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THEORY OF CONDUCTANCE AND NOISE ADDITIVITY IN PARALLEL MESOSCOPIC CONDUCTORS

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ABSTRACT

We present a theory of conductance and noise in generic mesoscopic conductors connected in parallel, and we demonstrate that the additivity of conductance and of shot noise arises as a sole property of the junctions connecting the two (or more) conductors in parallel.

INTRODUCTION

Since the pioneering work of van Wees and co-workers on conductance quantization [1], the ballistic transport regime has been the subject of widespread interest, both from a theoretical and an experimental point of view.

In this paper, we focus on transport and noise in mesoscopic conductors connected in parallel. It suffices to point out that all devices based on some kind of Aharonov-Bohm effect are based on such a topology.

In the case of macroscopic conductors connected in parallel it is well known that both the conductances and the thermal and shot-noise current power spectral densities add.

While transport properties of macroscopic conductors depend on local material properties, those of mesoscopic structures are obtained as the solution of a complex scattering problem, in which the shape of the boundaries and the potential profile over the whole device region play a relevant role. Therefore, the problem needs to be reformulated in these new terms.

Numerical studies of conductance additivity in sample structures made of two parallel constrictions, along with some analytical justifications, exist in the literature [2-4]. Furthermore, a numerical study has been presented showing additivity of shot noise in parallel constrictions [5].

MODEL

The structure considered is sketched in Fig. 1. It consists of 2 mesoscopic conductors Σ^u and Σ^d connected in parallel by means of two junctions, Σ^l and Σ^r. The junctions are ballistic systems with three leads, one connected to Σ^u, one to Σ^d, and the other used as an external lead of the whole structure. Phase coherence is mantained in the whole system.

The internal and external leads of the whole structure are numbered from 1 to 6, as sketched in Fig. 1. Let us define $\mathbf{a}_i$ and $\mathbf{b}_i$ ($i = 1, \ldots, 6$) as the column vectors whose N_i elements are the amplitudes of the modes in lead i entering and exiting the adjacent junction, respectively.

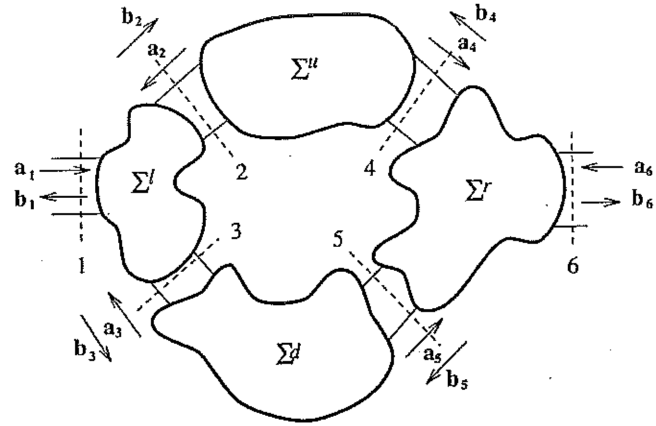

Figure 1: The structures consists of 2 mesoscopic conductors Σ^u and Σ^d connected in parallel by means of two junctions, Σ^l and Σ^r

Electron transport in each of the subsystems Σ^α $(\alpha = u, d, l, r)$ is completely described by the associated scattering matrix S^α, which is unitary and such as $S^{\alpha T}(\mathbf{B}) = S^\alpha(-\mathbf{B})$, where $\mathbf{B}$ is the applied magnetic field [6].

Let us first consider the case in which we have only the left junction Σ_l and leads 1, 2 and 3 are connected to different electron reservoirs. The relation between incoming and outgoing modes can be written as

$$\begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \mathbf{b}_3 \end{bmatrix} = S^l \begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \mathbf{a}_3 \end{bmatrix} = \begin{bmatrix} s^l_{11} & s^l_{12} & s^l_{13} \\ s^l_{21} & s^l_{22} & s^l_{23} \\ s^l_{31} & s^l_{32} & s^l_{33} \end{bmatrix} \begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \mathbf{a}_3 \end{bmatrix}, \tag{1}$$

where s^l_{ij} $(i, j = 1, 2, 3)$ is a $N_i \times N_j$ matrix, relating the amplitudes of the outgoing modes in lead i to the amplitudes of the incoming modes in lead j (as many evanescent modes as one desires can be considered).

