Università di Pisa

Unified approach for electron transport in resonant tunneling structures

Giuseppe Iannaccone
Dipartimento di Ingegneria dell'Informazione: Elettronica, Informatica, Telecomunicazioni, Università di Pisa

Bruno Pellegrini
Dipartimento di Ingegneria dell'Informazione: Elettronica, Informatica, Telecomunicazioni, Università di Pisa

Unified approach to electron transport in double-barrier structures

Giuseppe Iannaccone* and Bruno Pellegrini

*Dipartimento di Ingegneria dell'Informazione: Elettronica, Informatica e Telecomunicazioni,
Università degli studi di Pisa, Via Diotisalvi 2, I-56126 Pisa, Italy*

(Received 30 May 1995; revised manuscript received 28 August 1995)

In this paper we show an approach to electron transport in double-barrier structures that unifies the well-known sequential and resonant tunneling models in the widest range of transport regimes, from completely coherent to completely incoherent. In doing so, we make a clear distinction between "approaches" and "transport regimes," in order to clarify some ambiguities in the concept of sequential tunneling. Scattering processes in the well are accounted for by means of an effective mean free path, which plays the role of a relaxation length. Our approach is based on a recently derived formula for the density of states in a quantum well, as a function of the round-trip time in the well and of transmission and reflection probabilities for the whole structure and for each barrier.

I. INTRODUCTION

Tunneling in double-barrier structures has been extensively studied since the pioneering work of Tsu, Esaki, and Chang.[1,2] These structures promise many interesting device applications[3] and allow us to study problems, relating energy levels in quantum wells and tunneling, which are of general interest in condensed matter physics.

A proper description of electron transport in double barriers is required to understand the relevant phenomena affecting the electrical properties of these structures and to construct a suitable model for obtaining dc characteristics, high-frequency performances, and noise properties in agreement with the experimental measurements.

There are two well-known models for transport through a double barrier. The first has been basically proposed by Chang, Esaki, and Tsu[2] and is the "resonant tunneling" model, which is easy to understand by means of the analogy with a Fabry-Pérot resonator: the tunneling probability for the whole structure is resonantly enhanced for incident electron energies close to the discrete levels in the quantum well. Subsequent evolution of this model takes into account inelastic scattering in the quantum well by means of an inelastic contribution to the effective width of the energy range of well states, which broadens the resonance and lowers the peak of the tunneling probability as a function of energy.[4,5]

The other well-known model is that of "sequential tunneling," proposed by Luryi,[6] which consists in considering tunneling through the double barrier as a two-step process: first, the transition from one electrode into the well, then the transition from the well to the other electrode. The two sequential transitions are assumed to occur incoherently, so that scattering in the well region is accounted for implicitly; moreover, the amount of inelastic scattering in the well influences the width of the density of states in the well as a function of energy.

Zohta[7] has pointed out some ambiguity in the definition of sequential tunneling. In fact, in some papers sequential tunneling is intended as a transport mechanism, i.e., what happens when the electron traverses one barrier coherently, is scattered inelastically in the well, and loses phase memory, then escapes through the other barrier;[8] in other papers sequential tunneling is considered just as an alternative way to describe the resonant tunneling phenomenon.[7,9]

In order to clarify these ambiguities, in this paper we make a clear distinction between approaches and transport regimes. Here we discuss two different approaches to the study of transport in double-barrier structures: the "resonant tunneling" one, and the "sequential tunneling" one. On the other hand, we have a whole range of transport regimes, depending on the rate of collision processes in the well, i.e., on the value of the effective mean free path l: one limit is the completely coherent transport ($l \to \infty$), when no collisions occur in the well, and the other limit is completely incoherent transport ($l \to 0$), when the high collision rate cancels out any size effect in the well.

We wish to point out that, in this paper, the word "sequential" is used to denote both a transport regime and an approach, in order to follow the usual terminology adopted in the literature.[6–9] This fact should not be misleading to the reader: while the sequential "regime," discussed in Sec. IV B is a well-defined transport regime in which almost all electrons lose phase coherence in the well,[6,8] the sequential tunneling "approach" is just the method of considering the double-barrier structure as consisting of three isolated regions weakly coupled through the barriers (which have to be opaque so that the Bardeen tunneling Hamiltonian[10] can be used to obtain transition matrix elements).

