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NOISE IN PHYSICAL SYSTEMS AND 1/f FLUCTUATIONS

EFFECTS OF DISPLACEMENT CURRENT ON QUANTUM NOISE

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ABSTRACT

The electrokinematics theorem is applied to obtain the second quantization operator of the total current which allows us to take into account the displacement current and its effects on noise in ballistic mesoscopic devices. We show that they become appreciable for frequencies greater than the ratio of the transmission probability T to the particle dwell time τ_D in the device. This ratio, for devices with low T and/or long τ_D, may become smaller than the other characteristic frequencies of the noise spectrum.

INTRODUCTION

Very often displacement current effects on noise are neglected, even at extremely high frequencies for which such a quasistatic approximation does not hold true.

According to the Ramo-Shockley theorem [1] and, more in general, to the electrokinematics theorem [2,3], the motion of a charge carrier induces a current pulse at the device terminals, lasting the traversal time T_t across the device. Therefore, according to Carson's theorem [4], if the current pulses are independent of one another, full shot noise will be observed whose spectral density is constant for frequencies much lower than $1/T_t$ while, for higher frequencies, it depends on the current pulse shape, i.e., on the effects of displacement current.

In ballistic quantum devices, neglecting the displacement current leads to difficulties in defining the value of the current (because the conduction current depends on position) and limits the validity of the results to low frequencies [5,6].

The electrokinematics theorem [2] and its quantum mechanical extension [3], as we shall show here, allow us to solve these problems and to reach new results. As a matter of fact, it allows us to obtain a second quantization operator for the total current, and, unlike the preceding models [5-7], to take into account the displacement current too and its effects on noise in ballistic mesoscopic devices. We show that such effects become appreciable for frequencies greater than the ratio of the transmission probability T through the device to the particle dwell time τ_D. These ratio can be rather

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low in resonant-tunneling double-barrier structures (very high τ_D) or in very opaque single barriers (very low T).

SECOND QUANTIZATION OF THE TOTAL CURRENT

According to the electrokinematics theorem, the total current i across an electrode with surface S_2 of a two electrode device of volume Ω, total surface S and electric permittivity ϵ_r, is

$$i = \int_\Omega \mathbf{F} \cdot \mathbf{J}_c d^3r + \int_{S_A} \Phi \mathbf{J}_d \cdot d\mathbf{S} - \frac{1}{4\pi} \int_S \epsilon_r \frac{\partial A_0}{\partial t} \mathbf{F} \cdot d\mathbf{S}, \tag{1}$$

where $\mathbf{J}_c$ and $\mathbf{J}_d = -\epsilon_r(\partial \nabla A_0/\partial t)$ are the conduction and the displacement currents, respectively, $\mathbf{F} = -\nabla \Phi$ is an irrotational field such that $\nabla \cdot (\epsilon_r \mathbf{F}) = 0$ in Ω, being $\Phi = 1$ on S_2 and $\Phi = 0$ on the surface S_1 of the other electrode, $S_A = S - S_2 - S_1$ is the lateral surface, on which $\mathbf{J}_c = 0$, and A_0 is the scalar potential.

If the electrodes are kept at a constant potential, also the third integral has to be evaluated only on S_A and it becomes zero if $\mathbf{F}$ is chosen tangent to S_A in every point. On the contrary, it is the second integral that becomes zero if we choose $\Phi = 1$ on S_2 and $\Phi = 0$ elsewhere on S.

In any case the displacement current across the lateral surface of the device gives a contribution to the total current i, to the energy exchange with the external world and to noise. If such contributions become negligible, which is the case if $S_A \ll S_2$, and if the voltage V between the electrodes is constant, from (1) we have

$$i = \int_\Omega \mathbf{F} \cdot \mathbf{J}_c d^3r = -\int_{S_2} \mathbf{J}_c \cdot d\mathbf{S} - \int_\Omega \Phi \frac{\partial \rho}{\partial t} d^3r. \tag{2}$$

While the second integral gives the conduction current across S_2, the third one represents the contribution from the displacement current, due to local fluctuations of the charge density ρ. This term becomes zero only for the angular frequency $\omega \to 0$.

By expressing $\mathbf{J}_c$ and ρ in terms of the wave function Ψ of the particle system, from (2) we obtain a quantum mechanical expression of the current [3], which takes into account the displacement current effects through ρ.

If, for ballistic mesoscopic devices, we express Ψ, $\mathbf{J}_c$ and ρ in terms of the creation and annihilation operators $\hat{a}^\dagger$ and $\hat{a}$ [8], respectively, from (2) we obtain the current operator $\hat{i}$ in the second quantization form

$$\hat{i} = e \sum_{lm} \delta_{\sigma\xi} H_{lm} \exp[j(\omega_l - \omega_m)t]\hat{a}_l^+ \hat{a}_m, \tag{3}$$

in which we have $H_{lm} = A_{lm} + j(\omega_l - \omega_m)R_{lm}$, with

$$A_{lm} = \frac{j\hbar}{2m} \int_{S_2} [\psi_l^* \nabla \psi_m - (\nabla \psi_l^*)\psi_m] \cdot d\mathbf{S}, \qquad R_{lm} = -\int_\Omega \Phi \psi_l^* \psi_m d^3r, \tag{4}$$

where e and m are the particle charge and mass, respectively, and $\hbar\omega_l$ and ψ_l are the eigenvalue and the eigenfunction of the Hamiltonian, respectively, in state l, l being the set of quantum numbers characterising the state, including the spin σ and ξ.

