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## Simulation and Measurement of Shot Noise in Resonant Tunneling Structures

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**Abstract.** We present numerical simulations and measurements of shot noise in resonant tunneling structures. We show that when electron-electron interaction through Coulomb force and Pauli exclusion is properly taken into account, the main features of noise behavior of such devices can be correctly predicted. Electron-electron interaction is shown to be responsible for the suppression of shot noise in the positive differential resistance region of the I-V curve, and for the enhancement of shot noise in the negative differential resistance region.

### Key Words:

### 1. Introduction

One of the main consequences of aggressive scaling down of electronic devices is the increasing importance of noise, and in particular of “shot noise,” which has its origin in the granularity of charge.

In fact, while the signal power is proportional to the square of the current through a given device, the shot noise power spectral density is only proportional to the current. Therefore, as the number of charge carriers involved in device operation decreases, noise effects acquire relevance and, eventually, become predominant.

This consideration, by itself, justifies the recent growing interest in noise in nanoelectronic and mesoscopic devices. In addition, noise in such systems exhibits a behavior strongly dependent upon the details of device geometry: as a consequence, from noise characterization one can gain new insights into the structure and the transport properties of such devices. For instance, it is well known that most features of stationary transport can be reasonably described in terms of independent electrons, while electron-electron interaction plays a relevant role in determining noise properties. This means, on one hand, that noise characterization provides information on the collective behavior of electrons not easily obtainable otherwise, on the other hand, that a

realistic model of noise in such devices must take these effects into account.

Here, we focus on resonant tunneling structures, and compare numerical simulations based on a model presented elsewhere [1,2], with the results of experiments.

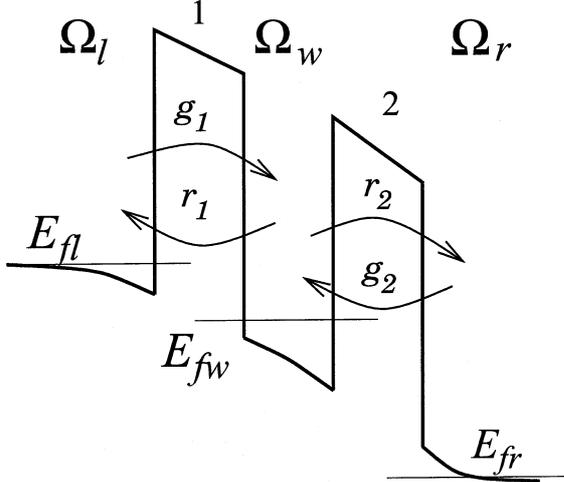
Since the pioneering work of Lesovik [3], and the first experimental results of Li and coworkers [4], many theoretical studies [1,5–9] and experimental results [4,9–12] have appeared in the literature, assessing that the power spectral density of the noise current  $S$  in such devices may be suppressed down to half of the “full” shot noise value  $S_{full} = 2q\langle i \rangle$ , i.e., that associated with a purely poissonian process.

The time dependent current  $i(t)$  consists of a series of current pulses, each corresponding to a single electron traversing the device, and therefore carrying a total charge equal to the electron charge  $q$ . If consecutive current pulses are negatively correlated, i.e., if the pulse distribution is sub-poissonian, suppressed shot noise is to be expected. Such correlation may be introduced by Pauli exclusion acting on electron levels in the well, or/and by Coulomb repulsion, depending on the details of the structure and of the dominant transport mechanism [1,8,9]. Analogously, if consecutive pulses are positively correlated, enhanced shot noise is to be measured.

## 2. Model

Let us consider the one-dimensional structure sketched in Fig. 1: it consists of three regions  $\Omega_l$ ,  $\Omega_w$ , and  $\Omega_r$ , i.e., the left reservoir, the well region, and the right reservoir, respectively, that are only weakly coupled through the two tunneling barriers 1 and 2. Moreover, we suppose that electron transport is well described in terms of sequential tunneling: an electron in  $\Omega_l$  traverses barrier 1, loses phase coherence and relaxes to a quasi-equilibrium energy distribution in the well region  $\Omega_w$ , then traverses barrier 2 and leaves through  $\Omega_r$ . Such hypothesis is very reasonable, except at millikelvin temperatures, when inelastic processes are strongly suppressed and no more effective in thermalizing electrons in the well.

