided by a numerical implementation of our model.

As is well known, the typical I-V characteristic of a resonant tunneling diode is due to the shape of the density of states in the well, which consists of a series of narrow peaks in correspondence with the longitudinal allowed energies in the well: for the $GaAs/Al_{0.36}Ga_{0.64}As$ material system considered here, there is a main narrow peak. In the negative differential resistance region of the I-V characteristic, the peak of the density of states is below the conduction band edge of the cathode: With increasing voltage, the density of states is moved downward, so that fewer states are available for tunneling from the cathode, and the current decreases.

The microscopic mechanism which allows for enhanced shot noise is the following (see Fig. 1): An electron tunneling into the well from the cathode raises the potential energy of the well by an amount $q/(C_1 + C_2)$, where q is the electron charge, C_1 and C_2 the capacitances between the well region and either contacts; as a consequence, the density of states in the well is shifted upwards by the same amount, with the result that more states are available for successive tunneling events from the cathode, and the probability per unit time that electrons enter the well increases. That means that electrons entering the well are positively correlated, so that enhanced shot noise is to be expected. The effect of a single electron entering the well is finite only if the diode transverse area is finite, implying finite C_1 and C_2. Actually, though less intuitive, the argument is perfectly valid if restated in terms of capacitances and number of electrons per unit area.

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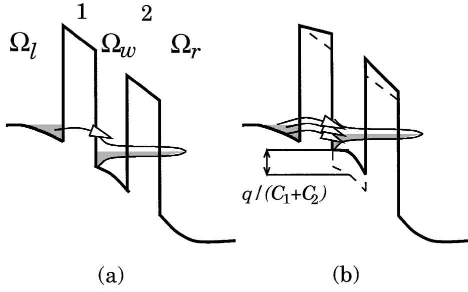

FIG. 1. Enhanced shot noise is obtained because an electron tunneling into the well (a) from the cathode raises the potential energy of the well by an amount $q/(C_1 + C_2)$ so that more states are available for tunneling from the cathode (b).

For a more analytical derivation we can consider the structure as consisting of three regions, Ω_l, Ω_w, and Ω_r, i.e., the left reservoir, the well region, and the right reservoir, respectively, that are only weakly coupled through the two tunneling barriers 1 and 2, as sketched in Fig. 1(a). In addition, we suppose that electron transport is well described in terms of sequential tunneling (which is reasonable, except for the case of temperatures in the millikelvin range): An electron in Ω_l traverses barrier 1, loses phase coherence, and relaxes to a quasiequilibrium energy distribution in the well region Ω_w, then traverses barrier 2 and leaves through Ω_r.

Since confinement is realized only in one direction (that of MBE growth), a state in Ω_s $(s = l, r, w)$ is characterized by its longitudinal energy E, its transverse wave vector $\mathbf{k}_\mathrm{T}$, and its spin σ, and tunneling can be treated as a transition between levels in different regions [15] in which E, $\mathbf{k}_\mathrm{T}$, and σ are conserved.

Following Davies *et al.* [6], we introduce "generation" and "recombination" rates through both barriers [8]: the generation rate g_1 is the transition rate from Ω_l to Ω_w, i.e., the sum of the probabilities per unit time of having a transition from Ω_l to Ω_w given by the Fermi "golden rule" over all pairs of occupied states in Ω_l and empty states in Ω_w. Analogously, we define r_1, the recombination rate through barrier 1 (from Ω_w to Ω_l), g_2, and r_2, generation and recombination rates through barrier 2.

Since negative differential resistance is obtained at high bias, when the electron flux is one directional, r_1 and g_2 can be discarded, while g_1 and r_2 are

$$g_1 = 2\,\frac{2\pi}{\hbar}\int dE\,|M_{1lw}(E)|^2\rho_l(E)\rho_w(E) \times \int d\mathbf{k}_\mathrm{T}\rho_T(\mathbf{k}_\mathrm{T})f_l(E,\mathbf{k}_\mathrm{T})[1 - f_w(E,\mathbf{k}_\mathrm{T})], \quad (1)$$

$$r_2 = 2\,\frac{2\pi}{\hbar}\int dE\,|M_{2rw}(E)|^2\rho_r(E)\rho_w(E) \times \int d\mathbf{k}_\mathrm{T}\rho_T(\mathbf{k}_\mathrm{T})f_w(E,\mathbf{k}_\mathrm{T})[1 - f_r(E,\mathbf{k}_\mathrm{T})], \quad (2)$$

