Quantum-Mechanical Simulation of Shot Noise in the Elastic Diffusive Regime

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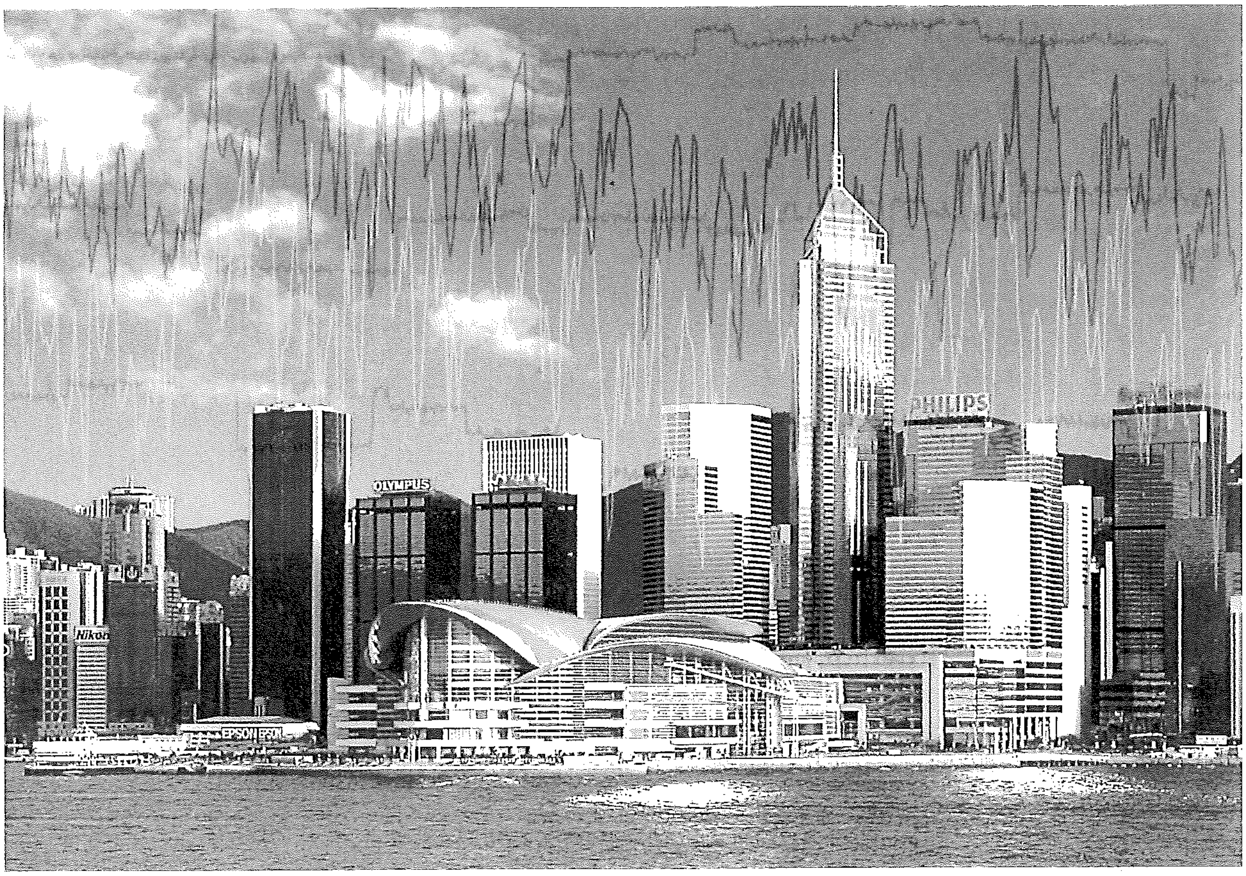

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Shot noise in the elastic diffusive regime has been computed numerically for several electron waveguides with a random distribution of obstacles. In particular, the shot noise suppression factor has been calculated as a function of conductor length, density and height of obstacles, number of propagating modes, and the results have been compared with existing analytical theories.

