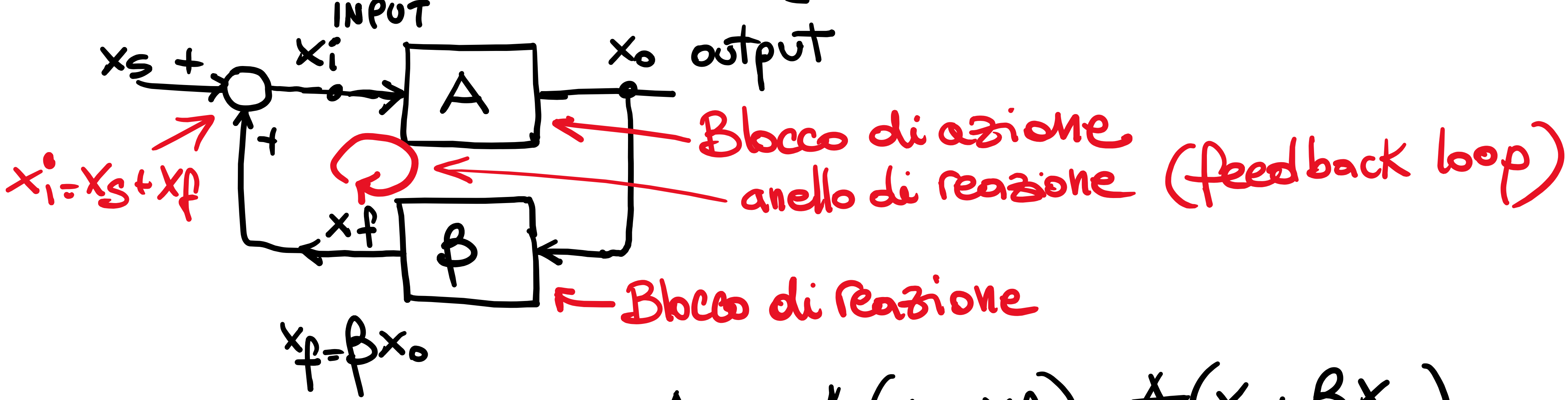
Circuiti in reazione

Sistemi lineare in reazione [Feedback systems]



$$A_F = \frac{x_o}{x_s}$$

$$x_o = A x_i = A(x_s + x_f) = A(x_s + \beta x_o)$$

$$x_o(1 - \beta A) = A x_s \Rightarrow \boxed{\frac{x_o}{x_s} = A_F = \frac{A}{1 - \beta A}}$$

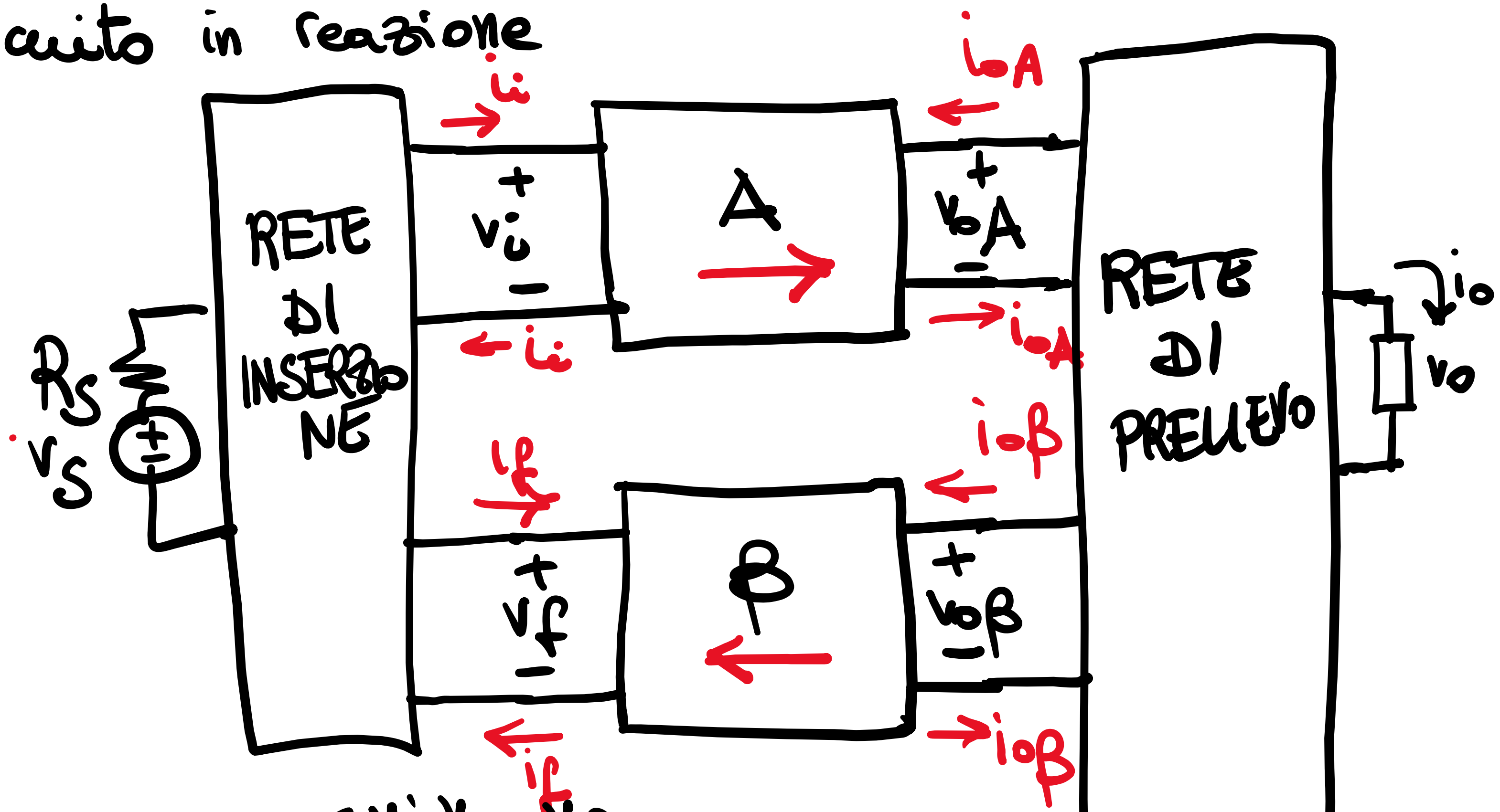
FUNZIONE DI TRASFERIMENTO DEL SISTEMA IN REAZIONE

- GLI ZERI DI A_F SONO GLI ZERI DI A
- I POLI DI A_F SONO GLI ZERI DI $(1 - \beta A)$
- SE $|\beta A| \gg 1$ ALLORA $A_F \sim -\frac{1}{\beta}$ NON DIPENDE DA A!

$(1 - \beta A) \leftarrow$ FATTORE DI REAZIONE

$\beta A \rightarrow$ GUADAGNO DELL'ANELLO DI REAZIONE

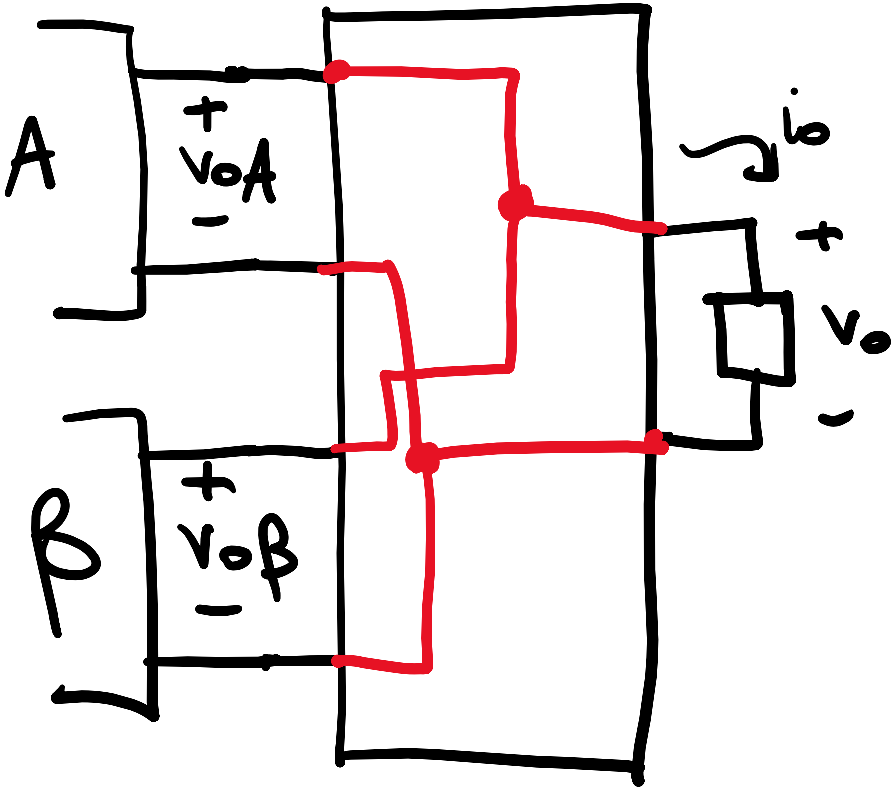
Circuito in reazione



$$x_i, x_s, x_f \begin{cases} v_i, v_s, v_f \\ i_i, i_s, i_f \end{cases} \qquad x_o \begin{cases} v_o \\ i_o \end{cases}$$

