

# *Shot noise in mesoscopic devices and quantum dot networks*

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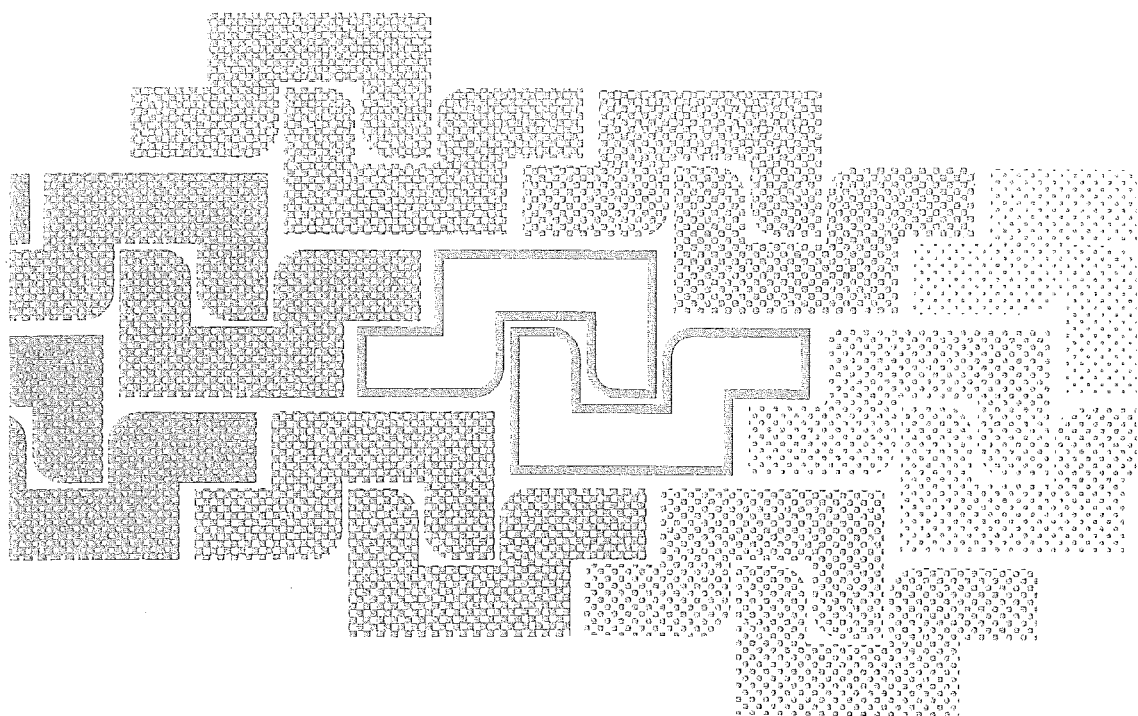
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# SHOT NOISE IN MESOSCOPIC DEVICES AND QUANTUM DOT NETWORKS

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**Abstract** We discuss specific noise phenomena in several ballistic mesoscopic structures and in networks of quantum dots or metallic dots interconnected by tunneling barriers, focusing on the techniques used for the numerical simulation and on the physical interpretation of the results.

**Keywords:** Shot noise suppression, Shot noise enhancement, Nanostructures, Numerical simulation

## 1. Ballistic structures without magnetic field

For the investigation of transport and noise in structures without magnetic field we use a technique based on the recursive evaluation of the overall Green's function, starting from the analytically computed Green's functions of elementary sections [1, 2]. From the transmission matrix it is then possible to obtain the conductance via the Landauer-Büttiker formula and the Fano factor, i.e. the shot noise suppression or enhancement factor, as will be detailed in the present section.

If we consider a section of the device such that the transverse potential can be considered constant along the direction of electron propagation, there will be no transverse mode mixing in it, thus the Green's function matrix (consisting in a representation over the transverse eigenmodes) will be diagonal and it will be possible to evaluate each element from an analytical expression [1], with the hypothesis of Dirichlet boundary conditions at the section ends. Starting from the semi-infinite section containing the output lead, we keep on

adding the preceding sections, one at a time, introducing a perturbation potential  $\hat{V}$  which "opens up" facing section ends and connects them. The Green's function of the perturbed structure (with the two sections joined) can be evaluated from Dyson's equation:

$$\hat{G} = \hat{G}^0 + \hat{G}^0 \hat{V} \hat{G}, \quad (1)$$

where  $\hat{G}^0$  is the unperturbed Green's function (with the two sections decoupled) and  $\hat{G}$  is the perturbed Green's function (with the application of  $\hat{V}$ ). This is an implicit equation, since  $\hat{G}$  appears on both sides: with some algebra, considering a representation of the Green's functions on the eigenmodes for the transverse direction and on the sites in real space for the longitudinal direction, it is possible to obtain explicit relationships that can be applied in the recursive procedure [2]. Once the Green's function of the overall structure has been computed, it is straightforward to obtain from it the transmission matrix  $t$ , following the procedure outlined in [1].