NOISE

The noise spectral density $S(\omega)$ of the current can be obtained by taking the Fourier transform of its autocorrelation function $\langle \hat{i}(t)\hat{i}(0) + \hat{i}(0)\hat{i}(t)\rangle/2 - \langle \hat{i}\rangle^2$ [5]. By taking into account that $H_{\mathrm{lm}} = H^*_{\mathrm{ml}}$ and by defining $F_{\mathrm{lm}} \equiv f_l(1 - pf_{\mathrm{m}}) + f_{\mathrm{m}}(1 - pf_l)$, where f_l (f_{m}) is the occupation factor of the state l (m) and $p = 1$ ($p = -1$) for fermions (bosons), for $\omega \geq 0$ we get

$$S(\omega) = 2\pi e^2 \sum_{\mathrm{lm}} \delta_{\sigma\varsigma} F_{\mathrm{lm}} |H_{\mathrm{lm}}|^2 \delta[\omega + (\omega_l - \omega_{\mathrm{m}})]. \tag{5}$$

If all of the eigenfunctions of the Hamiltonian can be factorized into a transverse and a longitudinal component, then any state l can be characterized by a subset of parameters l', which takes into account the spin and the transverse wave vector, by the electrode (a or b) from which the particle enters Ω, and by the longitudinal energy E. According to (4) and (5), only the states l and m characterised by the same subset l' of parameters associated to the spin and to the transverse wave vectors contribute to the noise. Therefore, we have

$$\begin{aligned} S(\omega) = &\frac{e^2}{h} \sum_{ab} \sum_{l'} \int dE \frac{F_{ab}(E, E + \hbar\omega)}{v_a(E)v_b(E + \hbar\omega)} \\ &\times |A_{ab}(E, E + \hbar\omega) + j\omega R_{ab}(E, E + \hbar\omega)|^2, \end{aligned} \tag{6}$$

where $v_a(E)$ is the longitudinal velocity of the particle in the state of longitudinal energy E, entering from the electrode a, and the other quantities are obtainable from the previous definitions by replacing l (m) with a (b). We have dropped, for notational convenience, the dependance on l' of all the quantities in (6).

As a check, we observe that (6), for $V = 0$ and $\omega = 0$, correctly gives the thermal noise spectral density, while for $V \neq 0$ and $\omega = 0$ it leads to the results obtained by other authors [6,7].

On the contrary, for $\omega \neq 0$, (6) contains new contributions associated with ωR_{ab} which is due to the displacement current.

Let us see how these contributions affect the noise spectral density $S(\omega)$. The power series expansion of $S(\omega)$ up to the second order in ω contains terms of the type

$$S_{aa}(\omega) = \frac{e^2}{h} \sum_{l'} \int dE \frac{F_{aa} A_{aa}^2}{v_a^2}(1 + \tau_{aa}^2 \omega^2), \tag{7}$$

where the quantities are those of (6) computed for $\omega = 0$ and $\tau_{aa} \equiv |R_{aa}/A_{aa}|$.

It is worth noticing that for planar devices with a symmetry plane for the probability density $|\psi_a(E)|^2$, $\tau_{aa}(E)$, according to (4), becomes

$$\tau_{aa}(E) = \frac{1}{2} \frac{\int_\Omega |\psi_a|^2 d^3 r}{|\frac{\hbar}{2m} \int_{S_2} [\psi_a^* \nabla \psi_a - (\nabla \psi_a^*) \psi_a] \cdot d\mathbf{S}|} = \frac{1}{2} \frac{\tau_{Da}(E)}{T(E)}, \qquad (8)$$

i.e. it is one half the ratio of the dwell time $\tau_{Da}(E)$ of the particle entering the device from the electrode a with a longitudinal energy E [9] to the trasmission probability $T(E)$ through the device.

In conclusion, in devices with very low trasmission probabilities (e.g., very opaque single barriers), or with large values of the dwell time (e.g., resonant-tunneling double-barriers), the displacement current effects on noise may become appreciable at frequencies much lower than those at which the other quantities F_{ab}, A_{ab}, v_a and v_b contributing to $S(\omega)$—and being included in previous models too [6]—start differing from their zero-frequency value.

The second quantization of the electrokinematics theorem allows one to take into account these new effects.

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REFERENCES

1. S. Ramo, *Proc. IRE* **27**, 584 (1939); W. Shockley, *J. Appl. Phys.* **9**, 636 (1938).
2. B. Pellegrini, *Phys. Rev. B* **34**, 5921 (1986).
3. B. Pellegrini, *Nuovo Cimento* **15 D**, 855 (1993); *ibid.* **15 D**, 881 (1993).
4. S. O. Rice, *Bell System Tech. J.* **23**, 282 (1944); *ibid.* **24**, 46 (1945).
5. G. B. Lesovik, *Pis'ma Zh. Eksp. Teor. Fiz.* **49**, 513 (1989) [*JEPT Lett.* **49**, 592 (1989)].
6. M. Büttiker, *Phys. Rev. B* **45**, 3807 (1992); *ibid.* **46**, 12485 (1992).
7. B. Yurke and G. P. Kochansky, *Phys. Rev. B* **41**, 8184 (1990).
8. A. S. Davydov, *Quantum Mechanics* (Pergamon Press: Oxford, 1955) 620.
9. G. Iannaccone and B. Pellegrini, *Phys. Rev. B* **49**, 15468 (1994); ib. **50**, 14660 (1994).