We can repeat the same considerations for the right junction: if leads 4, 5 and 6 are connected to different electron reservoirs we can write

$$\begin{bmatrix} \mathbf{b}_4 \\ \mathbf{b}_5 \\ \mathbf{b}_6 \end{bmatrix} = S^l \begin{bmatrix} \mathbf{a}_4 \\ \mathbf{a}_5 \\ \mathbf{a}_6 \end{bmatrix} = \begin{bmatrix} s^r_{44} & s^r_{45} & s^r_{46} \\ s^r_{54} & s^r_{55} & s^r_{56} \\ s^r_{64} & s^r_{65} & s^r_{66} \end{bmatrix} \begin{bmatrix} \mathbf{a}_4 \\ \mathbf{a}_5 \\ \mathbf{a}_6 \end{bmatrix}. \tag{2}$$

Analogously, for the conductors Σ_u and Σ_d we can write

$$\begin{bmatrix} \mathbf{a}_2 \\ \mathbf{a}_4 \end{bmatrix} = S^u \begin{bmatrix} \mathbf{b}_2 \\ \mathbf{b}_4 \end{bmatrix} = \begin{bmatrix} s^u_{22} & s^u_{24} \\ s^u_{42} & s^u_{44} \end{bmatrix} \begin{bmatrix} \mathbf{b}_2 \\ \mathbf{b}_4 \end{bmatrix}, \tag{3}$$

$$\begin{bmatrix} \mathbf{a}_3 \\ \mathbf{a}_5 \end{bmatrix} = S^d \begin{bmatrix} \mathbf{b}_3 \\ \mathbf{b}_5 \end{bmatrix} = \begin{bmatrix} s^d_{33} & s^d_{35} \\ s^d_{53} & s^d_{55} \end{bmatrix} \begin{bmatrix} \mathbf{b}_3 \\ \mathbf{b}_5 \end{bmatrix}. \tag{4}$$

We intend to demonstrate that conductances and noise in parallel conductors add if the scattering matrices of the junctions satisfy the following conditions:

$$s_{32}^{l}=0, \qquad s_{45}^{r}=0, \qquad s_{23}^{l}=0, \qquad s_{54}^{r}=0. \tag{5}$$

The physical meaning of (5) is that an electron injected into Σ^l from lead 3 does not exit from lead 2 (and vice-versa), and that an electron injected into Σ^r from lead 5 is not transmitted to lead 4 (and vice-versa).

Since $S^{l\dagger}S^l=1$ and $S^{r\dagger}S^r=1$ (i.e., the S-matrices are unitary), by multiplying row by column and using (5), we straightforwardly obtain

$$s_{21}^{l\dagger}s_{31}^{l}=0, \qquad s_{21}^{l}s_{31}^{l\dagger}=0, \qquad s_{64}^{r\dagger}s_{65}^{l}=0, \qquad s_{64}^{r}s_{65}^{r\dagger}=0. \tag{6}$$

In order to proceed with our demonstration, we have to consider the whole structure, and the associated scattering matrix S_{tot}. We can write

$$\begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_6 \end{bmatrix} = S_{\mathrm{tot}} \begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_6 \end{bmatrix} = \begin{bmatrix} s_{11} & s_{16} \\ s_{61} & s_{66} \end{bmatrix} \begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_6 \end{bmatrix}. \tag{7}$$

In addition, we have to consider the scattering matrices S_{up} and S_{down}, corresponding to the whole system with the conductor Σ^d or Σ^u removed, respectively. They are of the form

$$S_{\mathrm{up}} = \begin{bmatrix} s_{11}^{u} & s_{16}^{u} \\ s_{61}^{u} & s_{66}^{u} \end{bmatrix}, \qquad S_{\mathrm{down}} = \begin{bmatrix} s_{11}^{d} & s_{16}^{d} \\ s_{61}^{d} & s_{66}^{d} \end{bmatrix}. \tag{8}$$

Let us start by calculating s_{61}^{u}, which relates the amplitudes of the modes exiting from lead 6 to those of the modes entering from lead 1: since Σ_d has been removed the only path from lead 1 to lead 6 is that through the upper conductor. It is straighforward to write s_{61}^{u} in the form of the scattering series

$$\begin{aligned} s_{61}^{u} &= s_{64}^{r}s_{42}^{u}s_{21}^{l} + s_{64}^{r}s_{42}^{u}s_{22}^{l}s_{24}^{u}s_{44}^{r}s_{42}^{u}s_{21}^{l} + \ldots + s_{64}^{r}s_{42}^{u}(s_{22}^{l}s_{24}^{u}s_{44}^{r}s_{42}^{u})^{n}s_{21}^{l} \\ &= s_{64}^{r}s_{42}^{u}(1-s_{22}^{l}s_{24}^{u}s_{44}^{r}s_{42}^{u})^{-1}s_{21}^{l}. \end{aligned} \tag{9}$$

