Coulomb breach effect emerging in shot noise

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Abstract. Noise can be a unique probe of electron–electron correlations in nanoscale electron devices. While stationary or small signal transport can often be simply understood in terms of single particle behaviour, noise can be extremely sensitive to many-body effects (Landauer L 1996 *Physica* B **227** 156).
We present a new collective phenomenon emerging in electron transport in a resonant tunnelling diode, due to Coulomb repulsion dramatically magnified by the particular shape of the density of states in the quantum well. This phenomenon, for which the name of *Coulomb breach* is proposed, reveals itself by making shot noise several times greater than that expected in the absence of correlated electron motion. Experimental results showing shot noise enhancement of a factor 6.6 are reported.

1. Introduction

Shot noise, which has its origin in the granularity of charge, is particularly important in mesoscopic and nanoscale devices because its power spectral density is proportional to the current, while the power of the signal is proportional to the square of the current. Therefore, as devices are aggressively scaled down, and the number of electrons involved in device operation consequently decreases, shot noise may eventually become dominant.

The time dependent current $i(t)$ between two electrodes actually consists of a series of current pulses, each corresponding to a single electron traversing the device, and therefore carrying a total charge equal to the electron charge q. If electrons do not significantly interact with one another the distribution of pulses is Poissonian: this is the case in the 'full' shot noise regime, for which the power spectral density of the noise current is $S_{\mathrm{full}} = 2qI$, where I is the average value of $i(t)$ [2].

Deviations from full shot noise occur if correlation between pulses is introduced as a result of electron–electron interactions, such as Coulomb force, which limits the density of electrons in real space, and Pauli exclusion, which limits the density of electrons in phase space. The correlation is usually negative, therefore smooths out fluctuations, and leads to a power spectral density of the noise current smaller than S_{full} for a given I. In a charge-limited vacuum tube, for example, Coulomb interaction between electrons leads to a reduced shot noise. For a quantum point contact, theory [3–6] and experiments [7] show that Pauli exclusion totally suppresses shot noise at bias voltages corresponding to conductance plateaux, if propagating channels have unitary transmission probability.

A useful parameter for studying deviations from full shot noise is the so-called Fano factor γ, the ratio of the power spectral density of the noise current $S(\omega)$ to the full shot value $2qI$. We shall refer to the case $\gamma < 1$ as suppressed shot noise, and to the case $\gamma > 1$ as enhanced shot noise. From the point of view of noise, the Fano factor is open to an interesting interpretation: the actual interacting electrons behave as non-interacting quasi-particles of charge γq. This conjecture is strengthened by the fact that theoretical studies predict a Fano factor of 2 for a normal metal–superconducting junction [8], and of $\frac{1}{3}$ for a two-dimensional electron gas in the fractional quantum Hall regime [9] (the latter result has been confirmed by experiments [10]).

We present the case of a dramatic enhancement of shot noise in a resonant tunnelling diode biased in the negative differential resistance region of the I–V characteristic. This effect, which leads to a measured Fano factor as high as 6.6, is a result of the collective behaviour of electrons due to Coulomb repulsion, magnified by the particular shape of the density of states in the quantum well.

The typical I–V characteristic of a resonant tunnelling diode is due to the density of states in the well, which, for the structure considered here, has a main narrow peak in correspondence with the lowest longitudinal allowed energy in the well. When the diode is biased in the negative differential resistance region, the peak of the density of states is below the conduction band edge of the cathode (figure 1(a)): therefore, as the voltage is increased, fewer states are available for tunnelling from the cathode and the current decreases.