where ρ_s, f_s $(s = l, w, r)$, are the longitudinal density of states [16] and the equilibrium occupation factor in Ω_s (dependent on the quasi-Fermi level E_{fs}), respectively, and ρ_T is the density of transverse states; $M_{1lw}(E)$ is the matrix element for a transition through barrier 1 between states of longitudinal energy E: it is obtained in Ref. [14] as $|M_{1lw}(E)|^2 = \hbar^2\nu_l(E)\nu_w(E)T_1(E)$, where ν_s $(s = l, w, r)$ is the so-called attempt frequency in Ω_s and T_1 is the tunneling probability of barrier 1; $M_{2rw}(E)$ is analogously defined.

As is well known, in the negative resistance region of the I-V characteristic, the peak of ρ_w is below the conduction band edge of the left electrode. In such a way, as the voltage is increased, the number of allowed states for a transition from Ω_l to Ω_w is reduced, hence the current decreases. Since all electrons relax to lower energy states once they are in the well, it is reasonable to assume that Ω_w states with longitudinal energies above the conduction band edge of the left electrode E_cbl are empty, i.e., correspond to a zero occupation factor f_w (analogously, $f_r = 0$). In addition, if we discard size effect in the cathode, we have that $2\pi\hbar\rho_l\nu_l = 1$ if $E > E_\mathrm{cbl}$. Therefore we can rewrite g_1 and r_2 as

$$g_1 = 2\int_{E_\mathrm{cbl}}^{\infty} dE\,\nu_w(E)\rho_w(E)T_1(E)F_l(E), \quad (3)$$

$$r_2 = 2\int_{E_\mathrm{cbw}}^{\infty} dE\,\nu_w(E)\rho_w(E)T_2(E)F_w(E), \quad (4)$$

where $F_s(E)$ is the occupation factor of Ω_s $(s = l, w, r)$, integrated over the transverse wave vectors $F_s(E) \equiv \int d\mathbf{k}_\mathrm{T}\rho_T(\mathbf{k}_\mathrm{T})f_s(E,\mathbf{k}_\mathrm{T})$, and E_cbw is the bottom of the conduction band edge in the well.

Let us point out that g_1 and r_2 depend on the number of electrons N in the well region both through the potential energy profile, which is affected by the charge in Ω_w through the Poisson equation, and through the term F_w in (4) which depends on N through the quasi-Fermi level E_{fw}. It is worth noticing that in our case Pauli exclusion has no effect, since practically all possible final states are unoccupied. Following these considerations, g_1 and r_2 can be obtained as a function of N, at a given bias voltage V.

The steady state value $\tilde{N}$ of N satisfies charge conservation in the well, i.e., $g_1(\tilde{N}) = r_2(\tilde{N})$, and the steady state current is $I = qg_1(\tilde{N}) = qr_2(\tilde{N})$.

Following Ref. [8], it is worth expanding $g_1(N)$ and $r_2(N)$ around $\tilde{N}$ and defining the following characteristic times:

$$\frac{1}{\tau_g} \equiv -\left.\frac{dg_1}{dN}\right|_{N=\tilde{N}}, \qquad \frac{1}{\tau_r} \equiv \left.\frac{dr_2}{dN}\right|_{N=\tilde{N}}. \quad (5)$$

Our parameter of choice for studying deviations from full shot noise is the so-called Fano factor γ, the ratio of the power spectral density of the current noise $S(\omega)$ to the

full shot value $2qI$. From [8] we have, in this case, for $\omega\tau_g\tau_r \ll \tau_g + \tau_r$,

$$\gamma = \frac{S(\omega)}{2qI} = 1 - \frac{2\tau_g\tau_r}{(\tau_g + \tau_r)^2}. \quad (6)$$

From the definition (5), τ_g is positive in the first region of the I-V characteristic, when Pauli principle and Coulomb interaction make g_1 decreasing with increasing N. On the other hand, in the negative differential resistance region, the term which varies the most with increasing N is the longitudinal density of states, which shifts upwards by a factor of $q/(C1 + C2)$ per electron: since the peak is just below E_{cb1}, a slight shift of the peak sensibly increases the integrand in (3), yielding a negative τ_g. Note that, while from (6) we see that noise could also diverge if $\tau_g = -\tau_r$, this cannot physically happen, because the large deviation of N with respect to $\tilde{N}$ would make the linearization of g_1 and r_2 not acceptable.