As such, the sequential approach does not require loss of phase coherence in the well to be applicable; conversely, it can be used to describe also coherent transport. Of course, the amount of scattering in the well is important if we use the sequential tunneling approach: in fact, it affects the density of states in the well[7,11] which, in turn, affects the number of transitions per unit time between the well and any of the electrodes through Fermi's "golden rule."

In Sec. III we show that the approaches mentioned above, properly extended, are completely equivalent and either can be used to describe all possible transport regimes. This result has been obtained by Weil and Vinter in the case of completely coherent transport,[9] and by Zohta in the case of symmetric barriers and inelastic scattering in the well.[7] We extend this equivalence to the general case of arbitrary barriers, and arbitrary amount of scattering in the quantum well, so that the whole range of transport regimes is addressable. The derivation of this equivalence, as we shall show, is based on a recently derived formula for the density of states in the well region of a double-barrier structure.[11]

In Sec. IV we discuss three relevant transport regimes: the "coherent," "sequential," and "completely incoherent" regimes, for all of which, as said above, either approach can be used. However, for the coherent regime, where the prevalent contribution to the total current comes from electrons that conserve phase coherence and energy, the resonant tunneling approach is more straightforward. Regarding the sequential regime, the sequential tunneling approach is the best suited: in this regime practically all electrons lose phase memory and thermalize in the well, but size effects strongly affect the density of states in the well. Finally, also in the completely incoherent regime, where size effects are canceled out so that the density of states in the well approaches that in the bulk, the sequential tunneling approach is the most direct one.

II. TRANSPORT MODEL

Transport properties of ultrasmall structures strongly depend on both elastic scattering (due to impurities, crystal defects, and interface roughness) and inelastic scattering (due to phonons and electron-electron interactions). Elastic collisions conserve energy and phase coherence, while inelastic collisions do not.

However, if there is a sufficiently large number of impurities and defects randomly distributed, also elastic scattering has phase randomizing effects, due to averaging over many paths corresponding to different actions.[12-14] Therefore, we can account for dephasing effects of both elastic and inelastic collisions by means of a single characteristic length l, the effective mean free path, which is a phenomenological parameter and, as we shall see, plays the role of a relaxation length.

We assume that an electron traversing a length dx of the one-dimensional device structure has a probability dx/l of experiencing a collision, and that electrons emerge from collisions with a thermal quasiequilibrium energy distribution and a completely random phase, so that there is no quantum interference between these electrons and the ones that have not lost phase coherence.

As can be seen, energy relaxation and phase randomization processes are supposed to occur at the same time, and to be triggered by any kind of collision, regardless of whether it is elastic or inelastic. This is not rigorously true: in fact, electrons do not lose energy in elastic collisions, and inelastic collisions are presumably not so effec-

tive in relaxing energy as they are in randomizing phase. Anyway we shall use, for simplicity, a unique length scale for both processes. A more sophisticated model should distinguish between different scattering sources and incoherence phenomena. For instance, two different energy-dependent characteristic lengths for phase randomization and energy relaxation could be adopted.

Our model is close to the one proposed by Büttiker;[8,15] however, Büttiker's model is applicable when the differences between electrode chemical potentials are small and/or when energy relaxation is not accounted for. Moreover, in our model scattering phenomena can be spread—in principle—over the whole region of interest, and are not concentrated in a single inelastic scatterer coupling the well to an extra reservoir.

The idea of using a phenomenological mean free path to account for the loss of phase coherence in the well is not new.[7,12,13,16-18] As an improvement to previous similar models, we have associated to the effective mean free path also energy relaxing processes: in the following, we shall show that this step is important to take into account dissipation and to obtain the main result of this paper, i.e., the demonstration of the equivalence of the sequential and resonant tunneling approaches.

Other characteristic lengths usually considered in the study of transport in ultrasmall structures are not relevant to our discussion: all currents in the following sections are obtained after integration over all energies, so that thermal averaging due to the spreading of the Fermi-Dirac distribution at nonzero temperatures is already taken into account, making unnecessary the evaluation of the thermal diffusion length l_T.[26] Moreover, double-barrier structures are more extended in the transverse plane than in the longitudinal direction, therefore localization effects are negligible (and the localization length is not relevant).