1 Introduction

Electron transport is in the elastic diffusive regime if the length L of the conductor is much larger than the elastic scattering length l and much smaller than the inelastic scattering length l_i, so that inelastic scattering occurs only in the reservoirs. In this regime, a suppression of shot noise by a factor of one third has been predicted [1, 2] and experimentally measured in a not completely conclusive way [3]. In Ref. [1] the result has been obtained with a coherent quantum-mechanical model: according to random matrix theory the transmission eigenvalues have a bimodal distribution which leads to the 1/3 suppression factor, if $l \ll L \ll Nl$, where N is the number of propagating modes. On the other hand, the same suppression factor has been obtained by Nagaev [2] on the basis of a semiclassical one-dimensional model which includes the exclusion principle.

In both cases, the Coulomb interaction between electrons is not included, therefore the correlations responsible for the suppressed shot noise are those introduced by the exclusion principle. While the relationship between the results of both models has been investigated and justified to some detail [4, 5], some authors [6, 7] consider the same 1/3 factor arising with both models a mere numerical coincidence.

In this paper, we address the problem of suppressed shot noise in elastic diffusive conductors with a numerical simulation of a coherent quantum-mechanical model. The conductor is represented by an electron waveguide in which several obstacles are randomly placed. With respect to analytical models [1] this approach has the advantage of allowing an extension of the investigation to values of transport parameters beyond the range in which the approximations leading to the analytical results are valid ($l \ll L \ll Nl$). In addition, it allows a transition to a semiclassical model without the need of complex mathematical tools.

2 Model

We have represented the conductor as a 2-dimensional GaAs wire with a width of 200 nm, defined by hard walls and containing a uniform random distribution of scatterers. Each scatterer corresponds to an obstacle with a height u and occupies a square area 12 nm by 12 nm. The resulting potential landscape is represented in Fig. 1, where the x direction corresponds to that of current flow: black squares indicate the obstacles, which may also partially overlap. The coordinates of each obstacle are generated as a pair of random numbers, with the y coordinate uniformly distributed between zero and the wire width (200 nm) and the x coordinate uniformly distributed between zero and the wire length.

In order to evaluate the shot noise level and the conductance associated with the considered structure, we need to determine the transmission matrix t, containing the complex transmission coefficients from each input mode to each output mode [8]. This task is accomplished by means of the recursive Green's function technique [9, 10]: the

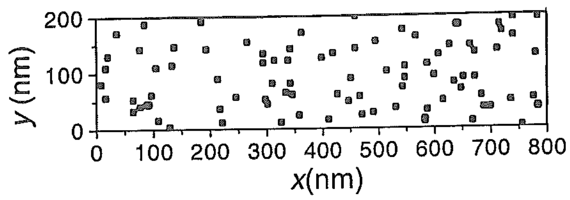

Figure 1: Structure of a conductor with L = 800 nm, W = 200 nm, and 90 randomly distributed obstacles

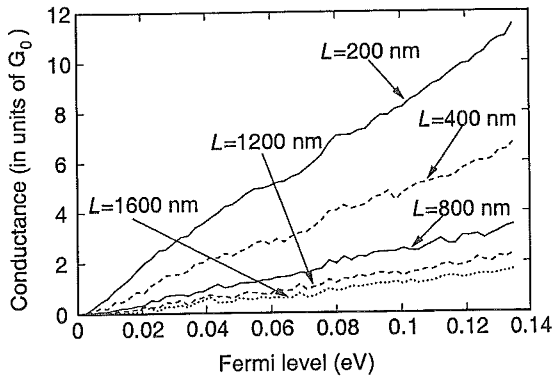

Figure 2: Normalized conductance as a function of the Fermi level for different conductor lengths. Each conductors is obtained from a portion of an electron wave guide of length 1600 nm and width 200 nm containing 180 obstacles. Results shown are averaged over 15 random distributions of obstacles.

quantum wire is subdivided into a number of transverse slices, in each of which the transverse confinement potential can be assumed constant, then the Green's functions for the Schrödinger equation are obtained analytically for each section, assumed to be isolated from the others. There is no mode mixing within each section, due to the longitudinal invariance of the confining potential, and coupling between the different modes takes place only at the interfaces between adjacent sections. By means of a simple procedure derived from the Dyson equation [9] it is possible to obtain the Green's functions for two coupled sections from the knowledge of those for the isolated sections. Such a procedure can be repeated recursively from one end of the conductor to the other, so that the Green's functions for the whole structure can be computed, and from them the transmission matrix is straightforwardly obtained [9].