We now focus on a particular structure, on which we have performed noise measurements and numerical simulations following the theory just described. Such structure has been fabricated at the TASC-INFM laboratory in Trieste and has the following layer structure: a Si-doped ($N_d = 1.4 \times 10^{18}$ cm^{-3}) 500-nm-thick GaAs buffer layer, an undoped 20-nm-thick GaAs spacer layer to prevent silicon diffusion into the barrier, an undoped 12.4-nm-thick AlGaAs first barrier, an undoped 6.2-nm-thick GaAs quantum well, an undoped 14.1-nm-thick AlGaAs barrier, a 10-nm GaAs spacer layer, and a Si-doped 500-nm-thick cap layer. The aluminum mole fraction in both barriers is 0.36 and the diameter of the mesa defining the single device is about 50 μm.

The barriers in our samples are thicker than in most similar resonant-tunneling diodes, for the purpose of reducing the current and, consequently, to increase the differential resistance, in order to obtain the best possible noise match with the measurement amplifiers (available ultra-low-noise amplifiers offer a good performance, with a very small noise figure, for a range of resistance values between a few kilohms and several megohms).

We have applied a measurement technique purposely developed for low-level current noise measurements, based on the careful evaluation of the transimpedance between the device under test and the output of the amplifier [17].

Our usual approach [17] also includes the subtraction of the noise due to the amplifier and other spurious sources, which is evaluated using a substitution impedance, equivalent to that of the device under test with known noise behavior. For the measurements in the negative differential resistance region, instead, we have evaluated an upper limit for the noise contribution from the amplifier in these particular operating conditions, since it is difficult to synthesize an appropriate substitution impedance. From experimental and theoretical considerations, we have verified that such a limit is always below 3% of the noise level from the device under test, so that corrections are not necessary. In Fig. 2 the measured current and the Fano factor γ at the temperature of liquid nitrogen (77 K) are plotted as a function of the applied voltage (the thicker barrier is on the anode side).

It can be noticed that as the voltage increases, the Fano factor decreases down to about 0.5 (which corresponds to the maximum theoretical suppression [8]), at the voltage corresponding to the current peak is exactly one, then increases again and reaches a peak of 6.6 at the voltage corresponding to the lowest modulus of the negative differential resistance, while, for higher voltages, it rapidly approaches one.

In Fig. 3 we show numerical results for the same structure at 77 K based on the theory discussed before and obtained by considering a relaxation length l of 15 nm [14,16]. The value of l is chosen in order to fit the peak-to-valley ratio of the diode current, and is the only fitting parameter used. As can be seen, there is an almost quantitative agreement between theory and experiment (the peak experimental current is 45 nA which corresponds to a current density of 23 A/m^2): We ascribe most of the difference to the tolerance in the nominal device parameters and to the simplistic inclusion of all phase-destroying mechanisms in a single, energy independent, relaxation length. All the relevant features of the Fano factor as a function of the applied voltage are reproduced, and can be easily explained in terms of our model.

The fact that γ is maximum at the voltage corresponding to the minimum negative differential resistance r_d of the device is readily justified once we recognize that r_d is practically proportional to $\tau_g + \tau_r$ [18]. In fact, from Fig. 4 it is clear that τ_r varies much more smoothly than τ_g with the applied voltage, so that, since τ_g and τ_r have opposite signs, the modulus of r_d is minimum when τ_g/τ_r approaches -1. At this point, we simply notice, from

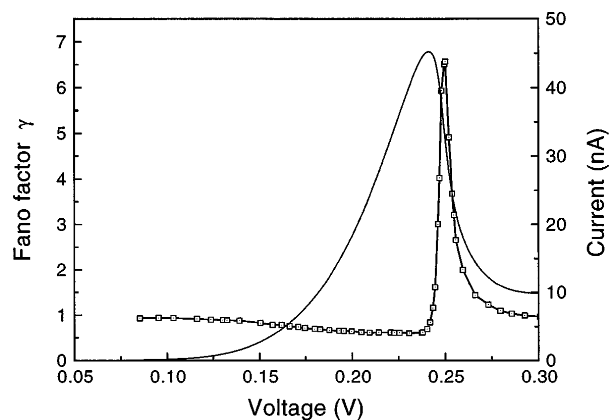

FIG. 2. Experimental current (solid line) and Fano factor γ (squares) as a function of the applied voltage, at the temperature of 77 K. The maximum value of γ is 6.6, while the minimum is close to 0.5.