From the transmission matrix  $t$  we obtain the conductance of the device via the Landauer-Büttiker formula:

$$G = \frac{2e^2}{h} \sum_{n,m} |t_{nm}|^2. \quad (2)$$

The low-frequency noise power spectral density is given by Büttiker [3] as

$$S_I = 4 \frac{e^3}{h} |V| \sum_n w_n (1 - w_n), \quad (3)$$

where the  $w_n$ 's are the eigenvalues of the matrix  $tt^\dagger$ ,  $V$  is the constant externally applied voltage,  $e$  is the electron charge, and  $h$  is Planck's constant. Since the full shot noise power spectral density is given by Schottky's theorem as  $S_{I_{fs}} = 2eI$  (where  $I$  is the average value of the current, given by the product of the applied voltage times the conductance), we obtain a simple expression for the ratio  $\gamma$ , usually defined as Fano factor, of the noise power spectral density to that expected for full shot noise:

$$\gamma = \frac{\sum_j w_j (1 - w_j)}{\sum_{n,m} |t_{nm}|^2} \quad (4)$$

Jalabert *et al.* have shown [4], using Random Matrix Theory, that a ballistic symmetric cavity delimited by apertures that are much smaller than the cavity size (such as the one shown in the inset of Fig. 1) is characterized by a Fano factor of 1/4. We have applied our numerical techniques to the investigation of shot noise suppression in such cavities in a variety of conditions, which can be treated easily with our approach.

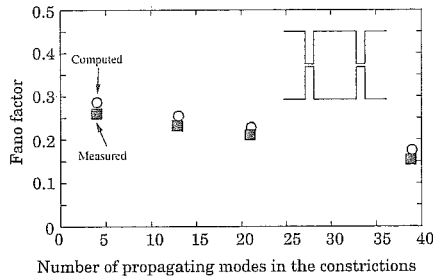


Figure 1. Fano factor in a chaotic cavity with no magnetic field

The Fano factor, which reaches the asymptotic value of  $1/4$  for narrow enough and symmetric input and output constrictions, decreases if the constrictions are made wider: in the limiting case of disappearing constrictions (when their width equals that of the wire) the Fano factor is known to drop to zero, since we reach the situation of a perfect, ballistic wire. Results for the Fano factor of a cavity with symmetric apertures as a function of the number of propagating modes in the constrictions are shown in Fig. 1. The width of the cavity is  $8 \mu\text{m}$  and its length is  $5 \mu\text{m}$ . Such results, although obtained with a coarse hard wall model, are in rather good agreement with the experimental data published by Oberholzer *et al.* in [5].

Our technique can be readily applied also to cascaded cavities, in particular we focus on identical cascaded cavities, finding that no appreciable variation of the Fano factor is observed with respect to that of a single cavity. Results for 2, 3, and 4 cavities are reported in Fig. 2 as a function of the Fermi energy. This conclusion is in sharp contrast with the analytical conclusion reached by Oberholzer *et al.* in [6], in which the authors state that the Fano factor for cascaded cavities should tend to the asymptotic limit  $1/3$ , in analogy with what happens for a series of potential barriers, each of which is orthogonal to the propagation direction. Further investigation is needed to understand the discrepancy between the two approaches, which cannot be explained simply on the basis that our model is fully coherent while that of [6] assumes decoherence between adjacent cavities: the recent literature seems to agree on the irrelevance of the presence or lack of coherence on the noise behavior of mesoscopic devices.

Another interesting structure to be investigated is an antidot lattice inserted in a quantum wire [7]. Let us first examine the noise behavior of a square antidot lattice: we consider a quantum wire  $1.2 \mu\text{m}$  wide with 7 layers along the longitudinal direction (see the inset of Fig. 3) of square antidots with a side of  $24 \text{ nm}$  and a distance between antidot centers of  $244 \text{ nm}$ . The resulting Fano factor is shown in Fig. 3 (solid curve) as a function of the Fermi energy, and it clearly settles around a value of about  $0.11$ . If we increase the size of