As we shall show, using a single effective mean free path for taking into account the effects of both the elastic and inelastic mean free paths, transport regimes in the considered structures depend only on the relations between l, the well width w, and the barrier transmission probabilities T_1 and T_2.

A. Double-barrier structure

We refer to the system shown in Fig. 1. The potential $V(x)$ defines two barriers and the well region. Given the electron longitudinal energy E, we can use the transfer matrix technique[19] and the multistep potential approximation[20] to calculate the transmission and reflection probabilities for the whole structure and for each single barrier.

Scattering in the well can be easily accounted for by using in the transfer matrices the complex wave vector $k_i(x) = k(x) + i/2l$, where $k(x) = \{2m[E - V(x)]\}^{1/2}/\hbar$, and m, $\hbar$ are the electron effective mass in the material of the well and the reduced Planck's constant, respectively.

Transport in the single barriers is assumed to be completely coherent; therefore, if T_i and R_i are the tunneling and reflection probabilities for barrier i ($i = 1, 2$), we have
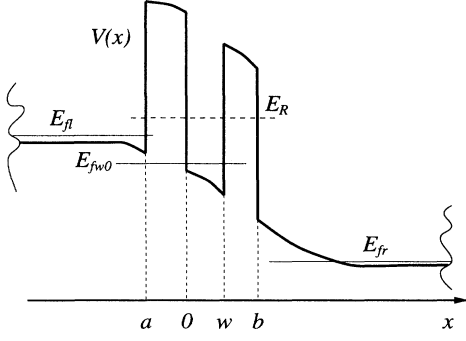
FIG. 1. The one-dimensional potential energy profile $V(x)$ defines the first barrier $(a, 0)$, the well region $(0, w)$, and the second barrier (w, b). E_{fl}, E_{fw0}, E_{fr} are the quasi-Fermi levels in the left electrode, well, and right electrode, respectively.

$T_i + R_i = 1$. On the other hand, collisions in the well make the continuity equation for the probability density current of a given state no longer applicable, and we have $T_{db}^l + R_{db}^l < 1$, where T_{db}^l and R_{db}^l are the transmission and reflection probabilities of the whole double-barrier structure for an electron coming from the left electrode (their expression is derived in Ref. 11). The same relation applies to T_{db}^r and R_{db}^r, where the superscript r stays for the right electrode.

In the absence of magnetic field, from time-reversal symmetry we have $T_{db} \equiv T_{db}^l = T_{db}^r$, while in general $R_{db}^l \neq R_{db}^r$.[21] By means of a general relation between the density of states and dwell times in mesoscopic systems,[22] in Ref. 11 we obtained that, on the assumption of a smooth potential in the well region, the density of states $\rho_w(E)$ in the effective well region (i.e., including states in the well and tail states penetrating the well sides of both barriers) can be written as

$$\rho_w(E) = \frac{1}{\pi\hbar}\tau_{rt}^i \left[\frac{1 - R_{db}^l}{T_1} + \frac{T_{db}}{T_2} \right],\qquad(1)$$

where both spin components have been considered and τ_{rt}^i is the round trip time in the well at the resonant energy. In the following sections we shall show the importance of this formula in unifying the resonant tunneling and the sequential tunneling approaches.

III. APPROACHES TO TRANSPORT IN DOUBLE-BARRIER STRUCTURES

A. Resonant tunneling approach

The introduction of the effective mean free path l has the effect that the current probability density for a given state is not conserved. Electrons that experience a collision seem to be "absorbed" in the well; in fact, they actually emerge with a thermal quasiequilibrium distribution, for which we need to introduce the density of states in the well and a quasi-Fermi energy level.