The typical resonant current peaks in the I-V curve are due to the shape of the density of longitudinal states in  $\Omega_w$ , which is strongly affected by confinement: for the material parameters considered here, it has a single narrow peak in correspondence of the allowed longitudinal energy level of  $\Omega_w$ ; the rate of inelastic scattering processes affects the width of such peak, and in our model this effect is taken into account through a phenomenological parameter, the mean free path  $l$ , which plays the role of a relaxation length. The density of states in the well is calculated using a compact formula derived in [13].



*Fig. 1.* A generic resonant tunneling structure consists of three isolated regions  $\Omega_l$ ,  $\Omega_w$ ,  $\Omega_r$  weakly coupled by tunneling barriers, here indicated with 1 and 2. Coupling between different regions is assumed to be small enough to be treated with first order perturbation theory.

A state in  $\Omega_s$  ( $s = l, r, w$ ) is characterized by its longitudinal energy  $E$ , its transverse wave vector  $\mathbf{k}_T$ , and its spin  $\sigma$ . Tunneling is treated as a transition between levels in different regions [14] in which  $E$ ,  $\mathbf{k}_T$  and  $\sigma$  are conserved.

Following Davies et al., we introduce the ‘‘generation’’ rates  $g_1$  and  $g_2$ , i.e., the transition rates from  $\Omega_l$  to  $\Omega_w$ , and from  $\Omega_r$  to  $\Omega_w$ , respectively, as

$$g_1 = \frac{4\pi}{\hbar} \int dE |M_{1lw}(E)|^2 \rho_l(E) \rho_w(E) \times \int d\mathbf{k}_T \rho_T(\mathbf{k}_T) f_l(E, \mathbf{k}_T) (1 - f_w(E, \mathbf{k}_T)) \quad (1)$$

$$g_2 = \frac{4\pi}{\hbar} \int dE |M_{2rw}(E)|^2 \rho_r(E) \rho_w(E) \times \int d\mathbf{k}_T \rho_T(\mathbf{k}_T) f_r(E, \mathbf{k}_T) (1 - f_w(E, \mathbf{k}_T)) \quad (2)$$

where  $\rho_s, f_s$ ,  $s = l, w, r$ , are the longitudinal density of states and the occupation factor in  $\Omega_s$ , respectively, and  $\rho_T$  is the density of transverse states;  $M_{1lw}(E)$  is the matrix element for a transition through barrier 1 between states of longitudinal energy  $E$ : it is obtained in [2] as  $|M_{1lw}(E)|^2 = \hbar^2 \nu_l(E) \nu_w(E) T_1(E)$  where  $\nu_s$  ( $s = l, r, w$ ) is the so-called attempt frequency in  $\Omega_s$  and  $T_1$  is the tunneling probability of barrier 1;  $M_{2rw}(E)$  is analogously defined.

In addition, we introduce the ‘‘recombination’’ rates  $r_1$  and  $r_2$ , i.e., the transition rates from  $\Omega_w$  to  $\Omega_l$  and from  $\Omega_w$  to  $\Omega_r$ , respectively, as

$$r_1 = \frac{4\pi}{\hbar} \int dE |M_{1lw}(E)|^2 \rho_l(E) \rho_w(E) \times \int d\mathbf{k}_T \rho_T(\mathbf{k}_T) f_w(E, \mathbf{k}_T) (1 - f_l(E, \mathbf{k}_T)) \quad (3)$$

$$r_2 = \frac{4\pi}{\hbar} \int dE |M_{2rw}(E)|^2 \rho_r(E) \rho_w(E) \times \int d\mathbf{k}_T \rho_T(\mathbf{k}_T) f_w(E, \mathbf{k}_T) (1 - f_r(E, \mathbf{k}_T)) \quad (4)$$

The total generation rate is  $g = g_1 + g_2$ , while the total recombination rate is  $r = r_1 + r_2$ .