$V_o = X_o$

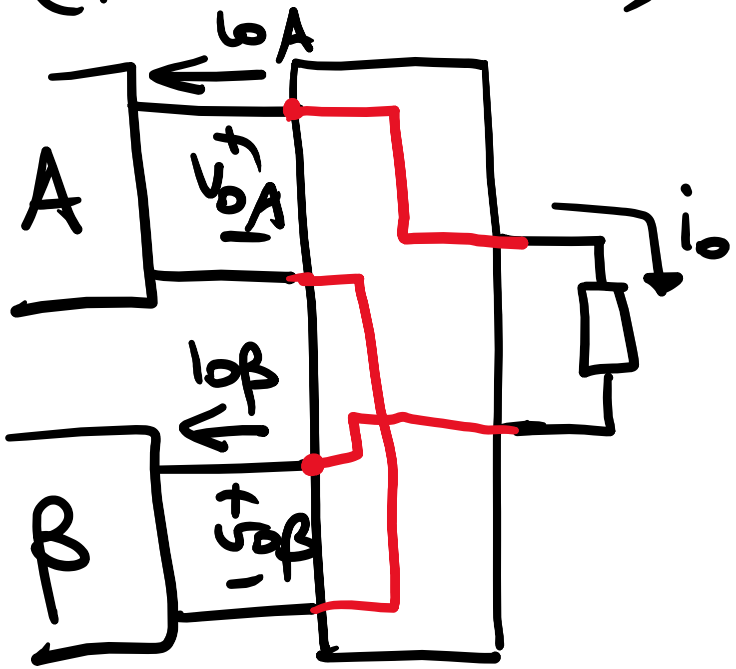
PRELIEVO DI TENSIONE
(PRELIEVO PARALLELO)



$v_o = v_{oA} = v_{oB}$

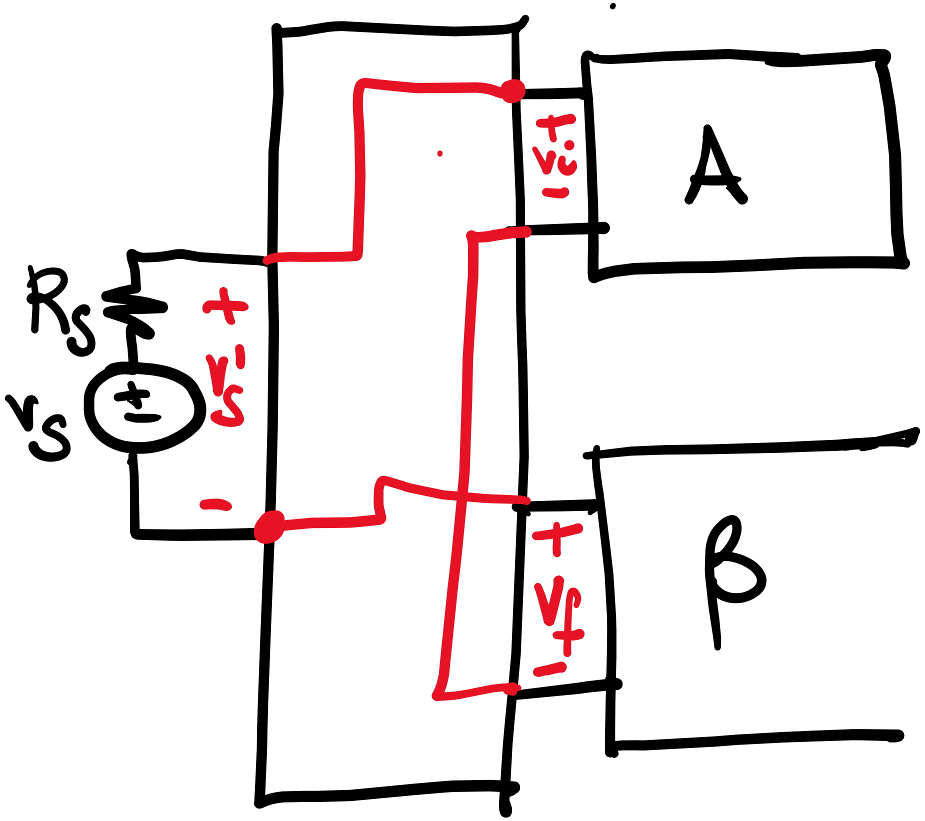
$i_o = X_o$

PRELIEVO DI CORRENTE
(PRELIEVO SERIE)



$i_o = -i_{oA} = i_{oB}$

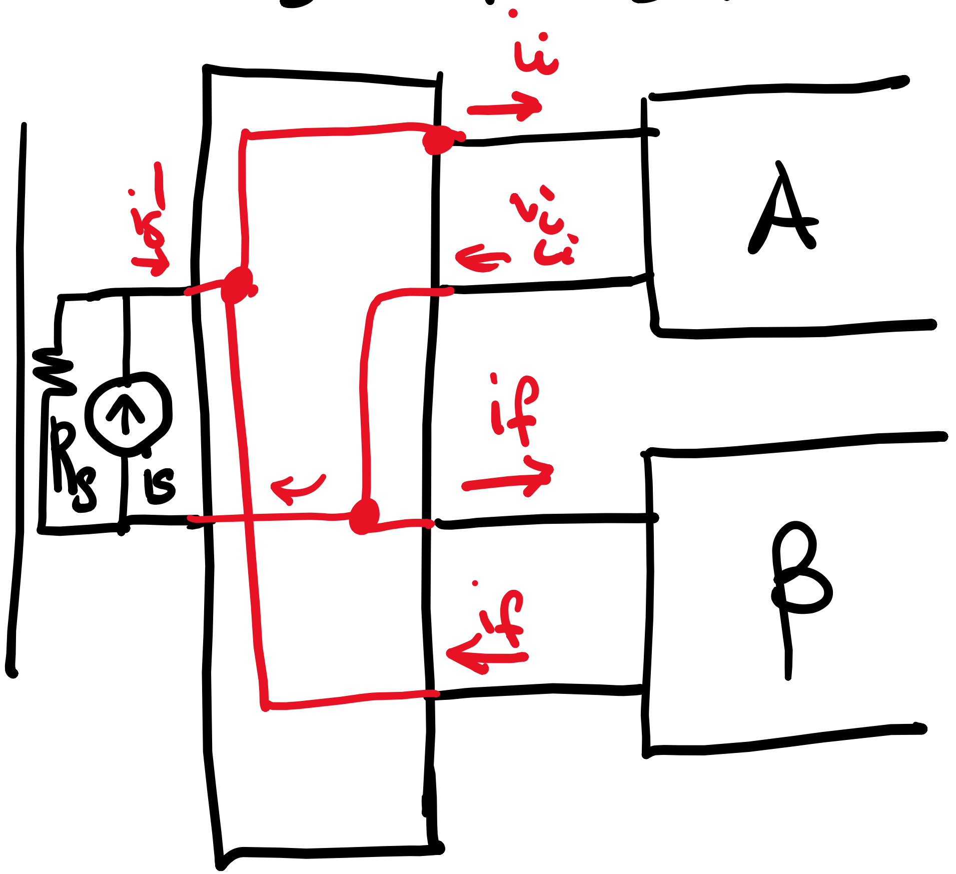
$x_i, x_s, x_f = v_i, v_s, v_f$



$v_i = v_s' + v_f$

INSERZIONE DI TENSIONE
(INSERZIONE SERIE)