The microscopic mechanism which allows for enhanced shot noise is sketched in figures 1(b) and (c) and is as follows: an electron tunnelling into the well from the cathode raises the potential energy of the well by an amount $q/(C_1+C_2)$, where C_1 and C_2 are the capacitances between the well region and either contact; as a consequence, the density of states in the well is shifted upwards by the same amount, with the result

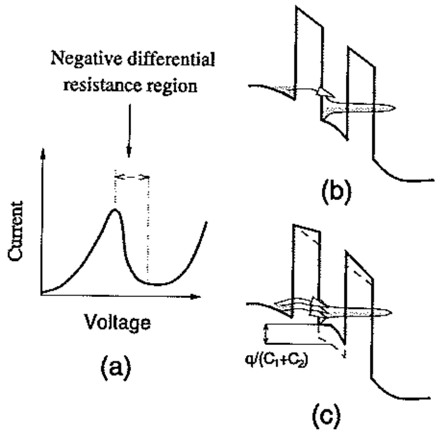

Figure 1. In a resonant tunnelling biased in the negative differential resistance region of the I–V characteristic (a), the main peak of the density of state lies below the conduction band edge of the cathode (b). Enhanced shot noise is obtained because an electron tunnelling into the well from the cathode raises the potential energy of the well by an amount $q/(C_1 + C_2)$, so that more states are available for tunnelling from the cathode (c).

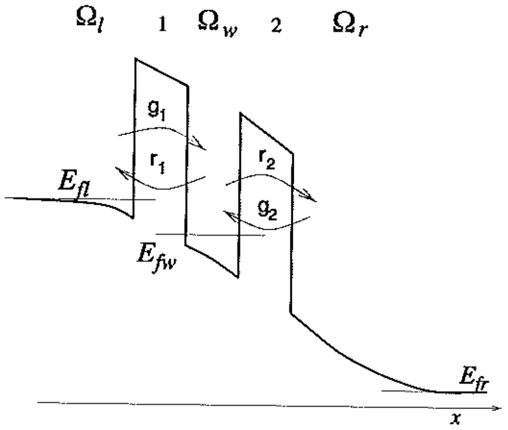

Figure 2. A generic resonant tunnelling structure consists of three isolated regions Ω_l, Ω_w and Ω_l weakly coupled by tunnelling barriers, here indicated with 1 and 2. Coupling between different regions has to be small enough to be treated with first-order perturbation theory.

that more states are available for tunnelling from the cathode, and the probability per unit time that electrons enter the well increases (figure 1(c)). That means that electrons entering the well are positively correlated, therefore enhanced shot noise is to be expected.

2. Model

Let us consider the one-dimensional structure sketched in figure 2: it consists of three regions Ω_l, Ω_w, and Ω_r, i.e. the left reservoir, the well region, and the right reservoir, respectively, that are only weakly coupled through the two tunnelling barriers 1 and 2. Moreover, we suppose that electron transport is well described in terms of sequential tunnelling: an electron in Ω_l traverses barrier 1, loses phase coherence and relaxes to a quasi-equilibrium energy distribution in the well region Ω_w, then traverses barrier 2 and leaves through Ω_r. Such a hypothesis is very reasonable, except at milli-kelvin temperatures, at which inelastic processes are strongly suppressed and no longer effective in thermalizing electrons in the well.

The typical resonant current peaks in the I–V curve are due to the shape of the density of longitudinal states in Ω_w, which is strongly affected by confinement. For the material parameters considered here, it has a single narrow peak in correspondence with the allowed longitudinal energy level of Ω_w; the rate of inelastic scattering processes affects the width of such a peak, and in our model this effect is taken into account through a phenomenological parameter, the mean free path l, which plays the role of a relaxation length. The density of states in the well is calculated using a compact formula derived in [11].

A state in Ω_s ($s = l, r, w$) is characterized by its longitudinal energy E, its transverse wavevector $\boldsymbol{k}_T$, and its spin σ. Tunnelling is treated as a transition between levels in different regions [12] in which E, $\boldsymbol{k}_T$ and σ are conserved.

