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**Massimo Macucci**

Dipartimento di Ingegneria dell'Informazione: Elettronica, Informatica, Telecomunicazioni,  
Università di Pisa

**Giuseppe Iannaccone**

Dipartimento di Ingegneria dell'Informazione: Elettronica, Informatica, Telecomunicazioni,  
Università di Pisa

**Paolo Marconcini**

Dipartimento di Ingegneria dell'Informazione: Elettronica, Informatica, Telecomunicazioni,  
Università di Pisa

# Shot noise behavior of cascaded mesoscopic structures

M. Macucci\*, G. Iannaccone\* and P. Marconcini\*

*\*Dipartimento di Ingegneria dell'Informazione  
Università di Pisa  
Via Caruso 2 - I-56122 PISA, Italy*

**Abstract.** We discuss shot noise suppression in cascaded mesoscopic cavities, motivated by the discrepancy existing between analytical and numerical results published in the last few years. We conclude that the noise suppression factor does not increase above the 0.25 value for a single cavity only if all cavities are identical (a case in which it may even drop below 0.25), while, otherwise, increases toward the analytically predicted limit of  $1/3$ , reaching even higher values. We comment on the numerical results, offering some possible explanations.

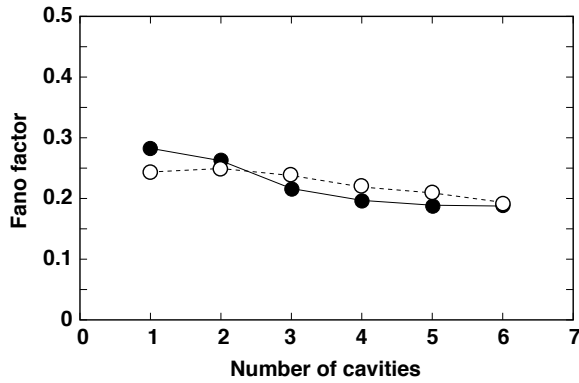
Shot noise in mesoscopic cavities has been the subject of intense theoretical and experimental investigation, since the seminal paper by Jalabert *et al.* [1], in which the suppression of noise down to  $1/4$  of the full shot value was predicted, based on random matrix theory. A semiclassical demonstration was devised by Blanter and Sukhorukov [2] and experimental evidence was provided by Oberholzer *et al.* [3]. The problem of cascaded cavities was investigated by M. Macucci *et al.* [4] with a numerical study of the case of identical cavities and by Oberholzer *et al.* [5] with a semiclassical model and an experiment on a double cavity. While Ref. [4] predicted a Fano factor (ratio of the shot noise power spectral density to that predicted for full shot noise by Schottky's theorem) of 0.25 (i.e. the same as that for a single cavity), Ref. [5] argues that shot noise should increase when more than one cavity are cascaded and, in the limit of an infinite number of cavities, should converge to  $1/3$ .

In order to understand the origin of this discrepancy, we have performed detailed numerical simulations and extended a simple analytical approach to an arbitrary number of transverse modes, which allows a better understanding of the underlying assumptions. Numerical calculations have been performed with an optimized recursive Green's function technique [6] that yields the transmission matrix for the cascaded cavities. From the transmission matrix, the conductance and the shot noise power spectral density are obtained following Büttiker [7], and from them the Fano factor is easily derived. Care must be taken to thermally average over a range of energies of approximately  $9kT$  (where  $k$  is the Boltzmann constant and  $T$  is the temperature), weighing with the derivative of the Fermi function. For all of our simulations we have assumed a temperature of 0.27 K, as in the

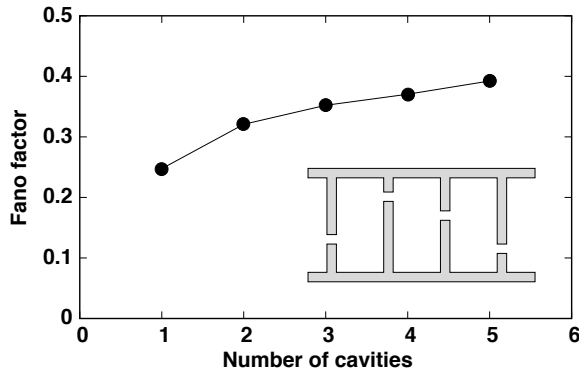
published experiments.