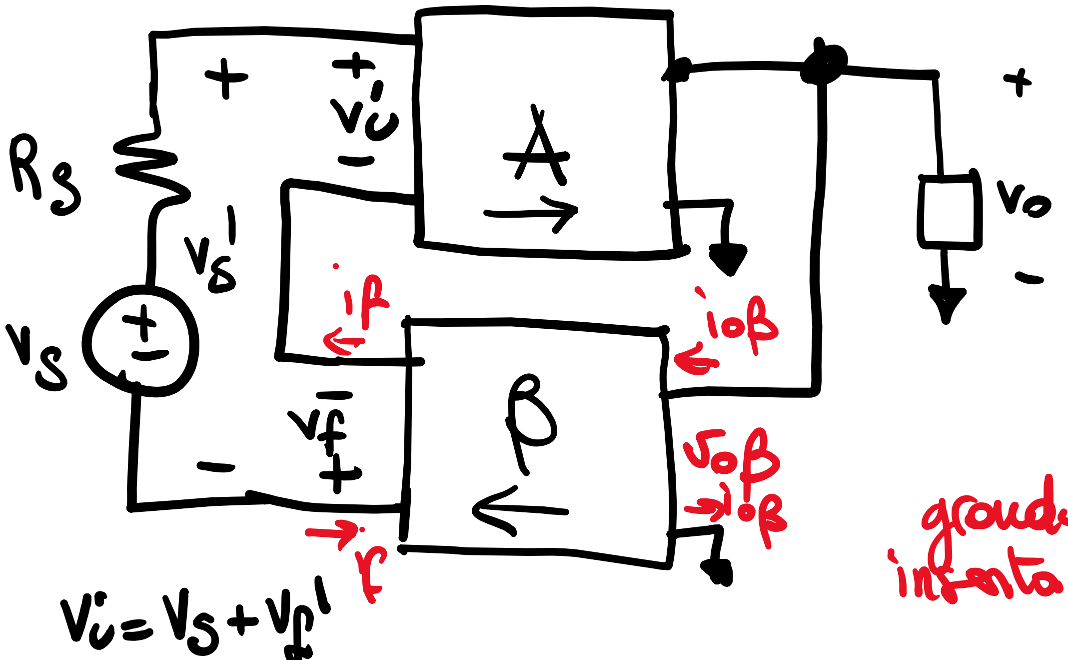
$x_i, x_s, x_f = i_i, i_s, i_f$



$i_i = i_s' + i_f$

INSERZIONE DI CORRENTE
(INSERZIONE PARALLELO)

① INSERZIONE DI TENSIONE, PRELIEVO DI TENSIONE (SERIE-PARALLELO)



$V_i = V_s + V_f'$

$X_s = V_s \qquad X_o = V_o$

$A_F = \frac{X_o}{X_s} = \frac{V_o}{V_s}$ ← Funzione di trasferimento del circuito in reazione

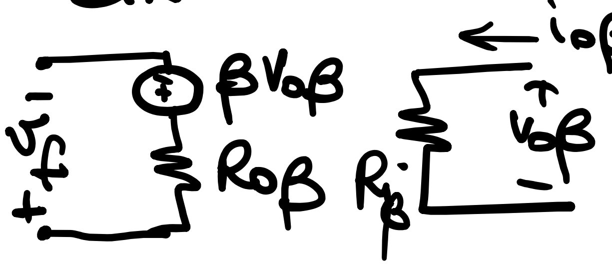
BLOCCO DI REAZIONE

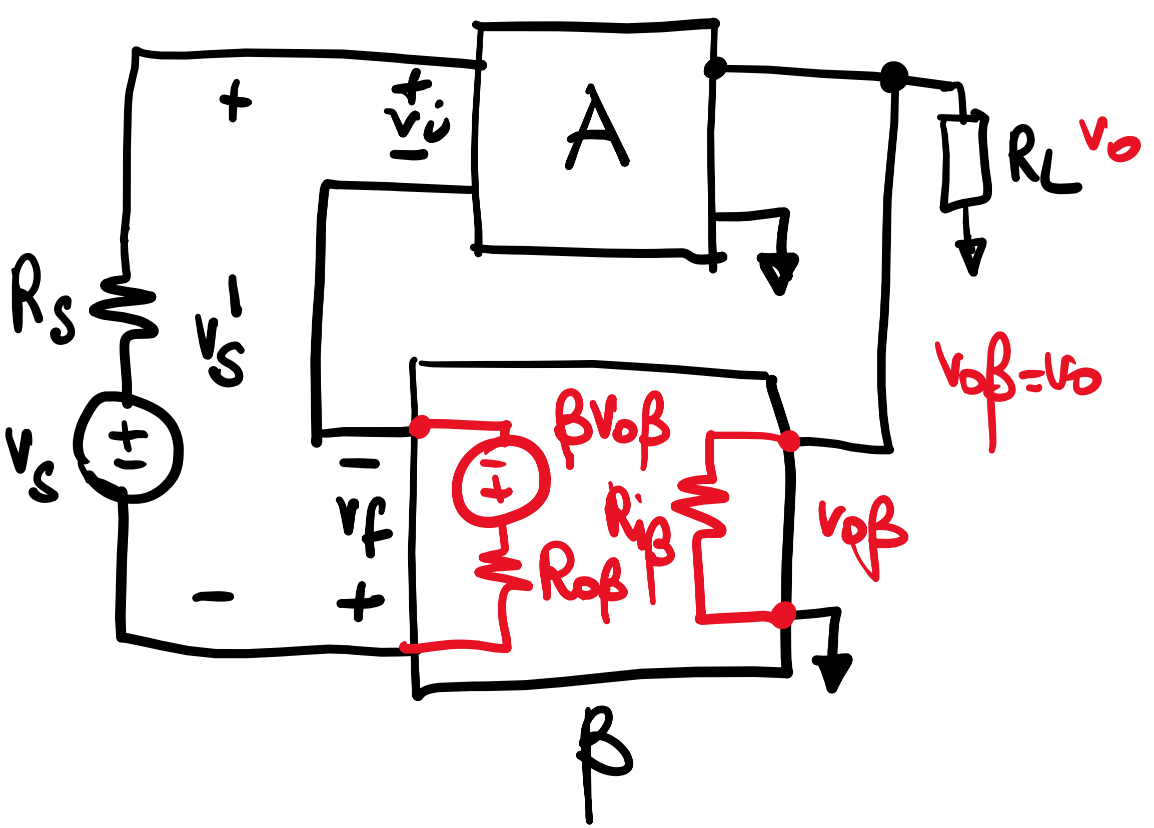
grandezza inserita $\begin{bmatrix} v_f \\ i_{o\beta} \end{bmatrix} = \begin{bmatrix} \beta & R_{o\beta} \\ \frac{1}{R_{i\beta}} & 0 \end{bmatrix} \begin{bmatrix} v_{o\beta} \\ i_f \end{bmatrix}$ ← grandezza prelevata

APPROSSIMAZIONE β UNIDIREZIONE

$\beta \triangleq \left.\frac{v_f}{v_o}\right|_{i_f=0}$; $R_{o\beta} \triangleq \left.\frac{v_f}{i_f}\right|_{v_o=0}$, $R_{i\beta} = \left.\frac{v_{o\beta}}{i_{o\beta}}\right|_{i_f=0}$

