Shot noise analysis in quasi one-dimensional Field Effect Transistors

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Abstract. We present a novel method for the evaluation of shot noise in quasi one-dimensional field-effect transistors, derived by means of a statistical approach within the second quantization formalism, which manages to include both the effects of Pauli exclusion and Coulomb interactions. The method has been applied to Carbon Nanotubes and Silicon Nanowire Transistors. We show that noise can significantly differ from that obtained by means of the Landauer-Büttiker's formula and that the main noise source is represented by the partition noise.

Keywords: Shot noise, FETs, nanowire transistors, carbon nanotube transistors.
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INTRODUCTION

In the last few years, an increasing interest has been directed to assess potential performance of quasi one-dimensional Field Effect Transistors (FETs) based on Carbon Nanotubes [1] (CNTs) and Silicon NanoWires [2] (SNWs) versus the ITRS requirements. However, an accurate investigation of electrical noise has been often neglected. In particular, Landauer-Büttiker's noise formula includes the effects of Pauli exclusion, but does not explicitly include electron-electron interaction through Coulomb force. From a numerical point of view, a self-consistent solution of the electrostatics and transport equations is mandatory in order to properly consider such effects. Here, we present a new method to compute the shot noise power spectral density, based on self-consistent Monte Carlo simulations of randomly injected electrons from the reservoirs, including Pauli and Coulomb interaction, which has been applied to ballistic CNT and SNW-FETs.

THEORY AND METHODOLOGY

As well known, Landauer-Büttiker's noise formula completely neglects Coulomb repulsion, since it does not include fluctuations in time of the potential profile and of transmission probabilities, due to randomly injected electrons through long-range Coulomb interaction. In order to consider the effect of Coulomb interaction, we have self-consistently solved the transport and the Poisson equation.

In particular, we have modelled the stochastic injection of carriers through the randomization of the occupation factors of states propagating from the contacts: the noise power spectrum can then be derived within a statistical approach and through the second quantization formalism. If $|\sigma\rangle$ is a many-particle state, the occupation number $\sigma_{\alpha m}(E)$

in the reservoir α ($\alpha = S, D$) in the channel m can be either 0 or 1, and can be expressed as $\sigma_{\alpha m}(E) = \langle a_{\alpha m}^{\dagger}(E) a_{\alpha m}(E) \rangle_{\sigma}$, where the operators $a_{Sm}^{\dagger}(E,t)$ and $a_{Sm}(E,t)$ create and annihilate, respectively, incident electrons in the source lead with total energy E in the transverse channel m. Since we are interested in current fluctuations, we need to consider an ensemble of many electrons states $\{| \sigma_1 \rangle, \cdots, | \sigma_N \rangle\}$ and to compute statistical averages $\langle \rangle$. Starting from the formula of the current operator at the source, at zero magnetic field, and taking advantage of the Milatz Theorem, the zero-frequency noise power spectral density can be expressed as

$$S(0) = S_{PN} + S_{TT} + S_{TR} + S_{IN}, \qquad (1)$$

where S_{PN}, S_{TT}, S_{TR} and S_{IN} read:

$$
\begin{aligned}
S_{PN} &= \left(\frac{e^2}{\pi\hbar}\right) \int dE \sum_{\alpha=S,D} \sum_{l \in \alpha} \langle [\tilde{\mathbf{t}}]_{\alpha;ll} \left(1 - [\tilde{\mathbf{t}}]_{\alpha;ll}\right) \sigma_{\alpha l} \rangle_s \\
S_{TT} &= -\left(\frac{e^2}{\pi\hbar}\right) \int dE \sum_{\alpha=S,D} \sum_{\substack{l,p \in \alpha \\ l \neq p}} \langle [\tilde{\mathbf{t}}]_{\alpha;lp} [\tilde{\mathbf{t}}]_{\alpha;pl} \sigma_{\alpha l} \sigma_{\alpha p} \rangle_s \\
S_{TR} &= -\left(\frac{2e^2}{\pi\hbar}\right) \int dE \sum_{k \in D, p \in S} \langle \left[\mathbf{t}'^{\dagger}\mathbf{r}\right]_{kp} \left[\mathbf{r}^{\dagger}\mathbf{t}'\right]_{pk} \sigma_{Dk} \sigma_{Sp} \rangle_s \\
S_{IN} &= \lim_{\Delta E \to 0} \left(\frac{e^2}{\pi\hbar\Delta E}\right) var \left[\int dE \left(\sum_{n \in S} [\tilde{\mathbf{t}}]_{S;nn} \sigma_{Sn} - \sum_{k \in D} [\tilde{\mathbf{t}}]_{D;kk} \sigma_{Dk}\right)\right], \qquad (2)
\end{aligned}
$$