The resonant tunnelling diode shares an interesting property with the bistable tunnel diode circuit studied by Landauer in [13]: provided the external voltage is fixed, the instantaneous state of the system is determined by the number of electrons N in the central region, i.e. the well region in our case, and the node connecting the two tunnel diodes in Landauer's case. Once the autocorrelation function of N is calculated, the current noise is readily obtained. The problem can therefore be simply recast in terms of a master equation problem: if $p(N)$ is the probability the N electrons are in the well region, we can write

$$\frac{\mathrm{d}p(N)}{\mathrm{d}t} = r(N+1)p(N+1) + g(N-1)p(N-1) \\ -[r(N)+g(N)]p(N), \qquad (1)$$

where $g(N)$ is the generation rate, i.e. the probability per unit time that the number of electrons in the well increases by one, and $r(N)$ is the recombination rate, i.e. the probability per unit time that the number of electrons in the well decreases by one. Following Davies *et al* [14], g is the sum of two partial 'generation' rates g_1 and g_2, i.e. the transition rates from Ω_l to Ω_w, and from Ω_r to Ω_w:

$$g_1 = \frac{4\pi}{\hbar}\int \mathrm{d}E\,|M_{1lw}(E)|^2\rho_l(E)\rho_w(E) \\ \times \int \mathrm{d}\boldsymbol{k}_T\rho_T(\boldsymbol{k}_T)f_l(E,\boldsymbol{k}_T)[1-f_w(E,\boldsymbol{k}_T)], \qquad (2)$$

$$g_2 = \frac{4\pi}{\hbar}\int \mathrm{d}E\,|M_{2rw}(E)|^2\rho_r(E)\rho_w(E) \\ \times \int \mathrm{d}\boldsymbol{k}_T\,\rho_T(\boldsymbol{k}_T)f_r(E,\boldsymbol{k}_T)[1-f_w(E,\boldsymbol{k}_T)]; \qquad (3)$$

where ρ_s, f_s, $s = l, w, r$, are the longitudinal density of states and the occupation factor in Ω_s, respectively, and ρ_T is the density of transverse states; $M_{1lw}(E)$ is the matrix element for a transition through barrier 1 between states of longitudinal energy E: it is obtained in [16] as $|M_{1lw}(E)|^2 = \hbar^2 \nu_l(E)\nu_w(E)T_1(E)$ where ν_s ($s = l, w, r$) is the so-called attempt frequency in Ω_s and T_1 is the tunnelling probability of barrier 1; $M_{2rw}(E)$ is analogously defined.

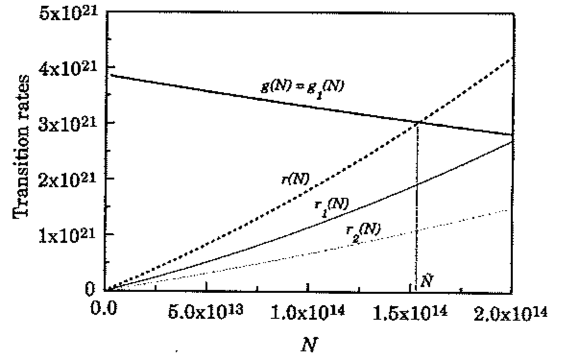

Figure 3. Transition rates as a function of the number of electrons in the well per unit area for the considered device with bias voltage 0.05 V at 77 K. The generation rate g_2 is not shown because it is several orders of magnitude smaller than the others.

Analogously, r is the sum of two partial 'recombination' rates r_1 and r_2, i.e. the transition rates from Ω_w to Ω_l and from Ω_w to Ω_r:

$$r_1 = \frac{4\pi}{\hbar}\int \mathrm{d}E\,|M_{1lw}(E)|^2\rho_l(E)\rho_w(E) \times \int \mathrm{d}k_T\,\rho_T(k_T)f_w(E,k_T)[1-f_l(E,k_T)], \tag{4}$$

$$r_2 = \frac{4\pi}{\hbar}\int \mathrm{d}E\,|M_{2rw}(E)|^2\rho_r(E)\rho_w(E) \times \int \mathrm{d}k_T\,\rho_T(k_T)f_w(E,k_T)[1-f_r(E,k_T)]. \tag{5}$$

We need to calculate the complete curves of g and r as a function of the number of electrons in the well N. For a given N, the Schrödinger equation and the Poisson equation are solved selfconsistently, assuming that electrons in the well obey a quasi-equilibrium distribution determined only by E_{fw}. After convergence is reached, the potential profile and E_{fw} are obtained, therefore the transition rates can be calculated according to (2)–(5).