Let dJ_1 be the contribution to the current through the first barrier due to electrons with longitudinal energies

between E and $E + dE$, referred to the conduction-band bottom of the left electrode. We can write dJ_1 as the sum of three terms, i.e.,

$$dJ_1 = dJ_1^l + dJ_1^w + dJ_1^r,\qquad(2)$$

where dJ_1^l, dJ_1^w, and dJ_1^r are the current contributions due to electrons that have suffered their latest collision (and have emerged with equilibrium energy distribution from) in the left region, the well, and the right region, respectively. Let k_T be the transverse wave vector, and $\rho_T(k_T)$ the density of transversal states. Moreover, let us indicate with $\rho_s(E)$, $f_s(E, k_T)$, $\nu_s(E)$ $(s = l, w, r)$, the one-dimensional density of states for longitudinal energies (including both spin contributions), the occupation factor, and the attempt frequency (i.e., the average number of bounces on each barrier per second), respectively: the subscripts l, w, and r refer to the left electrode, the well, and the right electrode, respectively. We also introduce the quasiequilibrium occupation factor in the well $f_{w0}(E, k_T)$ corresponding to the quasi-Fermi energy E_{fw0}. We can define the integral of the occupation factor over transversal wave vectors $F_s(E)$ $(s = l, w, r, w0)$, as

$$F_s(E) = \int f_s(E, k_T)\rho_T(k_T)dk_T.\qquad(3)$$

Therefore we can write

$$dJ_1^l = q(1 - R_{db}^l)\rho_l F_l \nu_l dE,\qquad(4)$$

$$dJ_1^w = -qT_1\rho_w F_{w0}\nu_w dE,\qquad(5)$$

$$dJ_1^r = -qT_{db}\rho_r F_r \nu_r dE,\qquad(6)$$

where we avoid writing explicitly the dependence of all the terms upon the longitudinal energy E. We also assume that the contacts are "ideal," in the sense that they absorb without reflection all electrons leaving the device, and inject electrons according to the thermal equilibrium distribution. This modeling is implicitly assumed when transport in quantum devices is described as a scattering event,[23] and, as long as we deal with stationary regimes, is applicable without any restriction.

The longitudinal density of states in the left region can be calculated with periodic boundary conditions as[24]

$$\rho_l(E) = \frac{L_l}{\pi\hbar v_l(E)},\qquad(7)$$

where L_l is the length of the left electrode and $v_l(E)$ is the longitudinal velocity corresponding to E. The attempt frequency (corresponding to periodic boundary conditions) is simply $\nu_l(E) = v_l(E)/L_l$; so we have $\rho_l(E)\nu_l(E) = 1/\pi\hbar$. The same considerations apply to the right region, therefore we have $\rho_r(E)\nu_r(E) = 1/\pi\hbar$. The attempt frequency in the well is just the inverse of the time required to complete a round trip of the well, i.e., $\nu_w(E) = 1/\tau_{rt}^i(E)$.

We can write the expression for the current through the second barrier in a similar way. The total current through barrier i $(i = 1, 2)$ is $J_i = \int dJ_i$. In stationary conditions we must have

$$J_1 = J_2. \tag{8}$$

By imposing this equality we can obtain the quasi-Fermi level E_{fw0} in the well.

B. Sequential tunneling approach

According to this approach, electron tunneling through the double barrier is considered as a two-step process, following Weil and Vinter's formalism.[7,9] If the tunneling probabilities of the two barriers are very small, i.e., $T_1, T_2 \ll 1$,[25] we can write $d(J_1)'$ and $d(J_2)'$, the contributions to the current through barriers 1 and 2 due to electrons with energies in the interval $(E, E + dE)$, as

$$d(J_1)' = \frac{\pi q}{\hbar}|M_1|^2 \rho_l \rho_w (F_l - F_w)dE, \tag{9}$$

and

$$d(J_2)' = \frac{\pi q}{\hbar}|M_2|^2 \rho_w \rho_r (F_w - F_r)dE, \tag{10}$$

where M_1 (M_2) is the matrix element for the transition from a state in the left (right) region to a state in the well.[10] We have (detailed derivation is shown in the Appendix)

$$M_1 = \hbar^2 T_1 \nu_l \nu_w, \tag{11}$$

$$M_2 = \hbar^2 T_2 \nu_w \nu_r. \tag{12}$$

In the original model,[6] the two transitions are assumed to occur incoherently, therefore energy relaxation and phase randomization in the well are implicitly accounted for. We can extend this model in order to include the effects of ballistic electrons and of coherent transport by simply considering the occupation factor f_w in the well as a superposition of three partial occupancy probabilities, a quasiequilibrium one, f_{w0}, and the occupancy probabilities f_w^r and f_w^l for electrons coming from the left and right electrodes, which have not been randomized, i.e.,