where $[\tilde{\mathbf{t}}]$ is defined as $\left[\mathbf{t}^{\dagger}\mathbf{t}\right]_{lp}$ if $\alpha = S$, $\left[\mathbf{t}'^{\dagger}\mathbf{t}'\right]_{lp}$ if $\alpha = D$, and $\mathbf{t}$, $\mathbf{t}'$ and $\mathbf{r}$ are the source-to-drain transmission, drain-to-source transmission and reflection amplitude matrices, respectively. ΔE is our energy step of choice. The noise power spectrum (1) is expressed as the sum of four terms. More in detail, S_{PN} corresponds to the partition noise contribution of uncoupled modes, i.e. when $\mathbf{t}^{\dagger}\mathbf{t}$ and $\mathbf{t}'^{\dagger}\mathbf{t}'$ are diagonal matrices and their diagonal elements represent the transmission probabilities in the basis of the eigen-channels. S_{TT} and S_{TR} are instead associated to the exchange correlations between transmitted and reflected states. The negative sign is a signature of the fermionic nature of the electrons, i.e. of the antisymmetric many-electrons wave-function under particle exchange. Finally, S_{IN} represents the injection noise contribution, related to the thermal random motions of carriers in the reservoirs. Note that eq. (1) differs from Landauer-Büttiker's formula, since the matrices $\mathbf{t}$, $\mathbf{t}'$ and $\mathbf{r}$ are expressed as a function of σ_{α}, i.e. the occupation factor of injected states from both contacts.

From a numerical point of view, we self-consistently solve the 3D Poisson and Schrödinger equations, within the NEGF formalism, and considering a large ensemble of injected states, through a Monte Carlo approach (SC-MC). In particular, a quantum fully ballistic transport model has been developed by means of a mode space approach, considering four modes [3], and extending our in-house developed open source simulator *NanoTCAD ViDES* [4].

RESULTS AND DISCUSSIONS

The simulated devices structures are depicted in Fig. 1a. We consider a (13,0) CNT embedded in SiO_2 and a SNWT with a cross section equal to 4×4 nm^2. In Fig. 1b, the transfer characteristics for different drain-to-source biases V_{DS} computed applying Landauer's formula and performing SC-MC simulation are plotted as a function of the gate overdrive $V_{GS} - V_{th}$, where V_{GS} and V_{th} are the gate and the threshold voltage, respectively. As can be seen in the inset of Fig. 1b, there is a very limited number of electrons inside the channel of a CNT and SNW-FET, which can make device operation extremely sensitive to charge fluctuations.

The contributions to the Fano factor of the four terms in eq. (1) are shown in Figs. 2a-d as a function of the gate overdrive for $V_{DS} = 0.5$ V for CNT-FETs and SNW-FETs. As can be seen, while in the sub-threshold regime the Poissonian noise is recovered, in the strong inversion regime, noise is greatly reduced with respect to the full shot value, for both SNW-FET and CNT-FET. It is important to remark that Landauer-Büttiker's formula does not allow to quantitatively evaluate noise, since overestimation up to 180% can be obtained for SNW-FETs ($V_{GS} - V_{th}= 0.4$ V).

In addition, the dominant noise sources in CNT-FETs is the partition noise S_{PN}, while the injection noise S_{IN} is equal to the 36% of the partition noise ($V_{GS} - V_{th} = -0.1$ V). For SNW-FETs, noise due to intrinsic thermal fluctuations of carriers gives a larger noise contribution, up to the 86% of the partition noise ($V_{GS} - V_{th} = -0.2$ V). For all the considered bias points, exchange correlations S_{TT} and S_{TR} are always negligible for both one-dimensional structures, since electrons flow along separate quantum channels without change their transverse wavevectors.