The occupation factor in the well, which, under the assumption of complete relaxation, depends only on the value of the quasi-Fermi level  $E_{fw}$  in the well, has to be calculated in the steady state condition, i.e., by imposing  $g = r$ . If we denote the steady state value of a generic quantity  $a$  as  $\langle a \rangle$ , the average current is

$$\langle i \rangle = q \langle g_1 - r_1 \rangle = q \langle r_2 - g_2 \rangle \quad (5)$$

The problem of transport is solved self-consistently, since the charge accumulated in the well affects the conduction band profile of the structure. Once  $g$  and  $r$  are calculated, we can expand them to first order in  $N$  around the steady state value  $\tilde{N}$  (such as  $g(\tilde{N}) = r(\tilde{N})$ ), and introduce the following characteristic times:

$$\frac{1}{\tau_{g1}} \equiv - \left. \frac{dg_1}{dN} \right|_{N=\tilde{N}} \quad \frac{1}{\tau_{g2}} \equiv - \left. \frac{dg_2}{dN} \right|_{N=\tilde{N}} \quad (6)$$

$$\frac{1}{\tau_{r1}} \equiv \left. \frac{dr_1}{dN} \right|_{N=\tilde{N}} \quad \frac{1}{\tau_{r2}} \equiv \left. \frac{dr_2}{dN} \right|_{N=\tilde{N}} \quad (7)$$

from which we can define

$$\tau_1^{-1} \equiv \tau_{g1}^{-1} + \tau_{r1}^{-1} \quad \tau_2^{-1} \equiv \tau_{g2}^{-1} + \tau_{r2}^{-1} \quad (8)$$

and

$$\tau^{-1} = \tau_1^{-1} + \tau_2^{-1} = \tau_g^{-1} + \tau_r^{-1} \quad (9)$$

The power spectral density  $S(\omega)$  of the noise current at low frequency ( $\omega\tau \ll 1$ ) can be written as

$$S(\omega) = 2q^2 \left( \frac{\tau^2 \langle g_1 + r_1 \rangle}{\tau_2^2} + \frac{\tau^2 \langle g_2 + r_2 \rangle}{\tau_1^2} \right) \quad (10)$$

a detailed derivation of this result can be found in [1]; it suffices here to say that no additional hypothesis is required to arrive at (10).

An important parameter is the noise suppression factor  $\gamma$ , also called ‘‘Fano factor’’, i.e., the ratio between  $S(\omega)$  and the ‘‘full shot’’ noise value  $S_{full} = 2q \langle i \rangle$ . From (5) and (8)–(10), it is apparent that it can reach a minimum of  $\gamma = 0.5$  if  $\tau_1 = \tau_2$ , and  $\langle g_2 \rangle \ll \langle r_2 \rangle$ .

### 3. Experiment

We have applied a measurement technique purposely developed for low-level current noise measurements, based on the careful evaluation of the transimpedance between the device under test and the output of the amplifier, and on the subtraction of the noise contributions due to the amplifier, the bias source and the biasing network [15]. This procedure allows us to measure noise levels that are up to 3 dB below

that of the available amplifiers with a maximum error around 10%.

For the successful application of our correction method it is necessary to precisely measure the noise due to the amplifier: to this purpose the measurement is repeated after replacing the resonant-tunneling device with an equivalent impedance (equivalent within the frequency range of interest). Such impedance is kept at a known constant temperature and in equilibrium conditions (no externally imposed current flowing through it), thus it exhibits only thermal noise, which can be precisely evaluated and subtracted from the result of the measurement, thereby yielding the noise due to the amplifier and the passive components of the biasing network. Therefore it is important that the noise due to the internal sources of the amplifier be the same during the original measurement on the resonant tunneling-device and that on the substitution impedance. This requirement can be satisfied, for the measurements at the lowest current levels, only by keeping also the temperature of the amplifier constant and, in particular, that of the feedback resistor, which, in these conditions, is the prevalent source of noise. Thus, the amplifier used for measurements at low bias currents has been enclosed in a sealed aluminum case that is kept at a constant temperature of 0°C in a bath of melting ice.