$$f_w = f_{w0} + f_w^l + f_w^r. \tag{13}$$

If we substitute (13) in (3) and then in (9), we can write the current contribution $d(J_1)'$ of electrons with longitudinal energies between E and $E + dE$ as (2), provided that we choose

$$d(J_1^l)' = q\frac{\pi}{\hbar}|M_1|^2 \rho_l \rho_w (F_l - F_w^l)dE, \tag{14}$$

$$d(J_1^w)' = -q\frac{\pi}{\hbar}|M_1|^2 \rho_l \rho_w F_{w0}dE, \tag{15}$$

$$d(J_1^r)' = -q\frac{\pi}{\hbar}|M_1|^2 \rho_l \rho_w F_w^r dE. \tag{16}$$

In $d(J_1^l)'$, $d(J_1^w)'$, and $d(J_1^r)'$ we have taken into account the contribution of electrons emerging with equilibrium distribution from the left, well, and right region, respectively.

C. Equivalence between the above approaches

In order to verify the equivalence between the resonant tunneling and sequential tunneling approaches we have to make explicit the conditions that guarantee that the values of dJ_1^s ($s = l, w, r$) of (4), (5), and (6) be equal to $d(J_1^s)'$ of (14), (15), and (16), respectively, and that the same relations apply to the corresponding terms for dJ_2 and $d(J_2)'$. Straightforward calculations yield the required conditions:

$$n_w^l \equiv \rho_w F_w^l = \frac{1}{\pi\hbar\nu_w}\frac{T_{db}}{T_2}F_l, \tag{17}$$

$$n_w^r \equiv \rho_w F_w^r = \frac{1}{\pi\hbar\nu_w}\frac{T_{db}}{T_1}F_r, \tag{18}$$

and

$$\rho_w = \frac{1}{\pi\hbar\nu_w}\left[\frac{1 - R_{db}^l}{T_1} + \frac{T_{db}}{T_2}\right]$$
$$\approx \frac{1}{\pi\hbar\nu_w}\left[\frac{1 - R_{db}^r}{T_2} + \frac{T_{db}}{T_1}\right]. \tag{19}$$

As long as (17)–(19) hold true, the two models can be thought of as completely equivalent. It is straightforward to see that conditions (17) and (18) are satisfied: in fact, given $\rho_l \nu_l = 1/\pi\hbar$, we can rewrite (17) as

$$n_w^l \nu_w T_2 = \rho_l F_l \nu_l T_{db}, \tag{20}$$

which can be read as follows: $\rho_l F_l$ is the number of electrons in the left region, ν_l the bounces on the first barrier per second, T_{db} is the coherent tunneling probability, therefore the right term is the number of electrons per second coherently traversing the double barrier. The left term has the same meaning, given that n_w^l is the number of electrons in the well that have come from the left region and have not lost phase coherence.

The condition given by (19) is exactly equal to (1), which has been obtained in Ref. 11, therefore the equivalence of the two approaches has been demonstrated.

IV. TRANSPORT REGIMES THROUGH THE DOUBLE BARRIER

Now, according to (2)–(7) and (19), we have

$$J_1 = \frac{q}{\pi\hbar}\int\left[(1 - R_{db}^l)F_l\right.$$
$$\left. - \left(1 - R_{db}^l + \frac{T_1 T_{db}}{T_2}\right)F_{w0} - T_{db}F_r\right]dE, \tag{21}$$

$$J_2 = \frac{q}{\pi\hbar}\int\left[T_{db}F_l + \left(1 - R_{db}^r + \frac{T_2 T_{db}}{T_2}\right)F_{w0}\right.$$
$$\left. - (1 - R_{db}^r)F_r\right]dE. \tag{22}$$

With these results, following the scheme by Büttiker,[8] we can discuss three relevant transport regimes in double-barrier structures, depending on the amount of inelastic scattering in the well: the coherent transport regime, the sequential regime, and the completely incoherent regime.

