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Effect of localization on the Fano factor of cascaded tunnel barriers

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Abstract. We study, by means of numerical and analytical quantum mechanical calculations, shot noise suppression in a series of tunnel barriers, finding results that strongly differ from the 1/3 limit for the Fano factor that would be expected from semiclassical models. The reason for the observed results is attributed to the presence of strong localization, which in the case of just one-dimensional disorder makes it impossible to reach the diffusive transport regime.

Keywords: tunnel barriers, shot noise suppression, strong localization
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INTRODUCTION

It is well known [1] that shot noise in diffusive conductors is suppressed, with respect to the value expected from Schottky's theorem, by a factor 1/3. De Jong and Beenakker [2, 3] showed that the same value for the shot noise suppression factor (Fano factor) is obtained for a series of tunnel barriers, using a semiclassical model based on the Boltzmann-Langevin equation. Here we show, by means of numerical and analytical calculations, that a quantum mechanical model yields different results and, in particular, the asymptotic 1/3 limit is not achieved, and we propose an explanation for these results.

NUMERICAL RESULTS

We focus on the case of unevenly spaced barriers (see Fig. 1(a)), i.e. of one-dimensional disorder, where (differently from the case of equidistant barriers) the transport behavior is not dominated by resonances between the interbarrier regions. As a result of the absence of mode-mixing in this structure, transport along the propagation direction x can be analyzed studying separately each propagating mode. We have therefore evaluated the transmission T_n of the generical n-th mode through the device, with the scattering matrix formalism. Then we have computed the conductance G, the shot noise power spectral density S_I and the Fano factor γ as [4, 5]

$$G = \frac{2q^2}{h}\sum_n T_n\,, \quad S_I = 4\frac{q^3}{h}|V|\sum_n T_n(1-T_n)\,, \quad \gamma = \frac{S_I}{2q|I|} = \frac{\sum_n T_n(1-T_n)}{\sum_n T_n} \qquad (1)$$

(q is the value of the elementary charge, h is Planck's constant, I is the average current flowing through the device and V is the externally applied voltage). The considered

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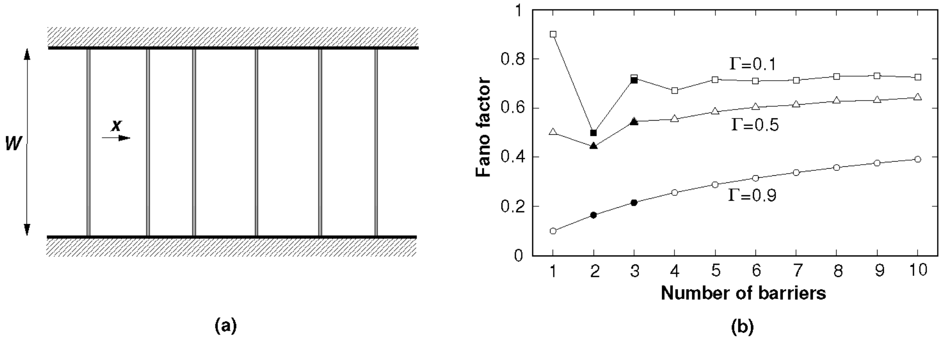

FIGURE 1. (a) Sketch of the considered structure. (b) Fano factor (averaged over 50 sets of interbarrier distances) for a series of ideal barriers, represented for three values of the transparency Γ. The empty symbols represent the numerical results, while the solid symbols come from the analytical calculation.

structures have a confinement width W of the order of some microns. In particular the values of the Fano factor and of the resistance that we show in the following have been obtained for $W = 8\ \mu$m, with a Fermi energy of 9.03 meV, which corresponds to 320 modes propagating in the structure; the results are properly averaged over a range of energy $qV = 40\ \mu$eV. In order to smooth out the fluctuations resulting from interference effects, we have averaged the results obtained from 50 different sets of interbarrier distances.

In Fig. 1(b) we show, as a function of the number of cascaded barriers, the values of the Fano factor achieved for three barrier transparencies, assumed to be independent of the longitudinal wave vector of the impinging electrons. It is apparent that these results (which are very close to those we have obtained with an exact description of the tunnel barriers) do not approach, increasing the number of barriers, the common 1/3 limit expected from semiclassical arguments.

STRONG LOCALIZATION

Indeed, due to the presence of just 1-D disorder, and consequently to the absence of mode-mixing, in this structure the transport problem is equivalent to a collection of intrinsically one-dimensional problems. Therefore, the localization length L_l is equal to the mean free path L_0 and it is impossible to satisfy the condition for diffusive transport ($L_0 << L_d << L_l$, with L_d being the length of the device). The presence of localization in the considered structure is confirmed by the exponential behavior of the resistance, as a function of the number of barriers, obtained from our numerical calculations (see Fig. 2(a), where we have considered ideal barriers and averaged over 50 different inter-barrier length sets). This is evident for opaque barriers, but also the nearly linear behavior observed for a series of highly transparent barriers actually represents a slowly exponential increase. Thus localization, which does not appear in common semiclassical models (not including phase coherence), does appear in quantum mechanical approaches and represents the key difference between the two types of theoretical analysis. In the ab-