CIRCUITO EQUIVALENTE





$$A_F = \frac{v_o}{v_s}$$

DEFINIAMO

$$A_e \triangleq \left.\frac{v_o}{v_s}\right|_{\beta=0}$$

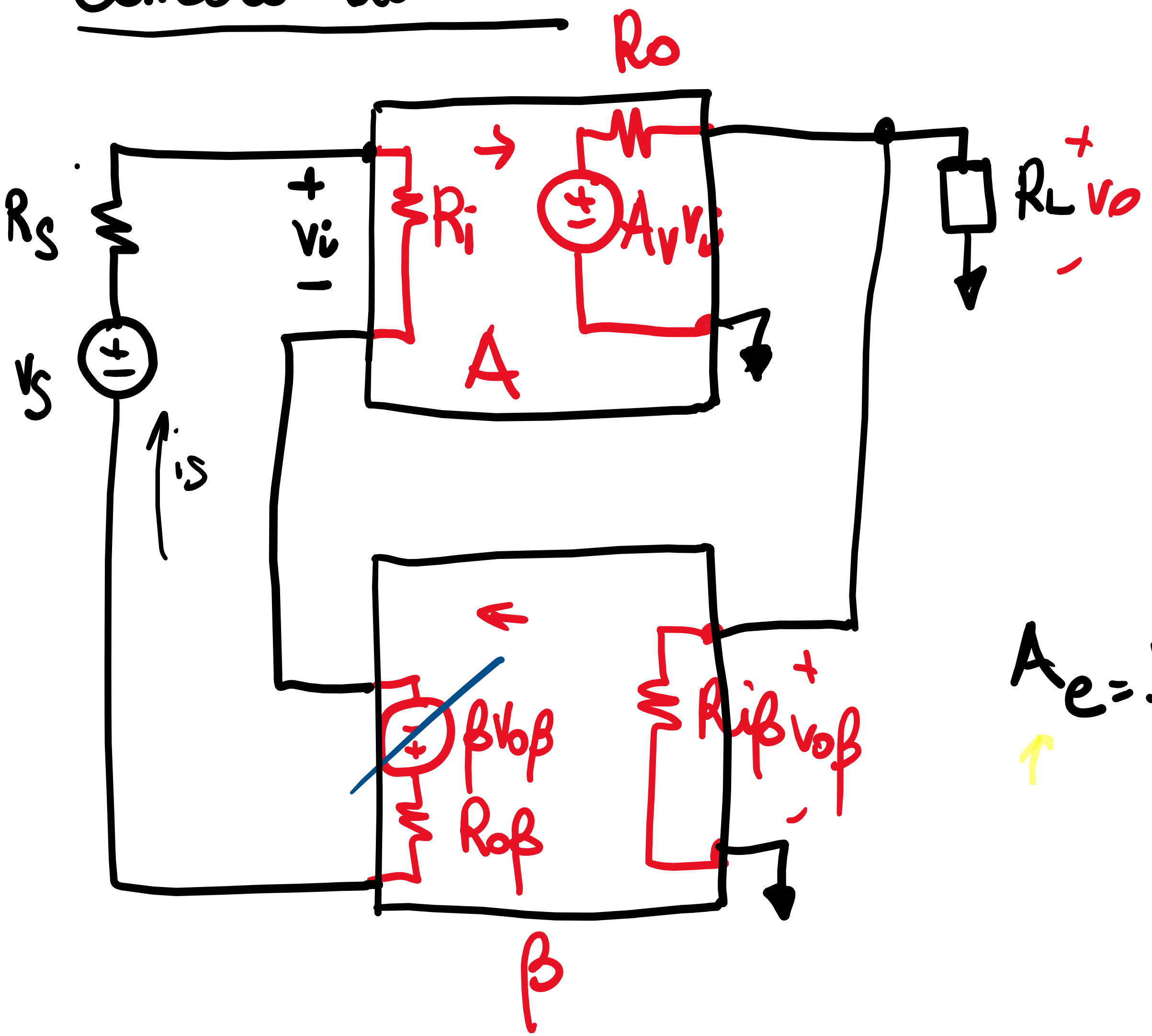
SE $\beta = 0$ ALLORA $v_o = A_e v_s$

SE $\beta \neq 0$ ALLORA $v_o = A_e(v_s + \beta v_o)$

$$v_o(1-\beta A_e) = A_e v_s$$

$$A_F = \frac{v_o}{v_s} = \frac{A_e}{1-\beta A_e}$$

Calcolo di Ae



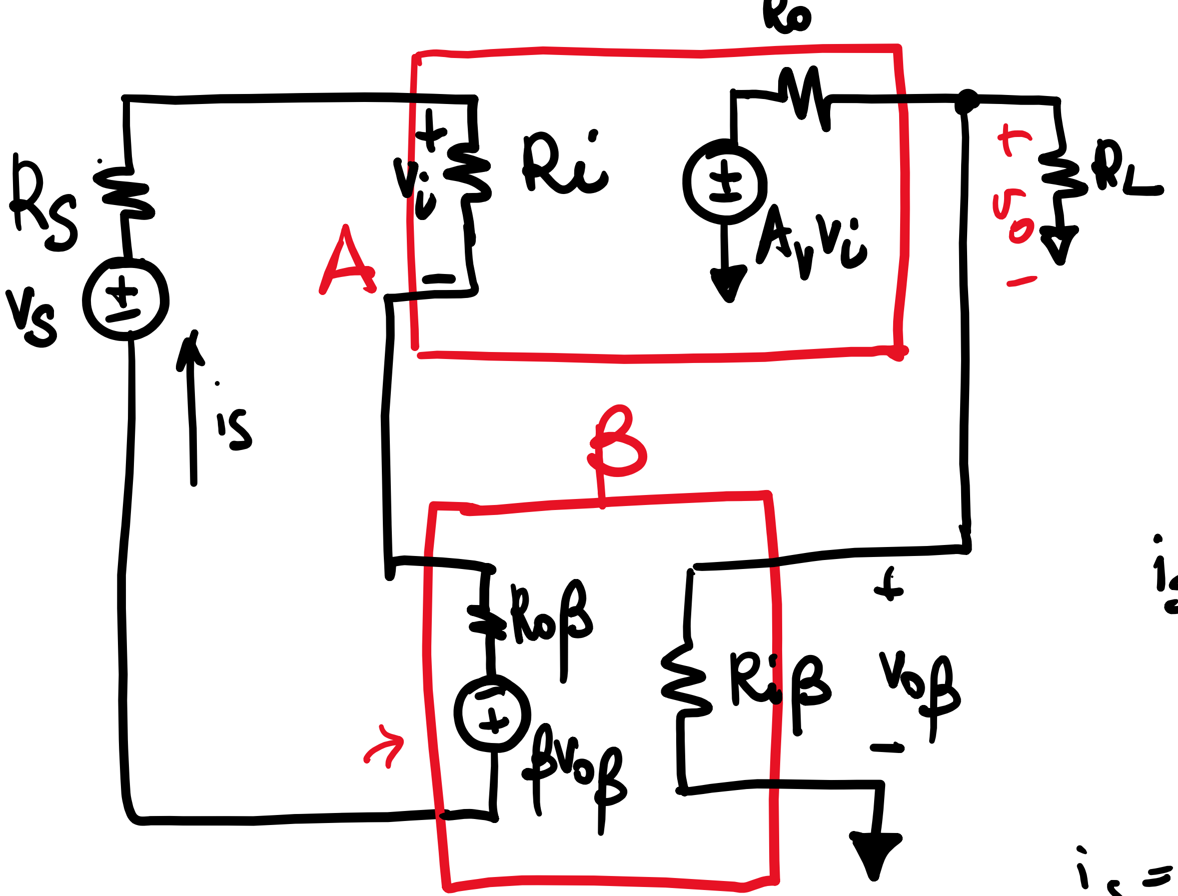
$$A_e = \left.\frac{V_o}{V_S}\right|_{\beta=0}$$

$$V_i = V_S \frac{R_i}{R_i + R_S + R_{o\beta}}$$

$$V_o = A_V V_i \frac{R_L \parallel R_{i\beta}}{R_o + R_L \parallel R_{i\beta}}$$

$$A_e = \left.\frac{V_o}{V_S}\right|_{\beta=0} = A_V \left[\frac{R_L \parallel R_{i\beta}}{R_o + R_L \parallel R_{i\beta}}\right] \frac{R_i}{R_i + R_S + R_{o\beta}}$$

IMPEDENZA DI INGRESSO



$$R_{IF} \triangleq \left.\frac{V_S}{i_S}\right|_{R_S=0}$$

Feedback

$$i_S = \frac{V_S + \beta V_o}{R_S + R_i + R_{o\beta}}$$

$$V_o = A_F V_S$$

$$V_o = \frac{A_e}{1-\beta A_e} V_S$$

$$i_S = \frac{V_S\left[1 + \frac{\beta A_e}{1-\beta A_e}\right]}{R_S + R_i + R_{o\beta}} =$$

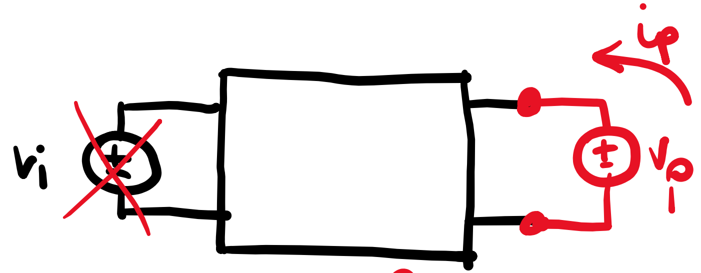
$$i_S = \frac{V_S}{(1-\beta A_e)(R_S + R_i + R_{o\beta})}$$

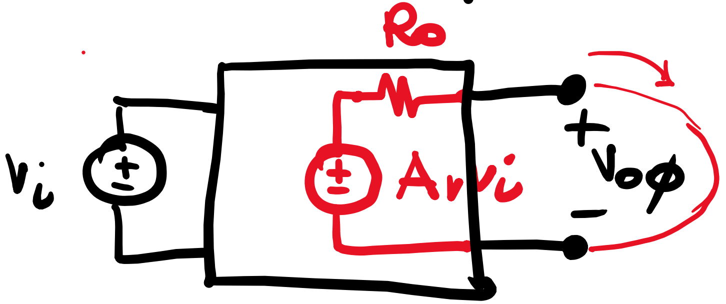
$$\boxed{R_{IF} = \left.\frac{V_S}{i_S}\right|_{R_S=0} = (R_i + R_{o\beta})\left(1 - \beta A_e\big|_{R_S=0}\right)}$$