A. Coherent transport regime

In this regime the prevalent contribution to the total current comes from "ballistic" electrons, which do not lose phase coherence and energy in the well. This happens if the probability that a particle suffers from collisions in the well is close to zero, i.e.,

$$T_{db} \approx 1 - R_{db}^l \approx 1 - R_{db}^r. \tag{23}$$

This condition is satisfied if the effective mean free path is long enough that almost any particle escapes from the barrier before undergoing scattering events, in other words $w/l \ll \min\{T_1, T_2\}$. Substitution of (23) in (21) and (22) easily yields $F_{w0} = 0$, as it has to be, and

$$J_1 = J_2 = \frac{q}{\pi\hbar} \int_0^\infty T_{db} (F_l - F_r) \, dE. \tag{24}$$

Therefore, according to this result and to (4)–(6), the resonant tunneling approach suits very well this transport regime, by simply taking $dJ_1^w = dJ_2^w = 0$, $dJ_1^l = dJ_2^l$, $dJ_1^r = dJ_2^r$.

B. Sequential transport regime

We call sequential regime the situation in which practically all electrons suffer from collisions before escaping from the well, i.e.,

$$T_{db} \ll 1 - R_{db}^l, \qquad T_{db} \ll 1 - R_{db}^r, \tag{25}$$

and, however, they complete at least a few round trips of the well before escaping, so that the density of states ρ_w is affected by the confinement. As can be seen from (5)–(7) of Ref. 11, it means that $T_1, T_2 \ll w/l < 1$, and implies that both $T_{db}T_2/T_1$ and $T_{db}T_1/T_2$ are much smaller than either $1 - R_{db}^l$ or $1 - R_{db}^r$. Taking into account (25) in (21) and (22) we obtain

$$J_1 = \frac{q}{\pi\hbar} \int (1 - R_{db}^l)(F_l - F_{w0}) dE, \tag{26}$$

$$J_2 = \frac{q}{\pi\hbar} \int (1 - R_{db}^r)(F_{w0} - F_r) dE. \tag{27}$$

Moreover, the density of states in the well given by (19) simply becomes $\rho_w \approx (\pi\hbar\nu_w T_1)^{-1}(1 - R_{db}^l)$, and from (17) and (18) we readily verify that ballistic electron concentrations n_w^l and n_w^r in the well are almost zero. As can be seen from (26) and (27), the double-barrier traversal becomes essentially a two-step process, and any effect of quantum interference is implicitly included in R_{db}^r and R_{db}^l. The sequential tunneling approach suits directly this regime, by simply substituting F_w with F_{w0} in (9) and (10).

C. Completely incoherent transport regime

For higher rates of incoherent processes, i.e., $w/l \gg 1$, we have that

$$1 - R_{db}^l \approx T_1, \qquad 1 - R_{db}^l \approx T_2, \qquad T_{db} \approx T_1 T_2, \tag{28}$$

as can be seen from (6) of Ref. 12. Therefore (26) and (27) become

$$J_1 = \frac{q}{\pi\hbar} \int T_1(F_l - F_{w0}) dE, \tag{29}$$

$$J_2 = \frac{q}{\pi\hbar} \int T_2(F_{w0} - F_r) dE. \tag{30}$$

Correlation between currents through barriers 1 and 2 is only due to the current conservation in the well, i.e., to the position of the quasi-Fermi level in the well. The density of states in the well, as can be seen from (19), becomes $\rho_w \approx (\pi\hbar\nu_w)^{-1}$, i.e., any size effect disappears, and ρ_w is equal to the density of states in the bulk of the material given by (7). The sequential tunneling approach suits easily this regime, by simply putting $F_w = F_{w0}$ and $\rho_w\nu_w = (\pi\hbar)^{-1}$ in (9) and (10).

V. SUMMARY

In this paper we have shown that the sequential tunneling and resonant tunneling approaches, properly extended, are completely equivalent for any rate of collision processes in the well, and can be used to describe the whole range of transport regimes in double-barrier structures. Let us point out again the extensions required.

In the resonant tunneling approach we account for incoherent transport by introducing a current due to electrons scattered in the well and emerging with equilibrium distribution (namely, dJ_1^w and dJ_2^w). Determining this current requires knowledge of the density of states and of the occupancy probability of the states in the well.

In the sequential tunneling description we account for electrons that have not been scattered in the well by means of the "ballistic" distributions n_w^l and n_w^r. Also in this description the density of states and the equilibrium distribution function are fundamental parameters.