We have considered cavities that are  $5\ \mu\text{m}$  long and  $8\ \mu\text{m}$  wide, cascaded with several different arrangements. If we consider rather narrow entrance and exit constrictions, 110 nm and 210 nm in our simulations, we observe that, as expected from theory, the conductance of a cavity is one half of that of each single constriction, because the complex behavior of electrons inside the cavity effectively decouples the two constrictions. The Fano factor for a single cavity is found to be, in agreement with theory, around 0.25. If more than one identical cavities are cascaded, the overall structure has quite different a behavior with respect to theoretical predictions [5]. In particular, the total resistance is significantly less than the sum of the resistances of the constrictions, and the Fano factor does not increase above 0.25; it, rather, decreases, as the number of cavities is increased, as shown in Fig. 1 with solid dots for cavities with 110 nm constrictions and with empty dots for 210 nm constrictions. This result has been checked, for a few selected cases, also for structures with gradual constrictions.

If even a small difference is introduced between the cavities, the behavior of the Fano factor as a function of the number of cavities changes abruptly, as reported in Fig. 2, where we present the results for the case of shifted constrictions (as shown in the inset of Fig. 2): the suppression factor increases as the number of cavities is increased, surpassing the theoretical limit of  $1/3$ . We have verified that a similar increase takes place if we differentiate the cavities by slightly changing their lengths, by varying the potential at the bottom of one of them by a fraction of a millielectronvolt, or by including an irregularity, such a small ( $0.5\ \mu\text{m}$ ) bump in the boundary.



**FIGURE 1.** Fano factor as a function of the number of identical cascaded cavities for 110 nm (solid dots) and 210 nm (empty dots) apertures



**FIGURE 2.** Fano factor as a function of the number of cascaded cavities with skewed 210 nm apertures; the inset contains a sketch of the simulated structure for 3 cavities

A simple model, similar to that in Ref. [5], can be devised to compute analytically the shot noise suppression of  $N - 1$  cavities in series, defined by  $N$  constrictions with the same *integer* number of propagating modes, if we can assume that the dwell time in each cavity is large enough that all states have equal occupation probability. In some sense, cavities are “quasi” reservoirs, except for the fact that the occupation factor in a cavity depends on the balance of fluxes entering and exiting the cavity at such an energy, and that the cavity is not in thermal equilibrium.

The equivalent circuit of a series of cavities at zero temperature therefore consists of a series of  $N$  noisy resistors (each representing a constriction), where each node  $i$  ( $i = 0 \dots N$ ) is at a potential corresponding to  $f_i V$ ,  $V$  being the very small voltage applied between the end reservoirs, and  $f_i$  is the occupation factor of the  $i$ -th cavity ( $f_i = i/N$ ).

Considering the shot noise associated with each constriction, the calculation of Ref. [5] can be easily gen-

eralized to an arbitrary number of propagating modes, obtaining, in the hypothesis of transmission coefficients that are either 1 or 0, the shot noise suppression factor  $F$

$$F = \frac{1}{3} \left( 1 - \frac{1}{N^2} \right), \quad (1)$$

which converges to  $1/3$  with increasing  $N$ .

The derivation of this result is based on the assumption of complete decoupling between successive constrictions introduced by the cavities: when cavities are identical such an assumption fails, probably due to equal resonances, whereby the first cavity selects a combination of modes that travels with limited scattering through the following cavities. Such an interpretation is consistent with the observed lack of additivity of the constriction resistances.

The increase over the theoretically predicted  $1/3$  limit for unequal cavities with narrow constrictions is less straightforward to explain, and may be associated with an effect similar to that observed in the simulation of diffusive conductors, where the Fano factor rises above the  $1/3$  limit for a limited number of propagating modes [8]. Further analysis is needed to establish the exact reason for this phenomenon.

## ACKNOWLEDGMENTS

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