POICHÉ TIPICAMENTE $|\beta A_e| \gg 1$

ABBIAMO $R_{IF} \gg R_i$

IMPEDENZA DI USCITA





DEFINIZIONE

$$R_o \triangleq \frac{v_p}{i_p}\Bigg|_{\text{tutti i generatori indipendenti SPENTI}}$$

metodo per calcolare la resistenza di uscita

Tensione di uscita a vuoto

$$v_{o\phi} = v_o\Big|_{i_o=0}$$

Corrente di uscita di corto circuito

$$i_{occ} = i_o\Big|_{v_o=0}$$

$$v_{o\phi} = A_v v_i$$

$$i_{occ} = \frac{A_v v_i}{R_o}$$

$$\rightarrow \boxed{\frac{v_{o\phi}}{i_{occ}} = R_o}$$

IMPEDENZA DI USCITA

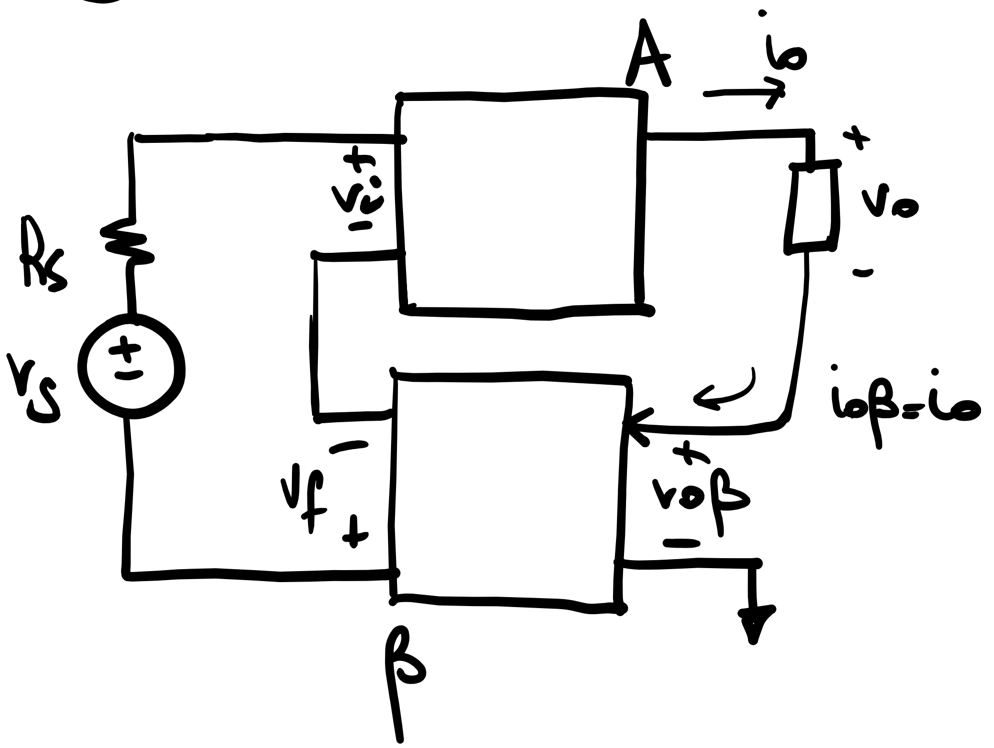
$$R_{oF} = \frac{v_{o\emptyset}}{i_{occ}} = \frac{\lim\limits_{R_L\to\infty}\left(v_o\right)}{\lim\limits_{R_L\to 0}\left(\frac{v_o}{R_L}\right)} = \frac{\lim\limits_{R_L\to\infty}\left(\frac{v_o}{v_s}\right)}{\lim\limits_{R_L\to 0}\left(\frac{1}{R_L}\frac{v_o}{v_s}\right)} = \frac{\lim\limits_{R_L\to\infty}\left[\frac{A_e}{1-\beta A_e}\right]}{\lim\limits_{R_L\to 0}\left[\frac{1}{R_L}\frac{A_e}{1-\cancel{\beta A_e}}\right]}$$

A_F

$$A_e = A_v \frac{R_L /\!/ R_{i\beta}}{\underbrace{R_o + R_L /\!/ R_{i\beta}}} \frac{R_i}{R_i + R_s + R_{o\beta}}$$

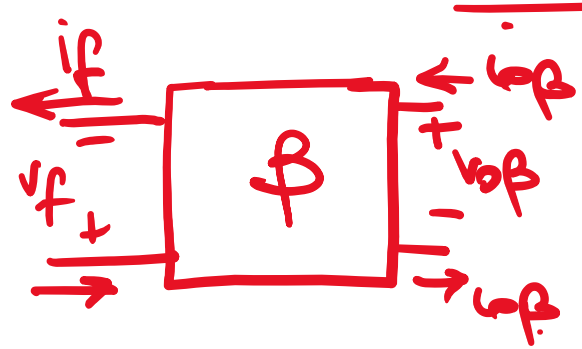
$$R_{oF} = \frac{\left[\dfrac{\dfrac{R_{i\beta}}{R_o + R_{i\beta}}}{1-\beta\lim\limits_{R_L\to\infty}(A_e)}\right]}{\lim\limits_{R_L\to 0}\left[\frac{1}{\cancel{R_L}}\frac{1}{R_o}\frac{\cancel{R_L}\cancel{R_{i\beta}}}{\cancel{R_L}+\cancel{R_{i\beta}}}\right]} = \frac{\dfrac{R_{i\beta}R_o}{R_o+R_{i\beta}}}{\left[1-\beta\lim\limits_{R_L\to\infty}(A_e)\right]} = \boxed{\frac{R_o /\!/ R_{i\beta}}{1-\beta A_e\Big|_{R_L\to\infty}}}$$

② INSERZIONE DI TENSIONE – PRELIEVO DI CORRENTE (SERIE - SERIE)



$x_o = i_o \quad x_s = v_s \quad \underbrace{A_F = \frac{x_o}{x_s} = \frac{i_o}{v_s}}$

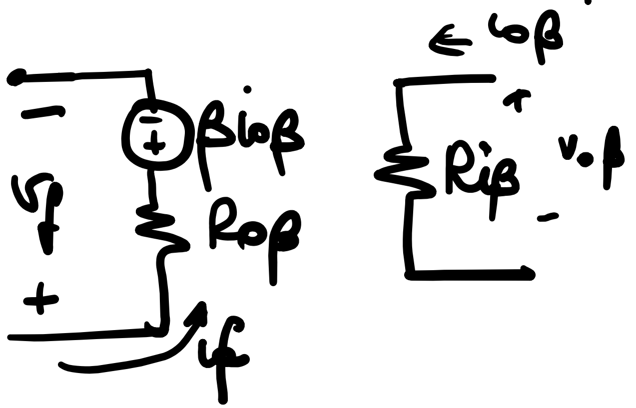
RETE DEL β

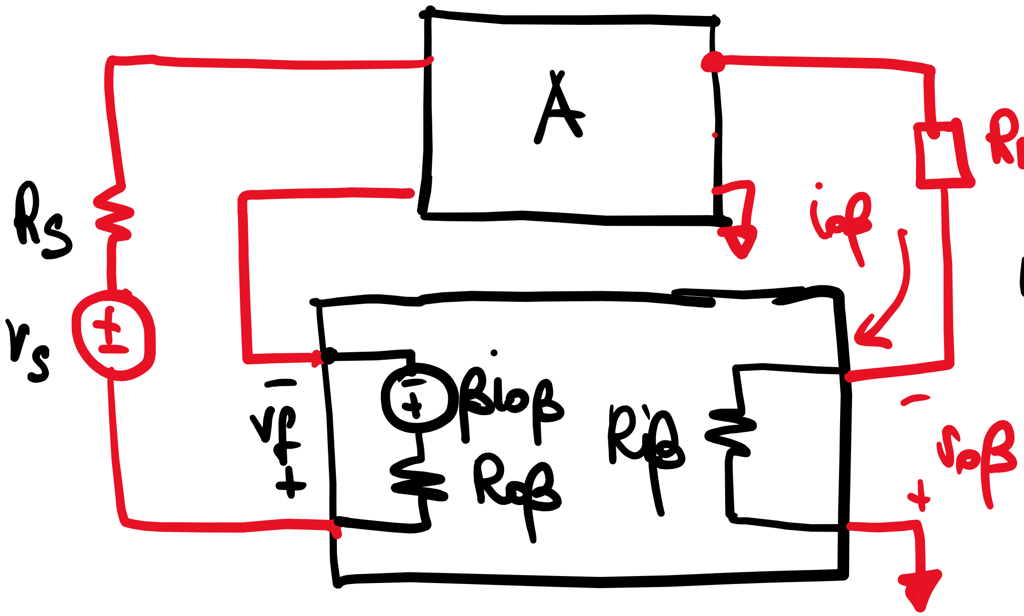


$$\begin{bmatrix} v_f \\ v_{o\beta} \end{bmatrix} = \begin{bmatrix} \beta & R_{o\beta} \\ R_{i\beta} & 0 \end{bmatrix} \begin{bmatrix} i_{o\beta} \\ i_f \end{bmatrix}$$

approx.

$$\beta = \frac{v_f}{i_{o\beta}}\bigg|_{i_f=0}, \quad R_{o\beta} = \frac{v_f}{i_f}\bigg|_{i_{o\beta}}, \quad R_{i\beta} = \frac{v_{o\beta}}{i_{o\beta}}\bigg|_{i_f=0}$$