Analogously, we can write s_{61}^{d} as

$$s_{61}^{d} = s_{65}^{r}s_{53}^{d}(1-s_{33}^{l}s_{35}^{d}s_{55}^{r}s_{53}^{d})^{-1}s_{31}^{l}. \tag{10}$$

From (5), (6), (9) and (10) we obtain

$$s_{61}^{d}s_{61}^{u\dagger}=0, \qquad s_{61}^{d\dagger}s_{61}^{u}=0, \qquad s_{61}=s_{61}^{u}+s_{61}^{d}, \tag{11}$$

where the last equality comes from (5), which warrants that no path from lead 1 to 6 is allowed passing through both conductors.

The conductance of the whole structure at 0 K is evaluated accoding to Landauer and Büttiker [7-9]:

$$G=\frac{2e^2}{h}\mathrm{tr}\{s_{61}^{\dagger}s_{61}\}. \tag{12}$$

Let $G_{\rm up}$ and $G_{\rm down}$ be the conductances of the system with the conductor Σ^d or Σ^u removed, respectively: we have

$$G_{\rm up} = \frac{2e^2}{h}\mathrm{tr}\{s_{61}^{u\dagger}s_{61}^{u}\}, \qquad G_{\rm down} = \frac{2e^2}{h}\mathrm{tr}\{s_{61}^{d\dagger}s_{61}^{d}\}. \tag{13}$$

From (11) we have

$$s_{61}^{\dagger}s_{61} = (s_{61}^{u\dagger} + s_{61}^{d\dagger})(s_{61}^{u} + s_{61}^{d}) = s_{61}^{u\dagger}s_{61}^{u} + s_{61}^{d\dagger}s_{61}^{d}, \tag{14}$$

which allows us to write, from (12) and (13), $G = G_{\rm up} + G_{\rm down}$. The additivity of conductances for parallel mesoscopic conductors implies also that of thermal noise current spectral densities, which are proportional to the conduntance.

The shot noise current spectral density in mesoscopic systems can be expressed in terms of the scattering matrix as derived by Büttiker: [10]

$$\langle(\Delta I)^2\rangle = 4|qV|\frac{q^2}{h}\mathrm{tr}\{s_{11}^{\dagger}s_{11}s_{61}^{\dagger}s_{61}\} = 4|qV|\frac{q^2}{h}(\mathrm{tr}\{s_{61}^{\dagger}s_{61}\} - \mathrm{tr}\{s_{61}^{\dagger}s_{61}s_{61}^{\dagger}s_{61}\}) \tag{15}$$

where V is the voltage applied between leads 1 and 6, and the last equality comes from the unitarity of the matrix $S_{\rm tot}$. The shot noise current spectral densities $\langle(\Delta I_{\rm up})^2\rangle$ and $\langle(\Delta I_{\rm down})^2\rangle$ of the system with Σ^d or Σ^u removed, respectively, are

$$\begin{aligned} \langle(\Delta I_{\rm up})^2\rangle &= 4|qV|\frac{q^2}{h}(\mathrm{tr}\{s_{61}^{u\dagger}s_{61}^{u}\} - \mathrm{tr}\{s_{61}^{u\dagger}s_{61}^{u}s_{61}^{u\dagger}s_{61}^{u}\}) \\ \langle(\Delta I_{\rm down})^2\rangle &= 4|qV|\frac{q^2}{h}(\mathrm{tr}\{s_{61}^{d\dagger}s_{61}^{d}\} - \mathrm{tr}\{s_{61}^{d\dagger}s_{61}^{d}s_{61}^{d\dagger}s_{61}^{d}\}). \end{aligned} \tag{16}$$

From (11) we have $s_{61}^{\dagger}s_{61}s_{61}^{\dagger}s_{61} = s_{61}^{u\dagger}s_{61}^{u}s_{61}^{u\dagger}s_{61}^{u} + s_{61}^{d\dagger}s_{61}^{d}s_{61}^{d\dagger}s_{61}^{d}$, which, along with (14), (15) and (16), allows us to finally write

$$\langle(\Delta I)^2\rangle = \langle(\Delta I_{\rm up})^2\rangle + \langle(\Delta I_{\rm down})^2\rangle. \tag{17}$$

We have shown that conductances, thermal and shot noise current spectral densities for parallel mesoscopic conductors add, provided a few conditions are satisfied by the junction scattering matrices. Our demonstration is trivially extended to the case of more than two conductors in parallel.

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