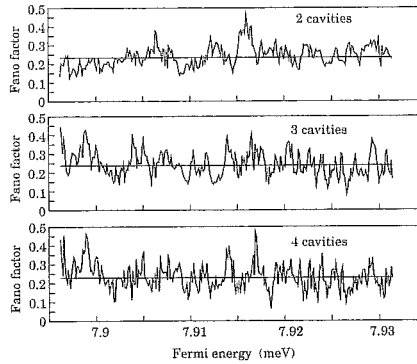


Figure 2. Fano factor for cascaded cavities

the antidots to 48 nm (keeping the same distance between antidot centers as in the previous case), the Fano factor raises to about 0.14, as visible in Fig. 3 (dashed curve). Further increases of the antidot size lead, for a square lattice, to a maximum Fano factor of about 0.15: as the antidot walls get closer along the transverse direction, which tends to increase the Fano factor, they also get closer along the longitudinal direction, thereby approaching the formation of regular, noiseless conduits. The result of these two competing effects is the saturation of the Fano factor to the mentioned limiting value. The dependence of the Fano factor on the number of layers exhibits a saturation, too: for 7 layers we have already reached the asymptotic condition [7].

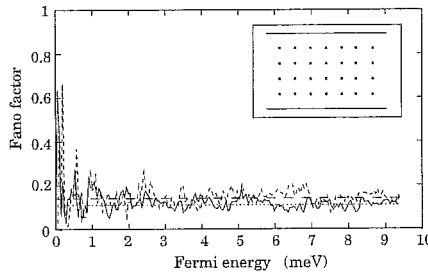


Figure 3. Fano factor for a square antidot lattice

If, instead of a square antidot lattice, we consider a rectangular one, with the longitudinal separation much larger than that along the transverse direction, we have a situation very similar to that of cascaded chaotic cavities, with the only difference that each cavity has multiple input and output apertures instead of a single one: as long as the total width of the resulting apertures is significantly

smaller than that of the wire, we achieve the conditions for shot noise suppression with a Fano factor of 0.25, as typical for chaotic cavities. This is shown in Fig. 4, where the results for shot noise suppression in the case of 188 nm square antidots separated by a distance of 278 nm along the transverse direction and of 1388 nm along the longitudinal direction are reported: the Fano factor settles around a value of about 0.25, as expected for chaotic cavities.

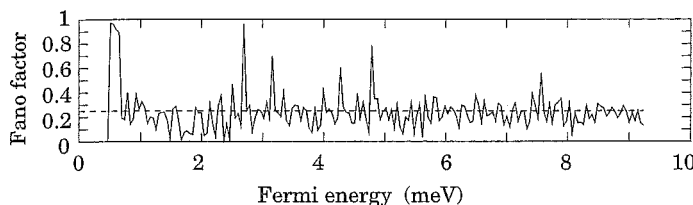


Figure 4. Fano factor for a rectangular antidot lattice

## 2. Chaotic cavities in a magnetic field

In order to study the behavior of chaotic cavities in the presence of an orthogonal magnetic field, we have selected a scattering matrix approach, which makes treatment of the magnetic field simpler, for several cases, than the formally similar Green's function approach, although being in principle slightly less efficient from the computational point of view.

We have chosen a gauge for the representation of the vector potential with a nonzero component only in the longitudinal direction  $x$  ( $\vec{A} = [-By \ 0 \ 0]^T$ ). We subdivide the structure into a number of transverse slices such that inside each of them the scalar and the vector potentials can be considered constant along the longitudinal direction. Then we consider the sections straddling from the middle of a slice to the middle of the following one and, from the transverse eigenfunctions and the longitudinal wave vectors in each slice, we compute the scattering matrix for each section by means of the mode-matching technique. We evaluate the overall scattering matrix of the device by recursively composing the scattering matrices of all of the slices, and, from its properly normalized transmission submatrix, we obtain the actual transmission matrix  $t$ . The most challenging task of this calculation is represented by the procedure to compute the transverse eigenfunctions and the longitudinal wave vectors in each slice, which is performed with the technique introduced by Tamura and Ando [8].

We have focused on the investigation of how the Fano factor varies in the presence of a magnetic field in a chaotic cavity. The results of our simulations show that, while for small or zero values of  $B$  the Fano factor in a symmetric



chaotic cavity is 0.25, such a value decreases for higher magnetic field values. We wish to point out that this result is independent of the dimensions of the cavity but strongly depends on the width of the constrictions which define it. The Fano factor as a function of magnetic field is shown in Fig. 5 for a few choices of the constriction width: we notice that, as the width is decreased, a larger magnetic field is needed to obtain the same Fano factor. Such a behavior can be explained by comparing the width of the constrictions  $W_c$  with the classical cyclotron radius of the electrons  $R_c = \sqrt{2m^*E_f}/(eB)$  (where  $m^*$  is the effective mass of the electron and  $E_f$  is the Fermi energy): in a confined mesoscopic structure in the presence of a magnetic field, edge states form, which can be classically explained as skipping orbits with radius  $R_c$ . For values of  $B$  such as to make  $R_c$  comparable with  $W_c$ , the edge states pass through the constrictions and mainly crawl along the walls of the cavity, thereby quenching the chaotic behavior of the cavity and, consequently, the value of the shot noise power spectral density.