$i_{o\beta} = i_o$

$$A_F = \frac{i_o}{v_s}$$

definiamo $A_e = \left.\frac{i_o}{v_s}\right|_{\beta=0}$

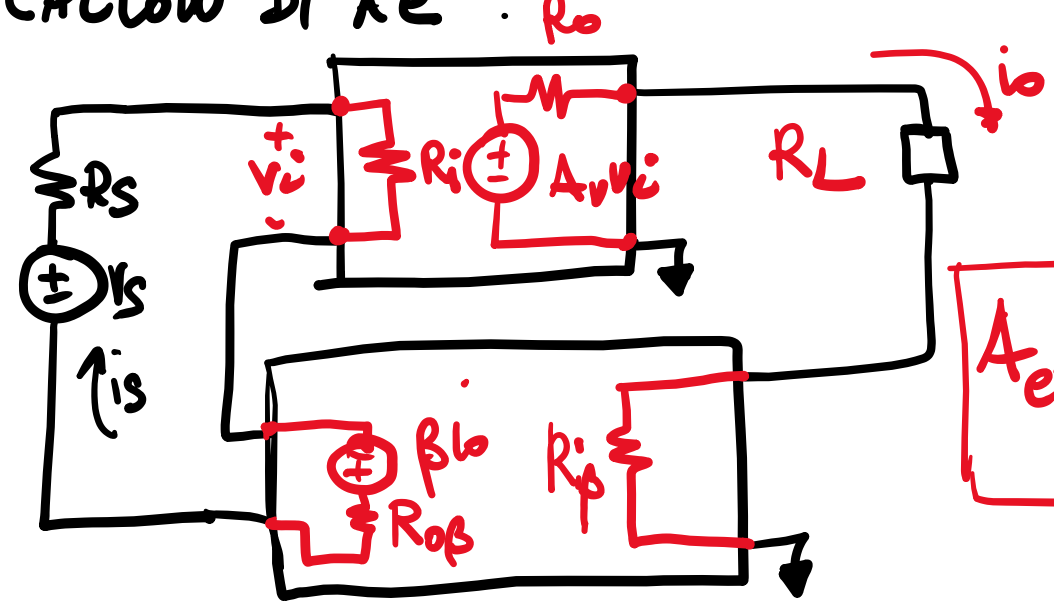
se $\beta = 0$ allora $i_o = A_e v_s$

se $\beta \neq 0$ allora $i_o = A_e(v_s + \beta i_o)$

; $i_o(1 - \beta A_e) = A_e v_s$

$$A_F = \frac{i_o}{v_s} = \frac{A_e}{1 - \beta A_e}$$

CALCOLO DI A_e .



$$v_i = v_s \frac{R_i}{R_s + R_i + R_{o\beta}} \qquad i_o = \frac{A_v v_i}{R_o + R_L + R_{i\beta}}$$

$$\boxed{A_e = \left.\frac{i_o}{v_s}\right|_{\beta=0} = A_v \frac{1}{R_o + R_L + R_{i\beta}} \cdot \frac{R_i}{R_s + R_i + R_{o\beta}}}$$

IMPEDENZA DI INGRESSO R_I

$$R_{IF} = \left.\frac{v_s}{i_s}\right|_{R_S=0}$$

$$\rightarrow \frac{i_o}{v_s} = A_F = \frac{A_e}{1-\beta A_e}$$

$$i_s = \frac{v_s + \beta i_o}{R_S + R_i + R_{o\beta}} = \frac{v_s\left[1 + \frac{\beta A_e}{1-\beta A_e}\right]}{R_S + R_i + R_{o\beta}} =$$

$$i_s = \frac{v_s}{(1-\beta A_e)(R_S + R_i + R_{o\beta})} \rightarrow \boxed{R_{IF} = \left.\frac{v_s}{i_s}\right|_{R_S\to 0} = (R_i + R_{o\beta})\left(1-\beta A_e\big|_{R_S=0}\right)}$$

IMPEDENZA DI USCITA

$$R_{OF} = \frac{v_{o\varnothing}}{i_{occ}} = \frac{\lim\limits_{R_L\to\infty}\left[\frac{i_o R_L}{v_s}\right]}{\lim\limits_{R_L\to 0}\left[\frac{i_o}{v_s}\right]} = \frac{\lim\limits_{R_L\to\infty}\left[\frac{A_e}{1-\cancel{\beta A_e}} R_L\right]}{\lim\limits_{R_L\to 0}\left[\frac{A_e}{1-\beta A_e}\right]} = \frac{\lim\limits_{R_L\to\infty}\left[\frac{1}{R_o + R_L + R_{i\beta}} \cdot R_L\right]}{\lim\limits_{R_L\to 0}\left[\frac{\frac{1}{R_o + \cancel{R_L} + R_{i\beta}}}{(1-\beta A_e)}\right]} =$$

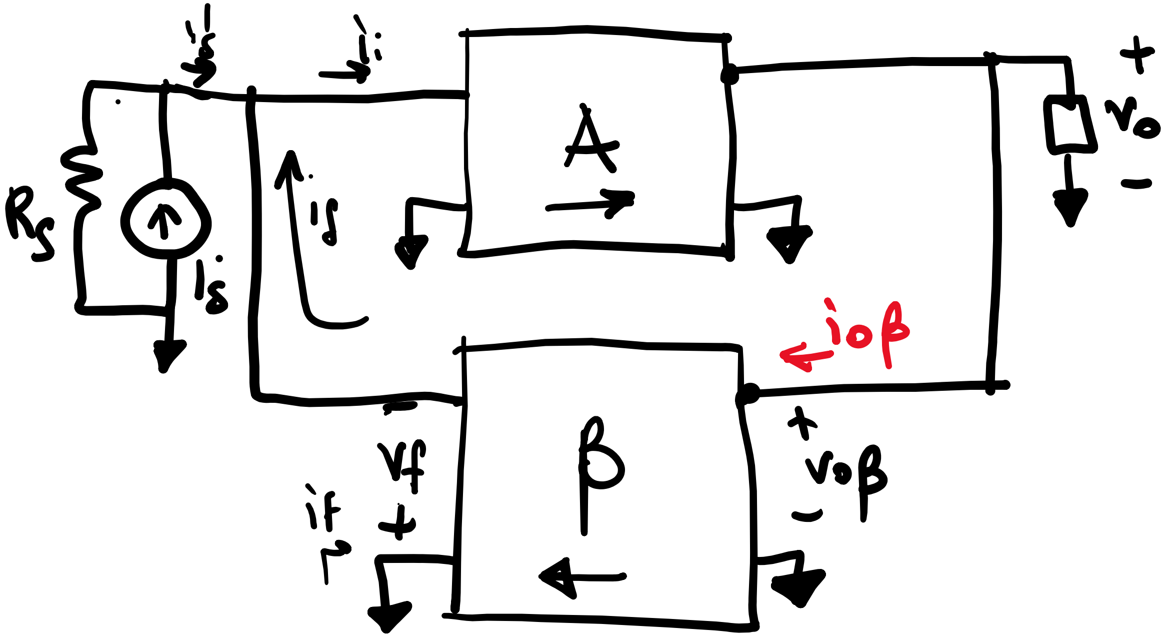
A_F A_F

$$R_{OF} = (R_o + R_{i\beta}) \lim_{R_L\to 0}(1-\beta A_e) = \boxed{(R_o + R_{i\beta})\left(1-\beta A_e\big|_{R_L\to 0}\right)}$$