A plot of the mentioned transition rates as a function of N at a given bias point for one of the devices studied is shown in figure 3 (both the rates and the number of electrons are given per unit area). We call $\tilde{N}$ the value of N for which $g = r$, and we can reasonably expand g and r linearly around $\tilde{N}$. Within the linear approximation, $\tilde{N}$ is also the mean value of N, and the mean value of a given transition rate is well approximated by its value for $N = \tilde{N}$.

If we denote the mean value of a generic quantity a as $\langle a\rangle$, the average current is

$$\langle i\rangle = q\langle g_1 - r_1\rangle = q\langle r_2 - g_2\rangle = q[g_1(\tilde{N}) - r_1(\tilde{N})] = q[r_2(\tilde{N}) - g_2(\tilde{N})]. \tag{6}$$

In addition, we introduce and calculate the following characteristic times, that are required for obtaining the power spectral density of the noise current:

$$\frac{1}{\tau_{g1}} \equiv -\left.\frac{\mathrm{d}g_1}{\mathrm{d}N}\right|_{N=\tilde{N}}, \qquad \frac{1}{\tau_{g2}} \equiv -\left.\frac{\mathrm{d}g_2}{\mathrm{d}N}\right|_{N=\tilde{N}}, \tag{7}$$

$$\frac{1}{\tau_{r1}} \equiv \left.\frac{\mathrm{d}r_1}{\mathrm{d}N}\right|_{N=\tilde{N}}, \qquad \frac{1}{\tau_{r2}} \equiv \left.\frac{\mathrm{d}r_2}{\mathrm{d}N}\right|_{N=\tilde{N}}; \tag{8}$$

we can also define

$$\tau_1^{-1} \equiv \tau_{g1}^{-1} + \tau_{r1}^{-1}, \qquad \tau_2^{-1} \equiv \tau_{g2}^{-1} + \tau_{r2}^{-1}, \tag{9}$$

and

$$\tau^{-1} = \tau_1^{-1} + \tau_2^{-1} = \tau_g^{-1} + \tau_r^{-1}. \tag{10}$$

The power spectral density $S(\omega)$ of the noise current at low frequency ($\omega\tau \ll 1$) can be written as

$$S(\omega) = 2q^2\left(\frac{\tau^2\langle g_1 + r_1\rangle}{\tau_2^2} + \frac{\tau^2\langle g_2 + r_2\rangle}{\tau_1^2}\right). \tag{11}$$

A detailed derivation of this result can be found in [17]; it suffices here to say that no additional hypothesis is required to arrive at (11).

An important parameter is the noise suppression factor γ, also called the 'Fano factor', i.e. the ratio between $S(\omega)$ and the 'full shot' noise value $S_{\text{full}} = 2q\langle i\rangle$. From (6) and (9)–(11), it is apparent that it can reach a minimum of $\gamma = 0.5$ if $\tau_1 = \tau_2$, $\langle g_2\rangle \ll \langle r_2\rangle$, $\langle g_1\rangle \gg \langle r_1\rangle$.

3. Experiment

We focus on a device fabricated at the TASC-INFM laboratory in Trieste with the following layer structure: a Si-doped ($N_d = 1.4\times10^{18}$ cm^{-3}) 500 nm-thick GaAs buffer layer, an undoped 20 nm-thick GaAs spacer layer to prevent silicon diffusion into the barrier, an undoped 12.4 nm-thick AlGaAs first barrier, an undoped 6.2 nm-thick GaAs quantum well, an undoped 14.1 nm-thick AlGaAs barrier, a 10 nm GaAs spacer layer and a Si-doped 500 nm-thick cap layer. The aluminium mole fraction in both barriers is 0.36 and the diameter of the mesa defining the single device is 50 μm.