We have optimized the double-barrier resonant-tunneling structures from the point of view of the differential resistance, by adjusting the barrier thicknesses and the cross-section, in order to obtain the best possible noise matching with the measurement amplifiers. Available ultra-low-noise amplifiers

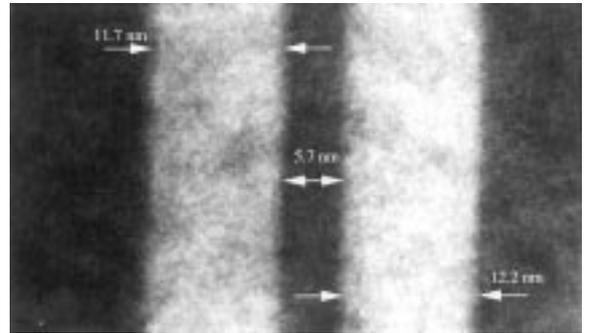


Fig. 2. TEM cross-section of the considered device: bright areas indicate the AlGaAs barriers, dark areas indicate GaAs layers.

offer a good performance, with a very small noise figure, for a range of resistance values between a few kilohms and several megaohms. For this reason the barriers in our samples are thicker than in most similar resonant-tunneling diodes, in order to reduce the current and, consequently, to increase the differential resistance. The diameter of the mesas defining the single devices has been shrunk down to about  $30\ \mu\text{m}$ , which represents a practical limit, because bonding would become too difficult for smaller diodes.

The structure which we focus on in this paper has been fabricated at the TASC-INFEM laboratory in Trieste and has the following layer structure: a Si-doped ( $N_d = 1.4 \times 10^{18}\ \text{cm}^{-3}$ ) 500 nm-thick GaAs buffer layer, an undoped 20 nm-thick GaAs spacer layer to prevent silicon diffusion into the barrier, an undoped 11.7 nm-thick AlGaAs first barrier, an undoped 5.7 nm-thick GaAs quantum well, an undoped 12.2 nm-thick AlGaAs barrier, a 10 nm GaAs spacer layer and a Si-doped 500 nm-thick cap layer. The aluminum mole fraction in both barriers is 0.36 and the diameter of the mesa defining the single device is  $30\ \mu\text{m}$ . The thicknesses of the two barriers and of the well region have been verified from TEM cross-sections, while the other dimensions are nominal.

#### 4. Results of the Simulation and Comparison with Experiments

The measured I-V curve in forward and reverse bias is shown in Fig. 3, while the simulation results are shown in Fig. 4. For the simulation, a mean free path of 15 nm has been considered, in order to obtain the best fit for the peak-to-valley ratio of the current. As can be seen, the agreement is rather good, given the tolerances on the parameters not observed by TE microscopy. A peak current of 260 nA in reverse bias corresponds to a peak current density of  $368\ \text{A/m}^2$ , sufficiently close to the calculated value of  $401\ \text{A/m}^2$ .

In Fig. 5, the measured  $\gamma$  in forward and reverse bias is plotted as a function of the applied voltage at the temperature of 77 K, while the calculated  $\gamma$  is shown in Fig. 6.

As expected, noise suppression is observed in the region of the I-V curve before the peak, and it reaches a minimum of 0.6. At the voltage corresponding to the current peak,  $\gamma$  is exactly one, meaning full shot noise. Then, in the negative differential resistance region,

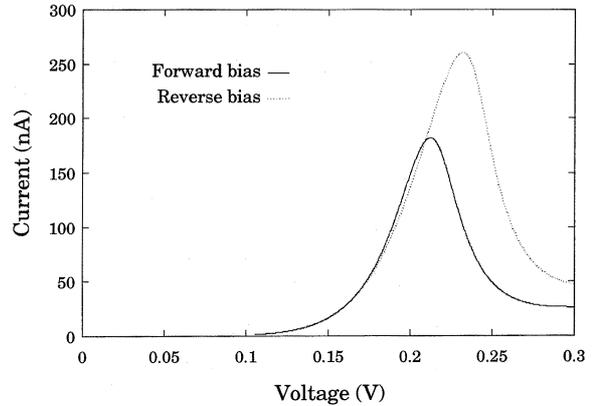


Fig. 3. Experimental I-V curves for forward and reverse bias at the temperature of 77 K.

enhanced shot noise is measured, and the noise suppression (enhancement, in this case) factor  $\gamma$  reaches a maximum value of 1.7 when the absolute value of the differential resistance is minimum. When the differential resistance is positive again, the shot noise suppression factor is close to 1, i.e., the process of electrons traversing the structure is purely poissonian. Also in this case, the results of the calculation are in reasonable quantitative agreement.