$\Leftarrow \Re|\beta A_e| \gg 1$

$$\underline{R_{OF} > R_o}$$

③ INSERZIONE DI CORRENTE, PRELIEVO DI TENSIONE

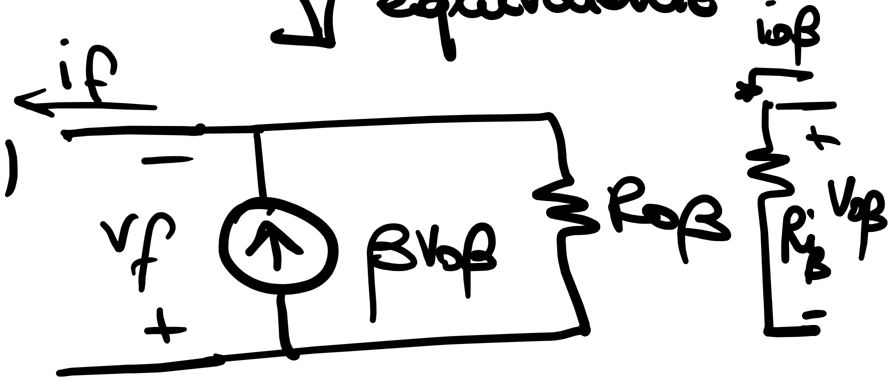


$x_o = v_o$ $\quad x_s = i_s$ $\quad A_f = \dfrac{v_o}{i_s}$

Rete del β

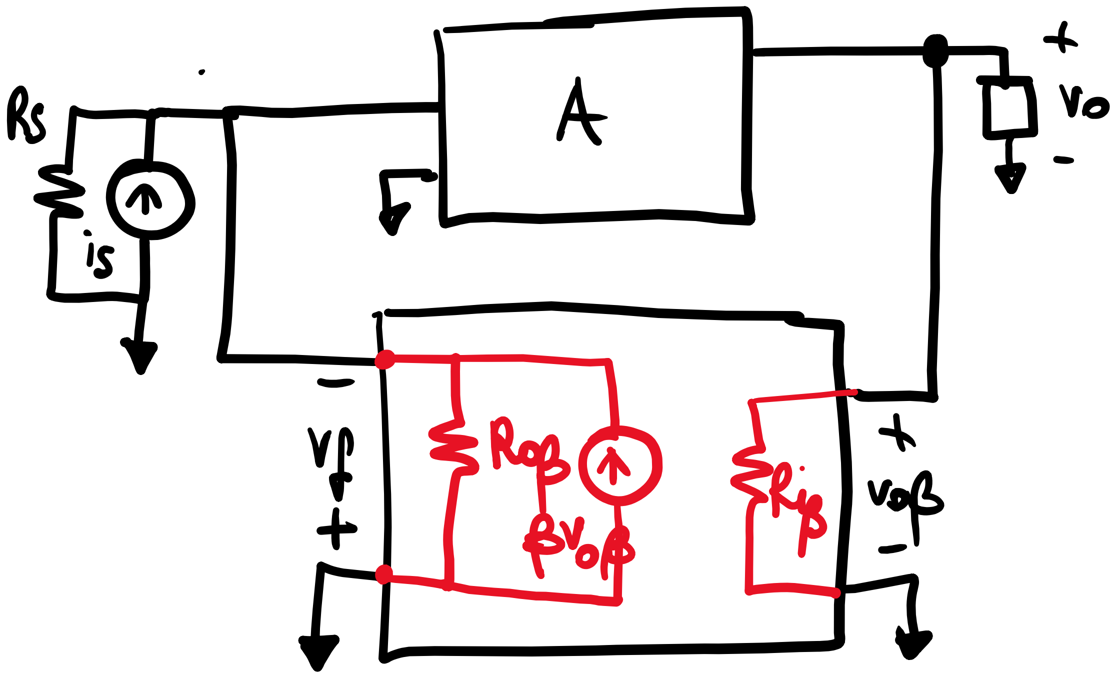
$$\begin{bmatrix} i_f \\ i_{o\beta} \end{bmatrix} = \begin{bmatrix} \beta & \frac{1}{R_{o\beta}} \\ \frac{1}{R_{i\beta}} & / \end{bmatrix} \begin{bmatrix} v_{o\beta} \\ v_f \end{bmatrix}$$

circuito equivalente



$\beta \triangleq \left.\dfrac{i_f}{v_{o\beta}}\right|_{v_f=0}$, $\quad R_{o\beta} \triangleq \left.\dfrac{v_f}{i_f}\right|_{v_{o\beta}=0}$, $\quad R_{i\beta} \triangleq \left.\dfrac{v_{o\beta}}{i_{o\beta}}\right|_{v_f=0}$,

Calcolo di A_e



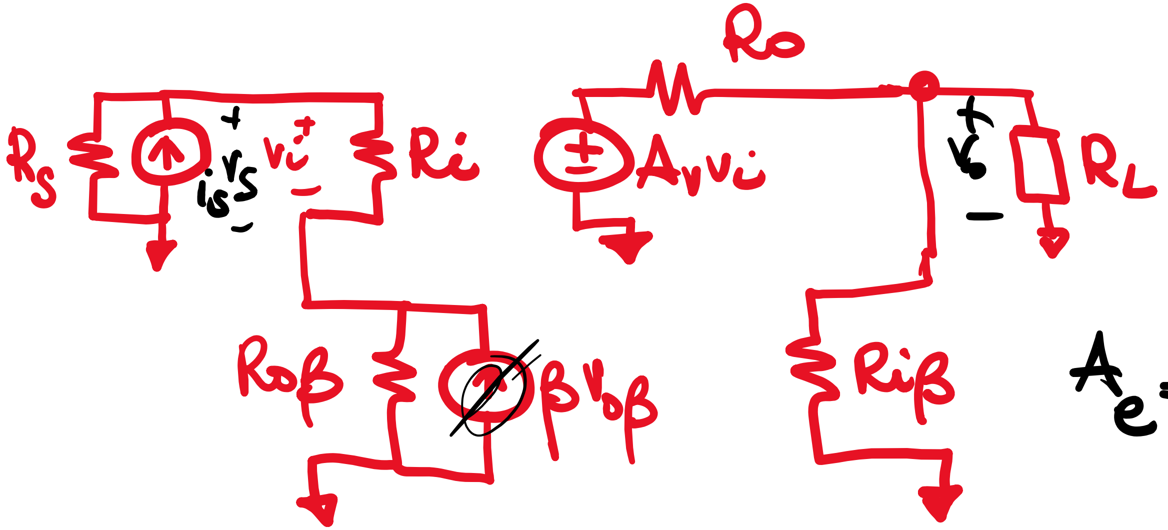
$$A_F = \frac{v_o}{i_s} \Rightarrow A_e \triangleq \frac{v_o}{i_s}\Big|_{\beta=0}$$

se $\beta = 0$: $v_o = A_e i_s$

se $\beta \neq 0$: $v_o = A_e(i_s + \beta v_o)$

$$v_o(1-\beta A_e) = A_e i_s$$

$$\boxed{A_F = \frac{v_o}{i_s} = \frac{A_e}{1-\beta A_e}}$$



$$v_i = i_s [R_s // R_i // R_{o\beta}]$$

$$v_o = A_v v_i \frac{R_L // R_{i\beta}}{R_L // R_{i\beta} + R_o}$$

$$A_e = \frac{v_o}{i_s}\Big|_{\beta=0} = A_v \frac{R_L // R_{i\beta}}{R_L // R_{i\beta} + R_o} [R_s // R_i // R_{o\beta}]$$

RESISTENZA DI INGRESSO

$$R_{IF} = \left.\frac{v_S}{i_S}\right|_{R_S\to\infty} \quad v_S = \left(i_S + \beta v_o\right)\left(R_S /\!/ R_i /\!/ R_{o\beta}\right) = i_S\left[1 + \frac{\beta A_e}{1-\beta A_e}\right]\left(R_S /\!/ R_i /\!/ R_{o\beta}\right)$$

$$\hookrightarrow v_o = A_F i_S = \frac{A_e}{1-\beta A_e} i_S$$

$$= \frac{1}{1-\beta A_e}$$

$$v_S = i_S \frac{R_S /\!/ R_i /\!/ R_{o\beta}}{1-\beta A_e} \Rightarrow R_{IF} = \left.\frac{v_S}{i_S}\right|_{R_S\to\infty} = \boxed{\frac{R_i /\!/ R_{o\beta}}{1-\beta A_e|_{R_S\to\infty}}}$$

RESISTENZA DI USCITA

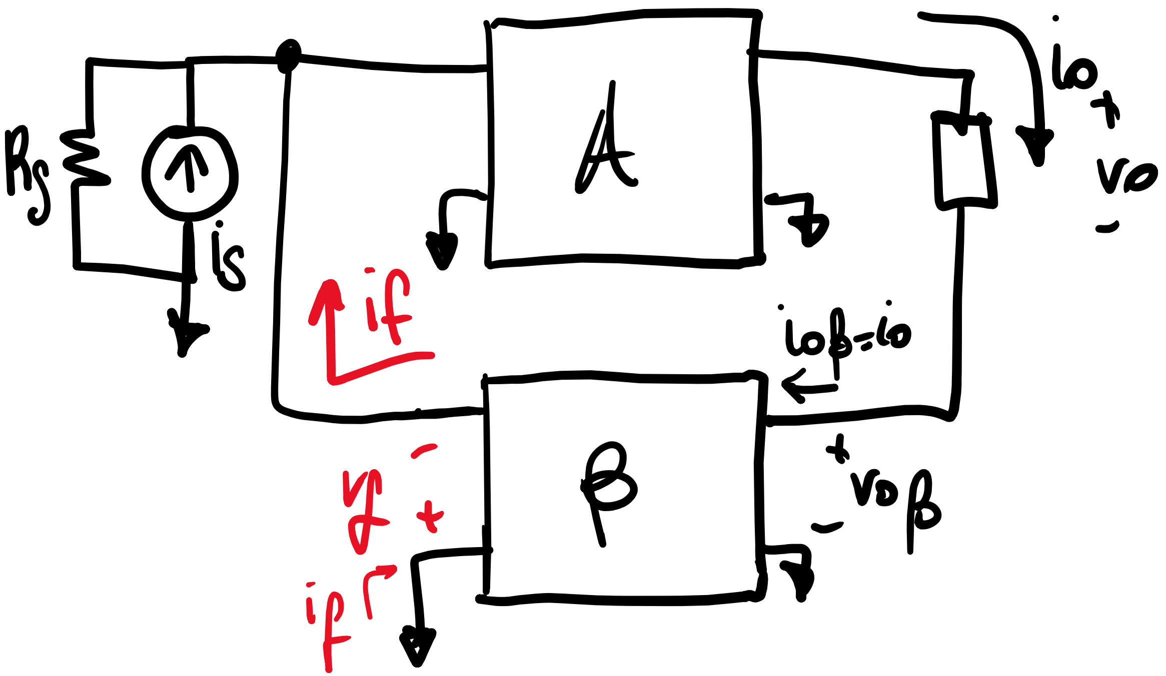
$$A_F$$

$$R_{oF} = \frac{v_{o\emptyset}}{i_{occ}} = \frac{\lim_{R_L\to\infty}\left[\frac{v_o}{i_S}\right]}{\lim_{R_L\to 0}\left[\frac{v_o}{R_L\, i_S}\right]} = \frac{\lim_{R_L\to\infty}\left[\frac{A_e}{1-\beta A_e}\right]}{\lim_{R_L\to 0}\left[\frac{1}{R_L}\frac{A_e}{1-\cancel{\beta A_e}}\right]} = \frac{\lim_{R_L\to\infty}\left[\frac{1}{1-\beta A_e}\right]\frac{R_{i\beta}}{R_{i\beta}+R_o}}{\lim_{R_L\to 0}\left[\frac{1}{\cancel{R_L}}\cdot\frac{\cancel{R_L}\cdot R_{i\beta}}{\cancel{R_L+R_{i\beta}}}\cdot\frac{1}{R_o}\right]}$$