CONCLUSION

We have developed a novel and general approach to study shot noise in nanoscale FETs. The derived analytical formula of the noise power spectrum manages to extend the validity of the Landauer-Büttiker formula including also Coulomb repulsion among electrons. Such an expression has been then exploited and implemented in a 3D self-consistent Poisson and Schrödinger solver within the NEGF formalism, in order to compute transport in one-dimensional devices, by means of Monte Carlo simulations.

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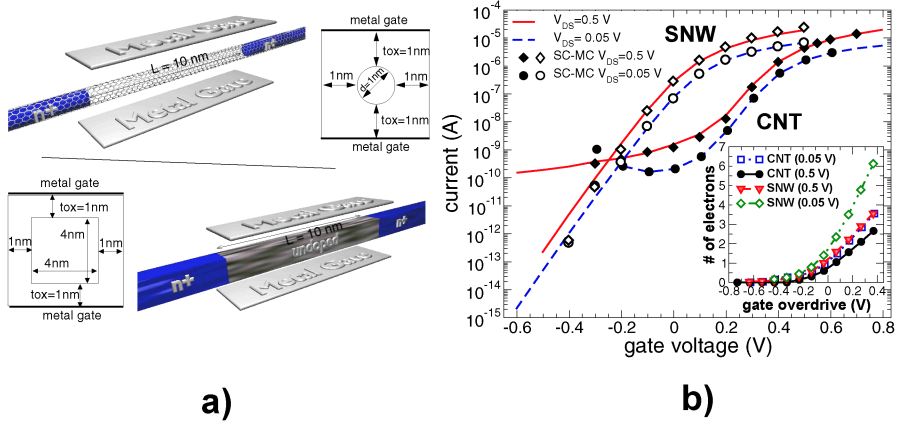
FIGURE 1. a) 3-D structures and transversal cross sections of the simulated CNT (top) and SNW-FETs (bottom). b)Transfer characteristics computed for V_{DS}= 0.5 V and 0.05 V, obtained by SC-MC simulations and applying Landauer-Büttiker's formula. Full dots refer to CNT, empty dots to SNW. Inset: average number of electrons in CNT-FETs and SNW-FETs channel, evaluated for V_{DS}= 0.5 V and 0.05 V.
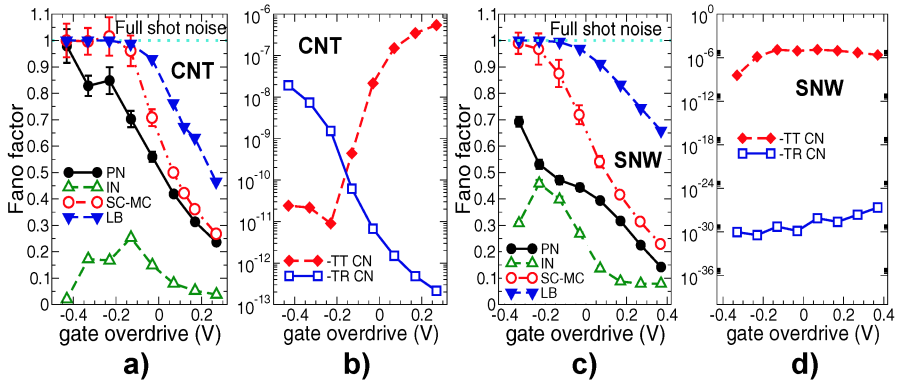
FIGURE 2. Contributions to the Fano factor F of the four terms in eq. (1) as a function of the gate overdrive for $V_{DS} = 0.5$ V. a)-c) Partition (PN, solid circles), injection (IN, open triangles up) and full noise (open circles) computed by means of SC-MC simulations and applying Landauer-Büttiker's formula (solid triangles down). b)-d) S_{TT} and S_{TR} contributions to F.

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