The current noise power spectrum is given by [8]:

$$S_I(0) = 4(q^2/h)|qV| \sum T_n(1 - T_n), \quad (1)$$

where q is the electron charge, h is Planck's constant, V the voltage applied across the conductor and T_n are the eigenvalues of the matrix $t^\dagger t$. The average current flowing in the conductor reads

$$I = VG = V(\frac{2q^2}{h}) \sum T_n, \quad (2)$$

where G is the conductance, as given by the Landauer formula. Considering that "full" shot noise has a power spectral density $S_{FS} = 2qI$, the suppression factor γ for the resulting noise is given by:

$$\gamma = \frac{S_I(0)}{S_{FS}} = \frac{\sum T_n(1 - T_n)}{\sum T_n}, \quad (3)$$

3 Results

In Fig. 2 the conductance G at equilibrium is plotted as a function of the the Fermi level E_f for different conductor lengths. Each conductor is obtained as a portion of an electron waveguide with a length L = 1.6 μm and width W = 200 nm in which 180 rectangular obstacles with height 1 eV are randomly distributed. Each curve is obtained by averaging the results from 15 random distributions of obstacles.

It is easy to show that transport occurs in regime of validity of the Drude model. In this regime the conductance is given by [4]

$$G = \frac{G_0 N}{L} \frac{\pi l}{2}, \quad (4)$$

where G_0 is the conductance quantum $2e^2/h$. Fig. 2 shows very clearly that G is inversely proportional to the conductor length L (only the shortest conductor departs somewhat from this behavior). We have also checked that the conductance is proportional to the number of propagating modes N. Using Eq. (4) with the data shown in Fig. 2 one obtains l = 40.7 nm, which is in good agreement with an estimate of l from $\sqrt{WL/n_o}$ = 42.2 nm, where n_o is the total number of obstacles in the conductor.

In Fig. 3 the shot noise suppression factor γ is plotted as a function of E_f for the cases described above. As can be seen, shot noise increases with increasing conductor length, but seems to saturate to a value γ around 1/3. Moreover, for

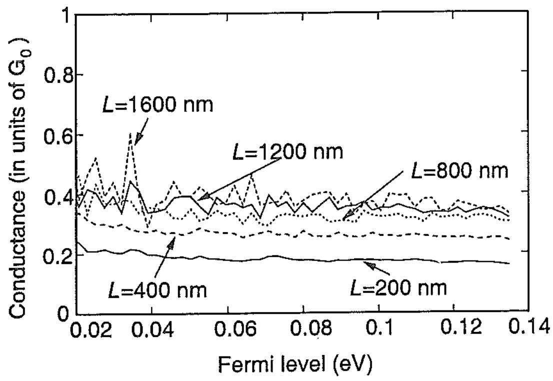

Figure 3: Shot noise suppression factor as a function of the Fermi level for the same cases shown in Fig. 2

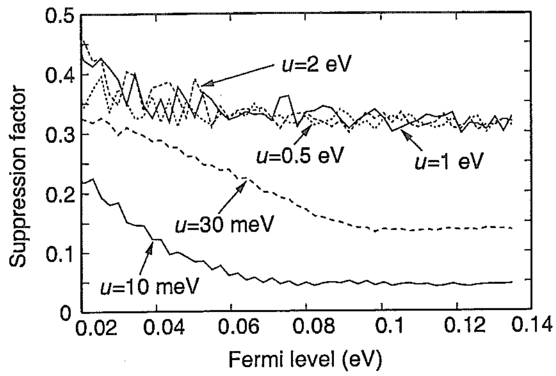

Figure 4: Shot noise suppression factor as a function of the Fermi level for a conductor of length 800 nm and width 200 nm in which 90 obstacles of height u are randomly distributed.

a given conductor length, γ reaches a saturation value with increasing energy (and, therefore, number of modes). This can be explained with the fact that as the electron wavelength becomes smaller than both the obstacle size and l the system behaves classically: the distribution of transmission coefficients is therefore independent from the electron energy, as long as the obstacles are sufficiently high. For $E_f = 0.06$ eV the electron wavelength in GaAs is 9.5 nm, already smaller than the obstacle size and l.

