Numerical Investigation of Shot Noise between the Ballistic and the Diffusive Regime

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Numerical Investigation of Shot Noise between the Ballistic and the Diffusive Regime

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Abstract. We investigate shot noise suppression in several mesoscopic structures by means of a numerical approach based on the computation of the transmission matrix with the recursive Green's function method. We retrieve the "universal" values of the suppression factor obtained with random matrix theory for chaotic cavities and diffusive conductors. We then extend the investigation to more complex structures, such as multiple cascaded cavities and partially diffusive systems, and discuss the consequences on the shot noise suppression factor. Finally, we analyze the behavior of shot noise in an electron waveguide containing a large number of scatterers as the spatial position of the scatterers is changed from a regular array to a random distribution.

Keywords: shot noise, mesoscopic, chaos, ballistic

1. Introduction

During the last few years remarkable theoretical (Lesovik 1989, Büttiker 1990, Beenakker and Büttiker 1992, Jalabert, Pichard and Beenakker 1994, González *et al.* 1998) and experimental (Kumar *et al.* 1996, Liefrink *et al.* 1994, Oberholzer *et al.* 2001) results have drawn significant attention to the issue of shot noise suppression in mesoscopic conductors. The most recent theoretical work in this field has been based on the random matrix approach (RMT), which has allowed prediction of the shot noise suppression down to $1/3$ of the full shot value in diffusive conductors (Beenakker and Büttiker 1992) and of the suppression down to $1/4$ in chaotic ballistic cavities (Jalabert, Pichard and Beenakker 1994). The RMT approach is quite powerful, but it cannot be easily extended to generic geometries; we have been interested in expanding the investigation of shot noise suppression to arbitrary mesoscopic structures, and, to this purpose, we have developed a numerical method based on an optimized recursive Green's function technique. With this method, we can treat generic structures, with the inclusion of the effects of atomistic distributions of dopants leading to a diffusive regime, and we can handle situations with a few hundreds of propagating modes. It is possible to show that the "universal" suppression factors $1/3$ and $1/4$ are easily retrieved, respectively, for a conductor with a large enough density of elastic scatterers and for a structure with a symmetric cavity with small enough input and output apertures. We study shot noise in nanostructures containing single and multiple cascaded cavities, noticing that the shot noise suppression is substantially independent of the number of cavities, and then take into consideration the case in which one of the cavities is filled with randomly distributed scatterers, arguing, on the basis of a simple circuit analogy, why the shot noise reduction factor becomes the same as for purely diffusive conductors. Finally, we investigate the transition that shot noise suppression in an electron waveguide containing a large number of scatterers undergoes as we move from a regular spatial distribution of such scatterers to a random distribution.

2. Model

Although our approach is general and can be applied to an arbitrary potential landscape, we consider, for the sake of computational simplicity, a device geometry defined by hard walls, with obstacles and boundaries characterized by right angles. The transmission matrix t, whose elements represent the transmission coefficient from each input mode to each output mode, is computed by means of the recursive Green's function approach (Sols *et al.* 1989, Macucci, Galick and Ravaioli 1995), which has been specifically optimized to guarantee sufficient numerical precision when handling up to a thousand of the slices characterized by constant transverse potential into which the structure has to be subdivided for the calculations that we will be presenting. Once t has been obtained, we compute the transmission coefficients in a representation in which the transmission matrix is diagonal, multiplying t by its hermitian conjugate $t^\dagger$ and finding the eigenvalues T_i of $tt^\dagger$. Following Lesovik (1989) the shot noise power density can be written as

$$S_I = 4\frac{q^2}{h}|qV|\sum_i T_i(1-T_i), \tag{1}$$

where h is the Planck constant, q is the electron charge and V is the applied voltage. Since the power spectral density of full shot noise is

$$S_{I_f} = 2qI = 2q\frac{2q^2}{h}|V|\sum_i T_i, \tag{2}$$

we can conclude that the Fano factor γ, i.e. the ratio of the actual shot noise power spectral density to the full shot noise, is given by

$$\gamma = \frac{\sum_i T_i(1-T_i)}{\sum_i T_i}, \tag{3}$$

which can be immediately evaluated once the T_i coefficients are known.