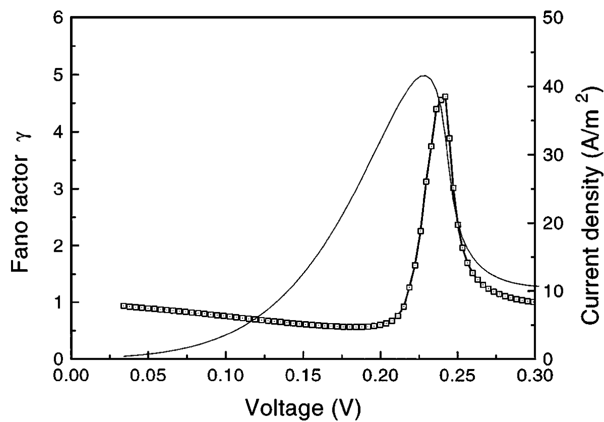

FIG. 3. Calculated current density (solid line) and Fano factor γ (squares) at 77 K as a function of the applied voltage for the considered structure.

(6), that γ gets larger as α_g/τ_r approaches -1, too (and would eventually diverge for $\tau_g = -\tau_r$).

Furthermore, from Figs. 3 and 4, and according to [18], we can notice that r_d and τ_g tend to infinity at about the same voltage, i.e., the one corresponding to the current peak. Therefore, according to (6), γ is 1 at the current peak bias, as can be verified from both experiments and calculations (Figs. 2 and 3, respectively).

In conclusion, we have demonstrated experimentally that Coulomb interaction, enhanced by the shape of the density of states in the well, can lead to a dramatic increase of shot noise in resonant tunneling diodes biased in the negative differential resistance region of the I-V characteristic. We have provided a model which leads to good numerical agreement with the experimental data, taking into account all the relevant physics involved in the phenomenon.

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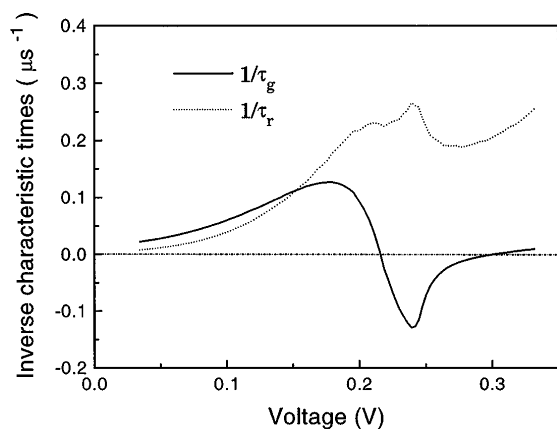

FIG. 4. Calculated $1/\tau_g$ (solid) and $1/\tau_r$ (dashed) as a function of the applied voltage at 77 K.

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[18] We just provide a brief argument,

$$r_d = \frac{dV}{dl} = \frac{1}{q}\left[\frac{d(E_{fl} - E_{fw}}{dI} + \frac{d(E_{fw} - E_{fr})}{dI}\right];$$

since electrons in the well obey a quasiequilibrium distribution, we can assume that the currents through each barrier essentially depend on the difference between the quasi-Fermi levels on both sides of the barrier. We therefore choose to leave E_{fl} and E_{fr} fixed, and let only E_{fw} vary. We use the fact that $I = qg_1(\tilde{N}) = qr_2(\tilde{N})$ and that E_{fw} depends on N, to write

$$r_d \approx \frac{1}{q^2}\left(\frac{-dE_{fw}/dN}{dg_1/dN} + \frac{dE_{fw}/dN}{dr_2/dN}\right) = \frac{dE_{fw}}{dN}\frac{\tau_g + \tau_r}{q^2},$$

where the last equality comes from (5) and all derivatives are evaluated for $N = \tilde{N}$.