The difference between these approaches is in the point of view that is given preference: the former assumes that electrons (except the ones that are inelastically scattered) traverse the structure coherently, and the latter assumes that electrons (except ballistic ones) thermalize in the well and obey an equilibrium energy distribution. Therefore, it is apparent that the resonant tunneling approach is better suited for describing coherent transport, and the sequential tunneling approach fits more directly incoherent transport, when practically every electron undergoes inelastic scattering in the well.

The equivalence of these two models, for the whole range of transport regimes, has been shown on the basis of a recently derived formula for the density of states in a quantum well as a function of the round-trip time

and of transmission and reflection probabilities for the whole structure and for each barrier. Energy relaxation and phase-breaking phenomena are accounted for, in our simple model, by means of a single parameter, the effective mean free path l, which plays the role of a relaxation length.

We wish to point out that while in the paper by Zohta[7] this equivalence was verified only for symmetric barriers, not accounting for energy relaxation in the well, and as long as the Breit-Wigner formalism was applicable, here the equivalence between resonant tunneling and sequential tunneling approaches is demonstrated for any double-barrier structure and for any degree of inelastic scattering, considered as a randomizing agent for both phase and energy.

ACKNOWLEDGMENTS

APPENDIX: MATRIX ELEMENT FOR THE TRANSITION THROUGH A POTENTIAL BARRIER

Let us consider the one-dimensional potential barrier sketched in Fig. 2, separating regions 1 and 2. The potential energy around the turning points a and b has been modified in order to have two flat steps, so that the wave functions on both sides of the potential barrier can be written as a superposition of plane waves. In fact we use a perturbed $V'(x)$ defined as

$$V'(x) = \begin{cases} V(a - \epsilon), & a - \epsilon < x < a + \epsilon \\ V(b + \epsilon), & b - \epsilon < x < b + \epsilon \\ V(x) & \text{otherwise} \end{cases} \quad \text{(A1)}$$

However, the step width 2ϵ can be arbitrarily small, therefore we do not lose generality.

Around a and b, the wave function ψ_1 of an electron of energy E coming from region 1 is

$$\psi_1 = \begin{cases} A_1[e^{ik_1(x-a)} + r^l e^{-ik_1(x-a)}], & a - \epsilon < x < a + \epsilon, \\ A_1 t^l e^{ik_2(x-b)}, & b - \epsilon < x < b + \epsilon, \end{cases} \quad \text{(A2)}$$

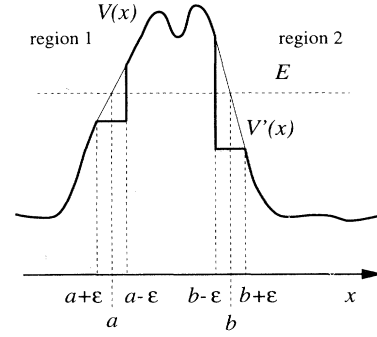where $k_1 = \{2m[E - V'(a)]\}^{1/2}/\hbar$, $k_2 = \{2m[E -$

FIG. 2. The one-dimensional potential energy $V(x)$ (fine line) separates regions 1 and 2. The potential energy profile $V'(x)$ (thick line) is equal to $V(x)$ except near the turning points a and b, where is has been modified in order to have flat steps, so that the wave functions on both sides of the potential barrier can be written as a superposition of plane waves: $V'(x) = V(a - \epsilon)$ for $a - \epsilon < x < a + \epsilon$, and $V'(x) = V(b + \epsilon)$ for $b - \epsilon < x < b + \epsilon$.