3. Numerical Results

We have first investigated the shot noise suppression in chaotic cavities (defined by apertures that are much narrower than the cavities themselves), retrieving (Macucci, Iannaccone and Pellegrini 2001) the value of 1/4 for the Fano factor, as predicted by Jalabert *et al.* (1994), if the number of propagating modes is larger than about 20. We have then studied a more complex structure, made up of two cascaded cavities, each 5 μm long, created in an electron waveguide with a width of 5 μm by delimiting them with diaphragms 250 nm thick and 1 μm wide, as shown in the inset of Fig. 1. We report the Fano factor for this structure in Fig. 1 as

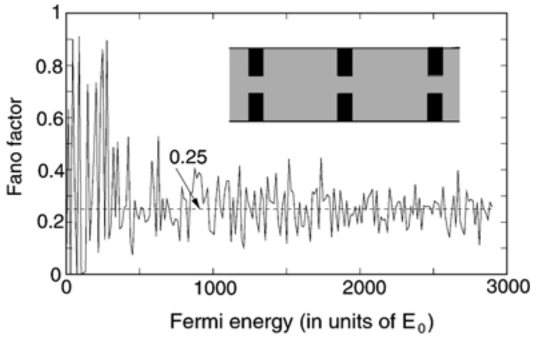

Figure 1. Fano factor for two cascaded chaotic cavities as a function of the Fermi energy, expressed in units of the threshold E_0 for propagation of the lowest mode in the empty waveguide. The inset contains a graphic representation of the confinement potential.

a function of the Fermi energy (expressed as a multiple of the threshold energy E_0 for propagation of the lowest mode in the empty waveguide), and notice that the average value is around 0.25, as in the case of a single cavity. The structures we are studying are relatively large, in order to allow propagation of a sufficiently large number of modes, to be in the regime in which the "universal" suppression factors are meaningful (Beenakker and Büttiker 1992).

We have also computed the Fano factor for three cascaded cavities, obtaining results that, although with larger fluctuations, are almost coincident with those for two cavities. The same happens for larger numbers of cascaded cavities, and even if we include intermediate diaphragms with different widths, as long as the rightmost and the leftmost apertures are symmetric. We notice that the actual shot noise suppression factor fluctuates rather widely as a function of the Fermi energy for all of the numerical results, and equals the asymptotic value predicted by random matrix theory only on the average.

A qualitatively different behavior is however observed if at least one of the cascaded cavities is filled with randomly distributed obstacles, which lead to a complex scattering pattern and to transport in the diffusive regime, i.e. a condition in which the elastic mean free path is much smaller than the device dimensions. In Fig. 2 we report the noise power spectral density as a function of the Fermi energy for two cascaded cavities, each with a length of 5 μm and a width of 5 μm, delimited by constrictions that are 1 μm wide and 0.25 μm long. Within the cavity region we have included 200 randomly distributed hard-wall 56.2 nm $\times$ 50 nm obstacles. Although the cavity is delimited by symmetric apertures, the Fano factor moves up to slightly less

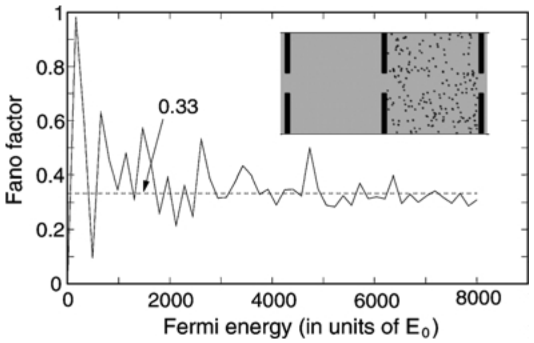

Figure 2. Fano factor for two cascaded chaotic cavities, one of which is filled with randomly distributed scatterers, as a function of the Fermi energy, expressed in units of the threshold E_0 for propagation of the lowest mode in the empty waveguide. The inset contains a graphic representation of the confinement potential.

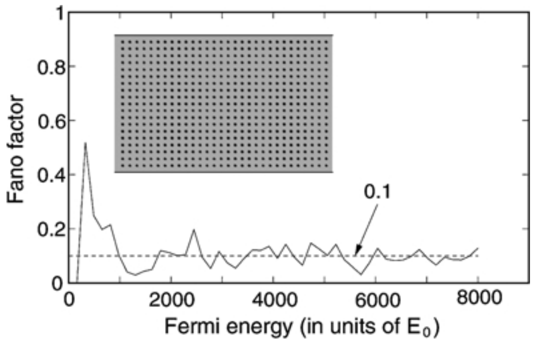

Figure 3. Fano factor for an electron waveguide filled with a square lattice of scatterers, as a function of the Fermi energy, expressed in units of the threshold E_0 for propagation of the lowest mode in the empty waveguide. The inset contains a graphic representation of the confinement potential.

than 1/3, significantly departing from the 1/4 result and reaching a typically diffusive behavior. The inset in the figure shows the device geometry, with the position of the obstacles.