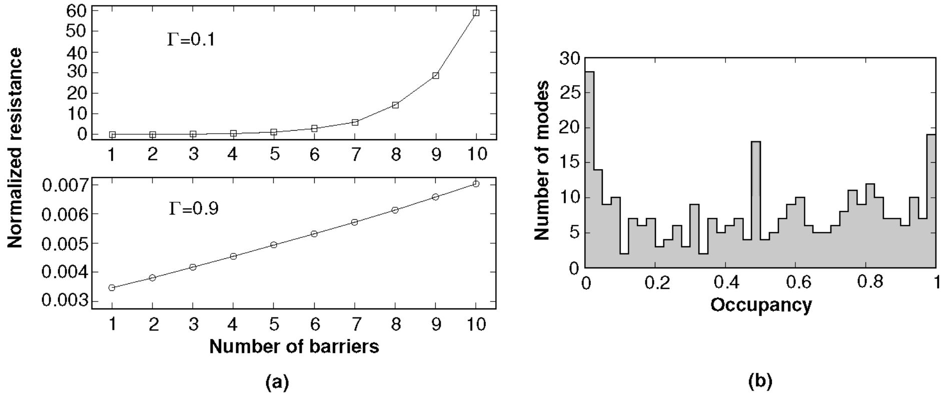

FIGURE 2. (a) Resistance (normalized with respect to the resistance quantum 12906.4 Ω) of a series of ideal barriers with $\Gamma = 0.1$ and 0.9, as a function of the number of barriers. (b) Distribution of the occupancy for the propagating modes in the 3rd interbarrier region of a series of six unequally spaced tunnel barriers with $\Gamma = 0.1$.

sence of a hypothetic mechanism leading to complete dephasing, the diffusive behavior can be recovered for a series of barriers only if strong mode-mixing is introduced in the structure, the number N of propagating modes is large, and the length of the device is such that the condition $L_0 << L_d << L_l$ (with $L_l = NL_0$) is satisfied.

The absence of mode-mixing makes it impossible also to define an occupancy depending only on energy in each interbarrier region, while this is one of the main assumptions that need to be made in semiclassical derivations. As an example, we have computed the occupancy of each propagating mode in the 3rd interbarrier region of a series of 6 unequally spaced barriers with an average transparency 0.1. This has been achieved for each mode dividing the partial density of states due to injection from the left lead by the total density of states. The distribution of the occupancies is shown in Fig. 2(b); the dispersion of these values demonstrates that it is impossible to define a unique value for the occupancy in the interbarrier region.

ANALYTICAL CALCULATIONS

Assuming ideal barriers with a transparency independent of the longitudinal wave vector of the impinging electrons, it has then been possible to find closed-form expressions for the Fano factor of the series of 2 and 3 tunnel barriers, as a function of their transparency. Indeed, from our simulations we have seen that the value of the overall transmission of the device (which appears at the denominator of γ in Eq. 1) is approximately identical in structures differing for the values of the interbarrier distances, due to the high number of propagating modes and to their averaging effect on transmission. Therefore it is reasonable, for this device, to separately average the numerator and the denominator of the Fano factor (over different sets of interbarrier distances), instead of averaging the values of the Fano factor for the different length sets. Therefore the mean Fano factor can be approximately expressed as $\gamma = \langle \sum_n T_n(1-T_n)\rangle / \langle \sum_n T_n \rangle = \sum_n (\langle T_n \rangle - \langle T_n^2 \rangle)/\sum_n \langle T_n \rangle$ (where the average is over sets of interbarrier distances). But, if the results are averaged over several sets of random interbarrier distances and the transparency of the barriers is identical for all the propagating modes, there is no dependence of the results on the longitudinal wave vector of the mode and therefore each mode gives the same contribution to the calculation. Therefore, dividing out the number of modes in the ratio, we have that $\gamma = (\langle T \rangle - \langle T^2 \rangle)/\langle T \rangle$, where each average is performed only on a single generical mode and depends only on the transparency Γ and on the number of the considered barriers. In particular, for each number of barriers, we have found, using the scattering matrix method, the expression of T and T^2 as a function of the phase contributions resulting from the traversal of each interbarrier region. Then we have averaged these two expressions over all possible values of the interbarrier distances, integrating each phase contribution between 0 and 2π. Following this procedure, we have obtained closed-form expressions for two and three cascaded barriers. If Γ is the barrier transparency, for a series of two barriers we have

$$\langle T \rangle = \frac{\Gamma}{2-\Gamma}, \quad \langle T^2 \rangle = \frac{\Gamma(2-2\Gamma+\Gamma^2)}{(2-\Gamma)^3} \quad \text{and thus} \quad \gamma = 1 - \frac{\langle T^2 \rangle}{\langle T \rangle} = \frac{2(1-\Gamma)}{(2-\Gamma)^2}, \tag{2}$$

and, for a series of three barriers,

$$\langle T \rangle = \frac{\Gamma^2}{\sqrt{\Gamma(4-3\Gamma)}}, \qquad \langle T^2 \rangle = \frac{\Gamma^2(4-6\Gamma^2+3\Gamma^3)}{\sqrt{\Gamma(4-3\Gamma)}(16-24\Gamma+9\Gamma^2)}$$

$$\text{and thus} \qquad \gamma = 1 - \frac{\langle T^2 \rangle}{\langle T \rangle} = \frac{3(4-8\Gamma+5\Gamma^2-\Gamma^3)}{16-24\Gamma+9\Gamma^2}. \tag{3}$$

This is coherent with our numerical results, as can be seen in Fig. 1(b), where the analytical results are shown with solid symbols. The first value is also equal to the one predicted semiclassically [2, 3]. Actually, for only 2 barriers a uniform occupancy in the middle region can be defined, with a value equal to the average between the occupancies in the input and output leads. In this special case, the phase averaging procedure is equivalent to the effect of a completely dephasing mechanism [6] and therefore the semiclassical results are recovered. However, already in the case of 3 barriers, results differ from the semiclassical ones [2, 3], as a consequence of localization.
Our results could be useful for the further investigation of experimental data obtained for superlattices [7] , which do significantly differ from the expected 1/3 limit.

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