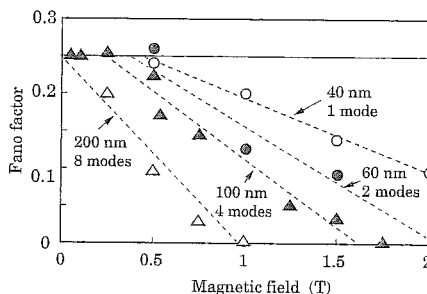


Figure 5. Fano factor as a function of the magnetic field

### 3. Shot noise suppression and enhancement in networks of metallic dots

Shot noise suppression and, more interestingly, enhancement have been predicted [9] to be observable in single electron circuits based on metallic islands connected by tunnel junctions and capacitors. The particular structure for which we have demonstrated the existence of shot noise enhancement is a Quantum Cellular Automaton cell [10] biased near the point of switching. The cell we have considered is shown in the inset of Fig. 6 and consists of four metal islands: tunneling is possible between the two upper ones or between the lower ones, but not between the upper and lower pairs, which are connected only through standard capacitors. The voltage sources connected to the dots via tunneling junctions make a current flow through the upper and lower pair of

dots, as long as the Coulomb blockade is lifted by adjusting the other voltage sources (which in the following we define bias voltages). In particular, we are interested in the condition in which an excess electron is present on each pair of dots. The Coulomb blockade is lifted when the chemical potentials in a pair of dots are aligned and electrons are therefore allowed to tunnel from one dot to the other.

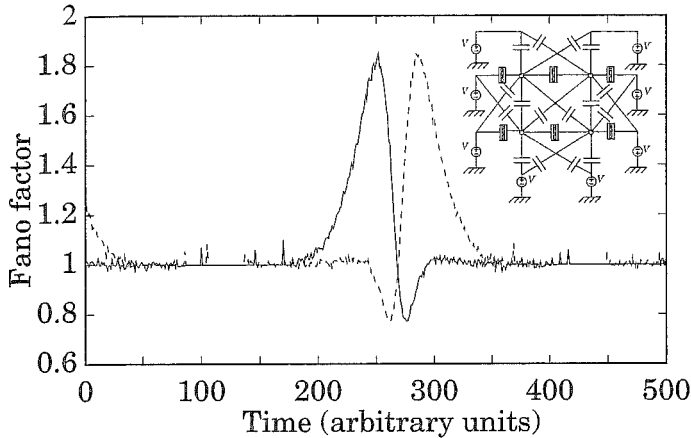


Figure 6. Fano factor for a circuit with 4 coupled dots

We compute the currents and noise in this structure by means of a Monte Carlo simulation, assuming that the bias voltages for each dot pair are varied with two linear ramps of opposite slope, so that at a specific instant the chemical potentials of the two dots line up. A small shift is introduced between the ramps applied to the lower dot pair and those applied to the upper dot pair, so that the Coulomb blockade for the two pairs is lifted at slightly different times. This leads to an interaction between the upper and the lower pair, in which the current through one pair “drives” the other, bringing the other pair closer to the alignment condition. The role of the “driver” and of the “driven” currents are interchanged when the “driven” current becomes larger than the “driver” one. In Fig. 6 we report the behavior of the Fano factor for the two currents as a function of time (and actually of the bias point, since we are considering the application of linear ramps): the driver current (dashed curve in the left part of the plot) exhibits suppressed shot noise, while the driven current (solid curve in the left part of the plot) is characterized by a Fano factor larger than 1. This is a result of positive electron correlations: as an electron tunnels through the dot pair through which the driven current flows, it favors tunneling of an electron in the opposite direction in the other dot pair, whose potentials are thus changed in such a way as to increase the likelihood of a further electron tunneling through the former pair.

#### 4. Conclusion

We have discussed shot noise suppression in several different nanostructures and a case of shot noise enhancement: it is apparent how important an analysis of the noise behavior is for a better understanding of correlations between charge carriers and of transport mechanisms in low dimensional devices. A common value of the Fano factor may hint at deeper analogies between apparently different structures, such as in the case of cascaded chaotic cavities and of transversally dense rectangular antidot lattices. Once again noise is shown to be a more sensitive probe of transport properties than other, more commonly considered, electrical quantities.

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