$V'(b)]\}^{1/2}/\hbar$, and r^l, t^l are the reflection and transmission coefficients of the barrier for an electron coming from the left. If ψ_2 is the wave function of an electron at the same energy in region 2 we can write

$$\psi_2 = \begin{cases} A_2 t^r e^{-ik_1(x-a)}, & a - \epsilon < x < a + \epsilon, \\ A_2[e^{-ik_2(x-b)} + r^r e^{ik_2(x-b)}], & b - \epsilon < x < b + \epsilon, \end{cases} \quad \text{(A3)}$$

where r^r and t^r are the reflection and transmission coefficients for an electron coming from the right. From Bardeen[10] we can calculate the matrix element from the transition between regions 1 and 2 at energy E, as

$$\begin{aligned} |M|^2 &= \frac{\hbar^4}{4m^2} |\psi_1^* \nabla \psi_2 - \psi_2 \nabla \psi_1^*|^2 \Big|_{x=a} \\ &= \hbar^2 T J_{1\text{inc}} J_{2\text{inc}}, \end{aligned} \quad \text{(A4)}$$

where $T = |t^r|^2 k_2/k_1$ is the transmission probability of the barrier; $J_{1\text{inc}}$ ($J_{2\text{inc}}$) is the probability current incident on the barrier associated to the state ψ_1 (ψ_2); for instance, from (A2) we find that $J_{1\text{inc}} = |A_1|^2 \hbar k_1/m$; however, $J_{1\text{inc}}$ is simply obtained as the integral of the probability density in region 1 times the attempt frequency on the barrier; i.e., if ψ_1 is normalized to unity, $J_{1\text{inc}} = \nu_1$. If we apply the same result to $J_{2\text{inc}}$ we can eventually write

$$|M|^2 = \hbar^2 T \nu_1 \nu_2. \quad \text{(A5)}$$

* Fax number: ++39-50-568522. Electronic address: ianna@pimac2.iet.unipi.it

[1] R. Tsu and L. Esaki, Appl. Phys. Lett. **22**, 562 (1973).

[2] L. L. Chang, L. Esaki, and R. Tsu, Appl. Phys. Lett. **24**,

593 (1974).

[3] *Physics of Quantum Electron Devices*, edited by F. Capasso (Springer-Verlag, Berlin, 1990).

[4] A. D. Stone and P. A. Lee, Phys. Rev. Lett. **54**, 1196

(1985).

[5] M. Jonson and A. Grincwajg, Appl. Phys. Lett. **51**, 1729 (1987).

[6] S. Luryi, Appl. Phys. Lett. **47**, 490 (1985).

[7] Y. Zohta, Jpn. J. Appl. Phys. **32L**, 177 (1993).

[8] M. Büttiker, IBM J. Res. Dev. **32**, 63 (1988).

[9] T. Weil and B. Vinter, Appl. Phys. Lett. **50**, 1281 (1987).

[10] J. Bardeen, Phys. Rev. Lett. **6**, 57 (1961).

[11] G. Iannaccone and B. Pellegrini, Phys. Rev. B (to be published).

[12] Y. Zohta, Solid State Commun. **72**, 931 (1989).

[13] S. Hershfield and V. Ambegaokar, Phys. Rev. B **38**, 7909 (1988).

[14] M. J. Laughton, J. R. Barker, J. A. Nixon, J. H. Davies, Phys. Rev. B **44**, 1150 (1991).

[15] M. Büttiker, Phys. Rev. B **33**, 3020 (1986).

[16] Yuming Hu, J. Phys. C **21**, L23 (1988).

[17] Y. Zohta, Solid State Commun. **72**, 845 (1990).

[18] A. Knäbchen, Phys. Rev. B **45**, 8542 (1992).

[19] P. Erdös and R. C. Herdnon, Adv. Phys. **31**, 64 (1983).

[20] Y. Ando and T. Itoh, J. Appl. Phys. **61**, 1497 (1987).

[21] See, for instance, A. Messiah, *Quantum Mechanics* (North-Holland, Amsterdam, 1962), Vol. II, p. 831.

[22] G. Iannaccone, Phys. Rev. B **51**, 4727 (1995).

[23] M. Büttiker, Phys. Rev. B **32**, 1846 (1986).

[24] Equation (7) is a simple textbook expression; see, for example, W. A. Harrison, *Solid State Theory* (McGraw-Hill, New York, 1970), pp. 268–269, and also Ref. 15.

[25] J. J. Sakurai, *Modern Quantum Mechanics* (Addison-Wesley, Redwood, 1985), pp. 325–333.

[26] C. W. J. Beenakker and H. van Houten, in *Solid State Physics*, edited by H. Ehrenreich and D. Turnbull (Academic, Boston, 1991), Vol. 44, pp. 7 and 53.