The suppression of shot noise in the first region of the I-V curve is due to the combined effect of Coulomb repulsion and Pauli exclusion: an electron entering the well both occupies a quasi-state in the well and raises the electrostatic potential of the well

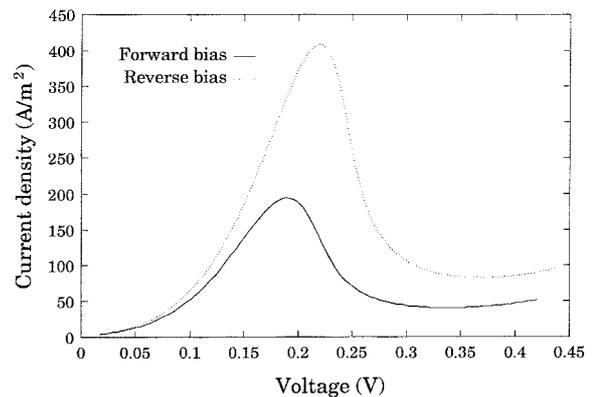


Fig. 4. Calculated I-V curves for forward and reverse bias at the temperature of 77 K.

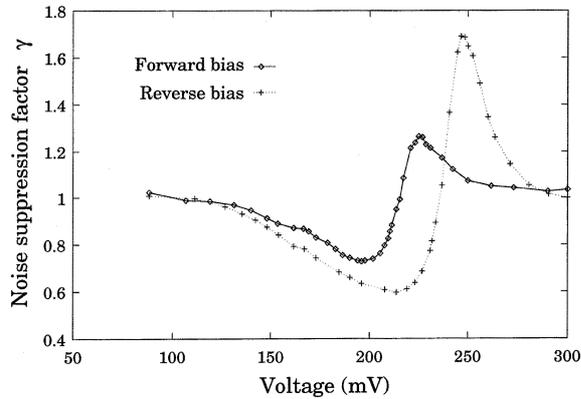


Fig. 5. Measured noise suppression factor as a function of the applied voltage for forward and reverse bias at 77 K.

region, therefore making less probable for another electron to enter the well. Since electron crossings are negatively correlated, sub-poissonian shot noise is to be expected.

The enhanced shot noise in the NDR region depends on the fact that the characteristic time  $\tau_{g_1}$  is negative, i.e., the transition rate  $g_1$  increases with increasing  $N$ . The reason is that the peak in the density of states is below the conduction band bottom of the cathode: when an electron enters the well, the conduction band bottom of the well is raised, and more states are available for tunneling from the left electrode, so that electron crossings through the whole structure are positively correlated.

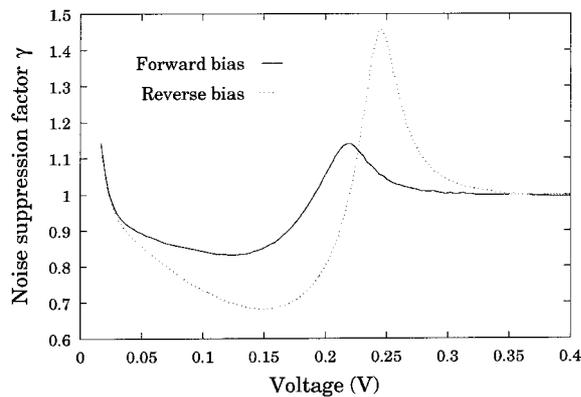


Fig. 6. Calculated noise suppression factor as a function of the applied voltage for forward and reverse bias at 77 K.

## 5. Discussion

We have shown that important features of shot noise in resonant tunneling devices can be reproduced numerically only if electron-electron interaction is properly taken into account. Within the tolerances of the device parameters, good agreement is obtained between experiments and simulations. On the other hand, we have shown that noise can be a unique tool for investigating correlated electron behavior and can provide insights into transport properties of nanoscale devices that are complementary to those given by DC and AC characterization.

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