The effect of the height of the obstacles can be assessed from Fig. 4: the case is considered of a conductor with $L = 800$ nm and $W = 200$ nm, in which 90 obstacles are randomly distributed; the height u of the obstacles ranges from 0.01 eV to 2 eV. If the electron energy is larger than u transport is very poorly affected by the presence of the obstacles, and shot noise is almost totally suppressed. On the other hand, if u is much higher than the electron energy, the scattering properties of the obstacles — and therefore the noise suppression factor — are substantially independent of u. Also in this case the results shown are averaged over 15 random distributions of obstacles.

Fig. 5 shows the dependence of the shot noise suppression factor on the number of obstacles for $L = 800$ nm and $W = 200$ nm. Again, it is evident that γ saturates with energy to a value which increases with increasing number of obstacles. These results does not allow us to draw conclusions on the $\gamma = 1/3$ hypothesis: the deviation from $1/3$ could depend on the fact that the condition $l \ll L \ll Nl$ is not fully fulfilled, since for 30 obstacles $l \approx 73$ nm, which is not much

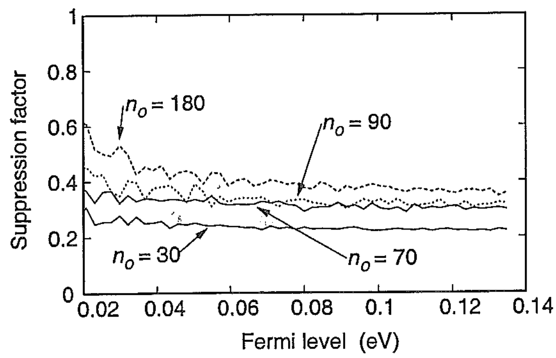

Figure 5: Shot noise suppression factor as a function of the Fermi level for a conductor of length 800 nm and width 200 nm and n_0 obstacles of height 2 eV.

smaller than L, while for 180 obstacles $l \approx 30$ nm and N in our calculations is at most 46, therefore $Nl \leq 1380$, which is not much larger than L.

Finally, in Fig. 6 we report the probability density of transmission eigenvalues, computed for the conductor just described with 180 obstacles, averaged over 754 random distributions of obstacles. The two peaks near $T = 0$ and $T = 1$ appear clearly. For the sake of comparison, the probability density obtained from random matrix theory [1, 5] is also plotted, and shows a very good agreement with the numerical calculation.

4 Discussion

The main result of this paper is the clear evidence of saturation of the shot noise suppression factor with increasing electron energy. As energy increases, the classical picture progressively be-

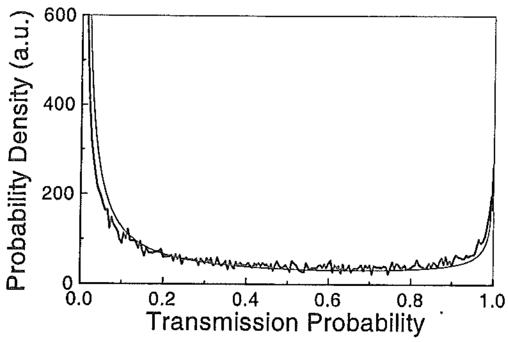

Figure 6: Probability density of transmission eigenmodes calculated for $L = 800$ nm, $W = 200$ nm, $n_o = 180$, $E_f = 0.135$ eV, averaged over 754 random distributions of obstacles (thick line) and those derived in Ref. [4] from random matrix theory (thin line).

comes adequate for describing transport in an elastic diffusive conductor. For this reason, it is not surprising that both quantum-mechanical and semi-classical models provide the same result for shot noise suppression.

On the other hand, the results shown are not sufficient to let us draw any final conclusion on the 1/3 "universal" suppression factor. Indeed, our results show that the γ factor increases with increasing density of obstacles, however without a clear saturation to 1/3. The reason for such a behavior could be that when l is very small the number of propagating modes considered is not sufficient to satisfy $L \ll Nl$, which is required in the analytical derivation; on the other hand, γ seems to saturate in many cases with increasing number of modes. Further investigation is needed on this issue, enlarging the size of our model from the point of view of the number of propagating modes and the number of obstacles.

5 Acknowledgments

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