The barriers in our samples are thicker than in most similar resonant-tunnelling diodes, for the purpose of reducing the current and, consequently, of increasing the differential resistance, in order to obtain the best possible noise match with the measurement amplifiers (available ultra-low-noise amplifiers offer a good performance, with a very small noise figure, for a range of resistance values between a few kilo-ohms and several mega-ohms).

We have applied a measurement technique purposely developed for low-level current noise measurements, based on the careful evaluation of the transimpedance between the device under test and the output of the amplifier [18].

In figure 4 the measured current and the Fano factor γ are plotted as a function of the applied voltage (the thicker barrier is on the anode side) at a temperature of 77 K.

As can be seen, in the first region of the I–V characteristic, when the current increases with increasing voltage, the Fano factor γ is smaller than one, and reaches a minimum value close to 0.5. This is in agreement with existing theories [14,17,19] according to which the combined action of Pauli exclusion and Coulomb repulsion may push γ down to 0.5, and has been confirmed by a few experiments [19–21].

At the voltage of the current peak the Fano factor is exactly one, then it increases again and reaches a peak of 6.6 close to the bias corresponding to the lowest absolute value of the negative differential resistance; for higher voltages, it rapidly approaches one.

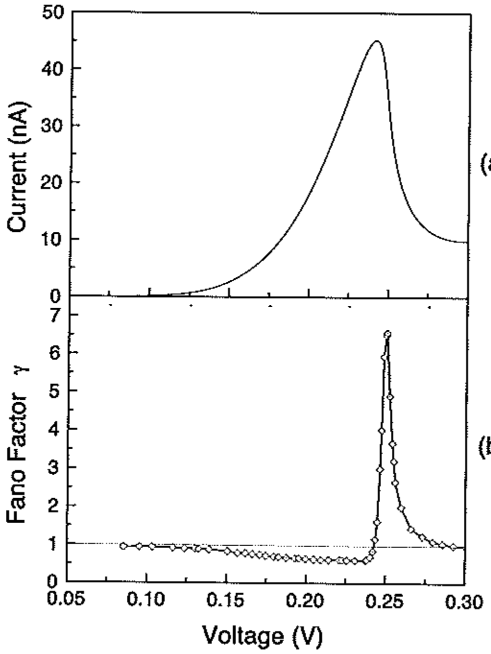

Figure 4. Experimental current (*a*) and Fano factor γ (*b*) as a function of the applied voltage at 77 K. The maximum value of γ is 6.6, while the minimum is close to 0.5.

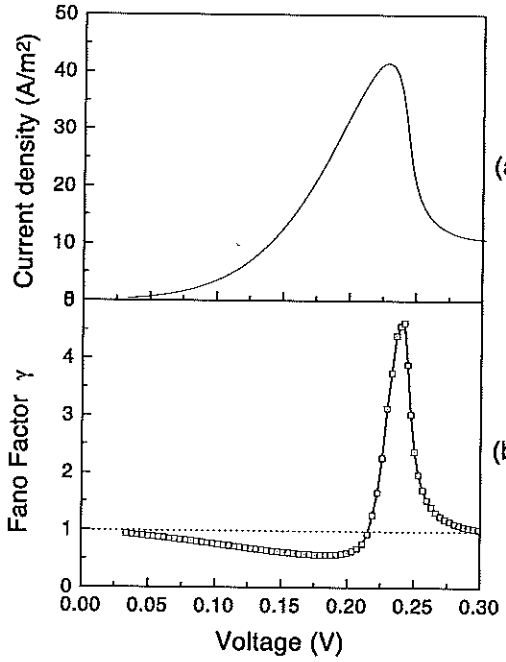

Figure 5. Calculated current density (*a*) and Fano factor γ (*b*) at 77 K as a function of the applied voltage for the considered structure.

As a remark, let us point out that, while the results shown in this paper are obtained from a single device, the same behaviour is found after thermal cycling and on different devices on the same wafer. In addition, measurements performed on resonant tunnelling diodes with a similar structure yield similar results [15].