An extremely simplified interpretation of this behavior can be derived from a circuit analogy. Let us consider a series of two current noise sources, with power spectral densities S_{I_1} and S_{I_2}, providing contributions of the same order of magnitude (as they are both of shot origin and share the same average current) and associated with different resistances R_1 and R_2, each of which is in parallel with the corresponding current noise source. If we want to determine the current noise power spectral density $S_{I_{out}}$ they produce on an external load R, we obtain $S_{I_{out}} = (S_{I_1}R_1^2 + S_{I_2}R_2^2)/(R_1 + R_2 + R)^2$, therefore the predominant contribution is the one associated with the larger resistance, which in our case corresponds to the diffusive region. Clearly, this is not an exact analogy, because the electron waveguide sections do not rigorously correspond to circuit elements in series, although the presence of a diffusive region has a strongly decoupling action between the different sections.

Another interesting aspect of the transition from ballistic to diffusive transport can be observed by applying our computational method to a quantum wire containing scatterers and looking at the dependence of the shot noise suppression factor on the position of such scatterers. If we have a regular pattern of scatterers, arranged in a square lattice, it has been shown (Macucci in press) that, at least for relatively small numbers of scatterers, shot noise is suppressed by a factor increasing with the portion of the waveguide surface occupied by the scatterers and saturating around 0.16. On the other hand, we know that, for a large number of randomly positioned scatterers, shot noise is suppressed by the universal factor 1/3 (Macucci, Iannaccone and Pellegrini 1999). We have performed a calculation of the Fano factor for a section of electron waveguide containing a very large number of square obstacles (570), each with a side 200 times smaller than the waveguide width, for two cases differing for the spatial arrangement of the scatterers, but not for their density. In one case we have a regular square lattice, with 19 rows and 30 columns, in the other case we generate the coordinates of the scatterers as randomly distributed variables over the same region of space. Results are shown in Fig. 3 (for the square lattice) and in Fig. 4 (for the random case), in which we report the Fano factor as a function of the Fermi energy, expressed as a multiple of the energy for propagation of the lowest mode in the empty waveguide. Each figure

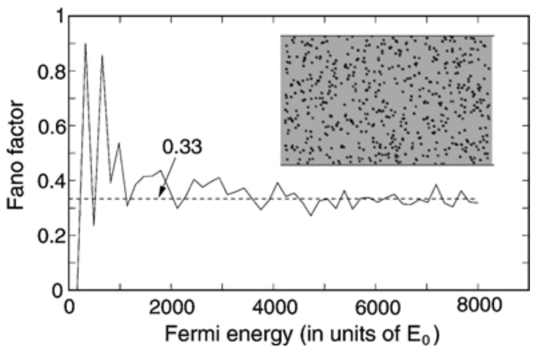

Figure 4. Fano factor for an electron waveguide filled with randomly distributed scatterers, as a function of the Fermi energy, expressed in units of the threshold E_0 for propagation of the lowest mode in the empty waveguide. The inset contains a graphic representation of the confinement potential.

contains an inset showing the position of the obstacles within the waveguide. It is apparent that, although the density of scatterers is the same in the two cases, the noise suppression sharply differs: for the regular lattice we observe an average value of the Fano factor around 0.1, which, considering the relatively low scatterer-to-waveguide area ratio, is in good agreement with the results obtained in Macucci (in press); when, instead, scatterers are distributed randomly, the $1/3$ "universal" suppression factor predicted by random matrix theory (Beenakker and Büttiker 1992) is immediately retrieved.

4. Conclusions

We have investigated shot noise suppression in mesoscopic conductors in a regime that varies from ballistic, with the inclusion of simple scattering geometries, to diffusive, observing how the shot noise suppression factor varies and fluctuates around the "universal" values $1/4$ and $1/3$ for the chaotic cavities and for the diffusive regime, respectively. We have also observed that the $1/4$ suppression factor is not influenced significantly by the characteristics and number of cascaded chaotic cavities, as long as the leftmost and rightmost apertures are of the same width. Furthermore, we have shown that the presence of a diffusive region within an electron waveguide leads to a Fano factor around $1/3$ with little influence from the other geometrical details of the structure, and we have justified this result on the basis of a simple circuit analogy. Finally, we have investigated the change in the shot noise suppression factor in an electron waveguide, as the position of a large number of scatterers is varied from regular to random without varying their spatial density: a transition is observed from transport in a periodic structure to the diffusive regime. Further work is planned to better understand this transition as the scatterer arrangement is gradually changed from regular to random and as a function of the actual statistical distributions used for the scatterer coordinates.

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