4. Numerical simulation

Based on the model described above, we have performed a numerical simulation of shot noise in the considered device: the results at a temperature of 77 K, with a relaxation length of 15 nm are shown in figure 3 [16]. The value of l is chosen in order to fit the peak-to-valley ratio of the diode current, and is the only fitting parameter used. As can be seen, there is an almost quantitative agreement between theory and experiment (the peak experimental current is 45 nA which corresponds to a current density of 23 A m^{-2}): we ascribe most of the difference to the tolerance in the nominal device parameters and to the simplistic inclusion of all phase-destroying mechanisms in a single, energy independent, relaxation length. All the relevant features of the Fano factor as a function of the applied voltage are reproduced, and can be easily explained in terms of our model.

5. Conclusion

In conclusion, we would like to stress the point that the described effect can be viewed in opposition to the Coulomb blockade. In the case of Coulomb blockade, an electron entering the well prevents other electrons from tunnelling into the well, since it occupies a low energy level and raises the energy of the lowest available level in the well because of Coulomb repulsion. Instead, in the case described here, an electron entering the well opens up a breach in the cathode barrier, making much easier electron tunnelling into the well. That is why we propose the name of Coulomb breach for this effect. Coulomb breach emerges only in noise, and is therefore much more difficult to observe and measure than Coulomb blockade, which can even be seen in DC transport. Similarly to the Coulomb blockade, the Coulomb breach is expected to play an important role in the transport and noise properties of few-electron systems and devices.

As a final remark, within the realm of validity of the quasi-particle interpretation of the Fano factor, noise in the considered resonant tunnelling diode biased in the negative differential resistance region may be described in terms of independent quasi-particles of charge up to $6.6q$.

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References

[1] Landauer L 1996 *Physica* B **227** 156
[2] Schottky W 1918 *Ann. Phys., Paris* **57** 541
[3] Lesovik G B 1989 *JETP Lett.* **49** 592 (*Pis. Zh. Teor. Fiz.* **49** 513)
[4] Yurke B and Kochanski G P *Phys. Rev.* B **41** 8184
[5] Büttiker M 1990 *Phys. Rev. Lett.* **65** 2901

[6] Büttiker M 1992 *Phys. Rev.* B **46** 12 485
[7] Kumar A, Saminadayar L, Glattli D C, Jin Y and Etienne B 1996 *Phys. Rev. Lett.* **76** 2778
[8] de Jong M J M and Beenakker C W J 1994 *Phys. Rev.* B **49** 16 070
[9] Kane C L and Fisher M P A 1994 *Phys. Rev. Lett.* **72** 724
[10] de-Picciotto R, Reznikov M, Heiblum M, Umansky V, Bunin G and Mahalu D 1997 *Nature* **389** 162
[11] Iannaccone G and Pellegrini P 1996 *Phys. Rev.* B **53** 2020
[12] Bardeen J 1961 *Phys. Rev. Lett.* **6** 57
[13] Landauer R 1962 *J. Appl. Phys.* **33** 2209
[14] Davies J H, Hyldgaard P, Hershfield S and Wilkins J W 1992 *Phys. Rev.* B **46** 9620
[15] Iannaccone G, Lombardi G, Macucci M and Pellegrini B 1998 *Analog Integrated Circuits and Signal Processing* to be published
[16] Iannaccone G and Pellegrini B 1995 *Phys. Rev.* B **52** 17 406
[17] Iannaccone G, Macucci M and Pellegrini B 1997 *Phys. Rev.* B **55** 4539
[18] Macucci M and Pellegrini B 1991 *IEEE Trans. Instrum. Meas.* **40** 7
[19] Brown E R 1992 *IEEE Trans. Electron Devices* **39** 2686
[20] Liu H C, Li J, Aers G C, Leavens C R and Buchanan R 1995 *Phys. Rev.* B **51** 5116
[21] Ciambrone P, Macucci M, Iannaccone G, Pellegrini B, Sorba L, Lazzarino M and Beltram F 1995 *Electron. Lett.* **31** 503