$$R_{oF} = \frac{R_o R_{i\beta}}{R_o + R_{i\beta}}\,\frac{1}{1-\beta \lim_{R_L\to\infty}(A_e)} = \frac{R_o /\!/ R_{i\beta}}{1-\beta A_e|_{R_L\to\infty}}$$

④ INSERZIONE DI CORRENTE, PRELIEVO DI CORRENTE
(PARALLELO - SERIE)

$x_o = i_o$, $x_s = i_s$ $\quad A_F = \frac{i_o}{i_s}$

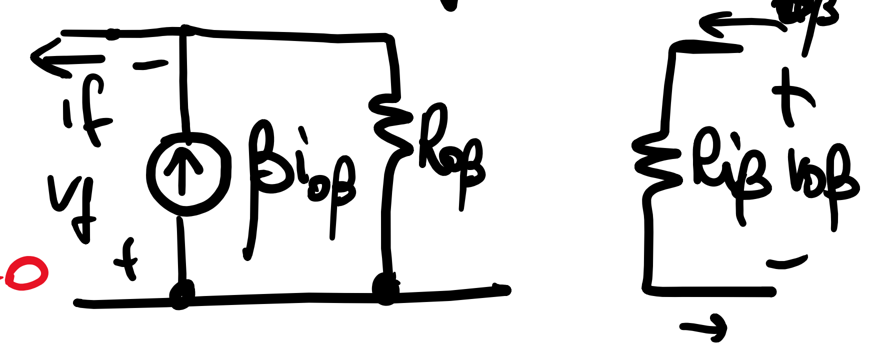


Rete del β

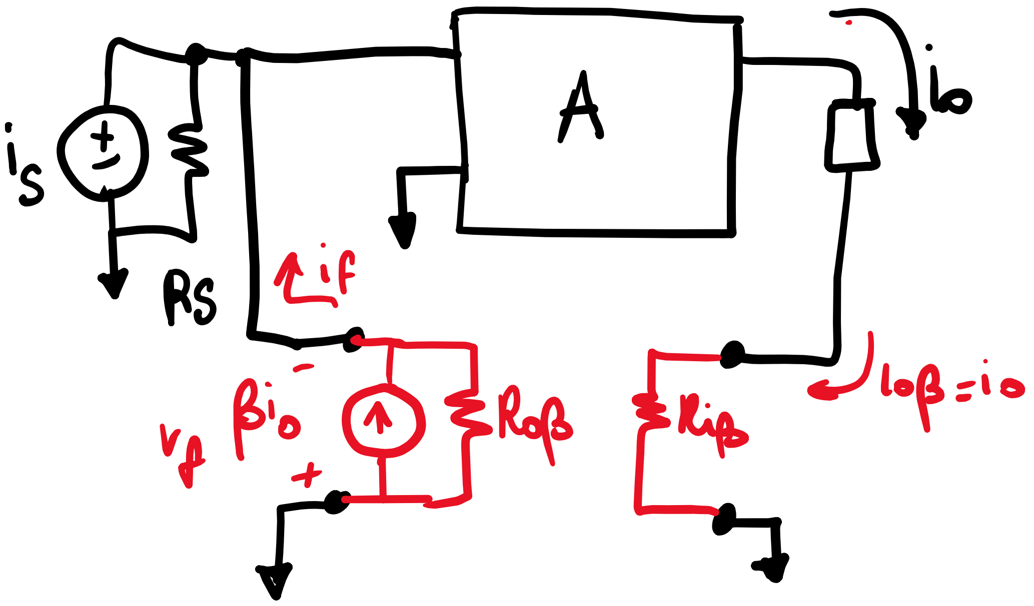
$$\begin{bmatrix} i_f \\ v_{o\beta} \end{bmatrix} = \begin{bmatrix} \beta & \frac{1}{R_{o\beta}} \\ R_{i\beta} & 0 \end{bmatrix} \begin{bmatrix} i_{o\beta} \\ v_f \end{bmatrix}$$

approx unidirezionale

circuito equivalente



$$\beta \triangleq \left.\frac{i_f}{i_{o\beta}}\right|_{v_f=0} \qquad R_{o\beta} \triangleq \left.\frac{v_f}{i_f}\right|_{i_{o\beta}=0} \qquad R_{i\beta} \triangleq \left.\frac{v_{o\beta}}{i_{o\beta}}\right|_{v_f=0}$$



$$A_F = \frac{i_o}{i_s} \Rightarrow A_e \triangleq \left.\frac{i_o}{i_s}\right|_{\beta=0}$$

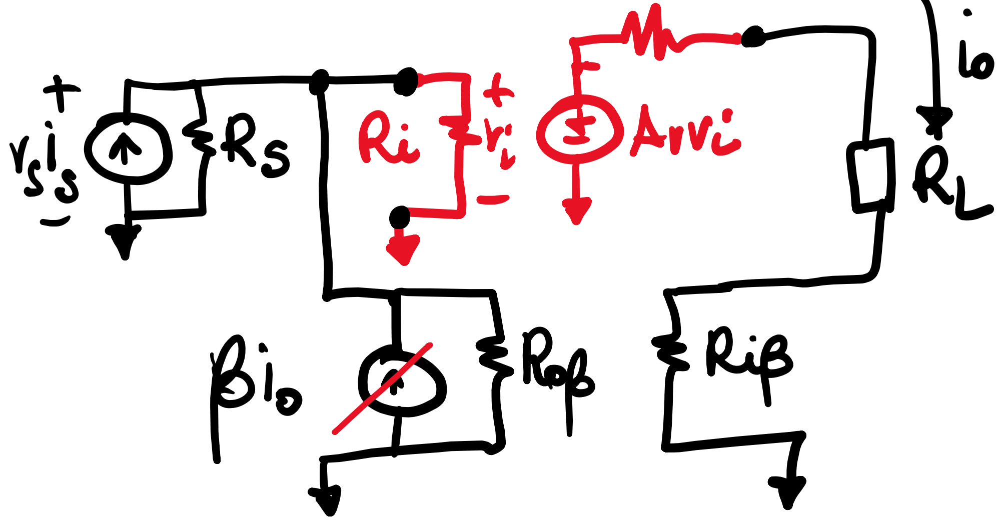
se $\beta = 0$ $\quad i_o = A_e i_s$

se $\beta \neq 0$ $\quad i_o = A_e (i_s + \beta i_o)$

$$i_o(1-\beta A_e) = A_e i_s$$

$$\boxed{A_F = \frac{i_o}{i_s} = \frac{A_e}{1-\beta A_e}}$$

Calcolo di A_e R_o



$$v_i = i_s \left[R_s // R_i // R_{o\beta}\right] \qquad i_o = \frac{A_v v_i}{R_o + R_L + R_{i\beta}}$$

$$A_e = \left.\frac{i_o}{i_s}\right|_{\beta=0} = \frac{A_v (R_s // R_i // R_{o\beta})}{R_o + R_L + R_{i\beta}}$$

Impedenza di ingresso

$$R_{IF} = \left.\frac{v_S}{i_S}\right|_{R_S=0}$$

$$v_S = (i_S + \beta i_o)(R_S /\!/ R_i /\!/ R_{o\beta}) = i_S\left[1 + \frac{\beta A_e}{1-\beta A_e}\right](R_S /\!/ R_i /\!/ R_{o\beta})$$

$$\frac{i_o}{i_S} = \frac{A_e}{1-\beta A_e}$$

$$= i_S \frac{(R_S /\!/ R_i /\!/ R_{o\beta})}{(1-\beta A_e)}$$

$$\boxed{R_{IF} = \left.\frac{v_S}{i_S}\right|_{R_S=0} = \frac{(R_i /\!/ R_{o\beta})}{1-\beta A_e|_{R_S\to 0}}}$$

Impedenza di uscita

$$A_F = \frac{A_e}{1-\beta A_e}$$

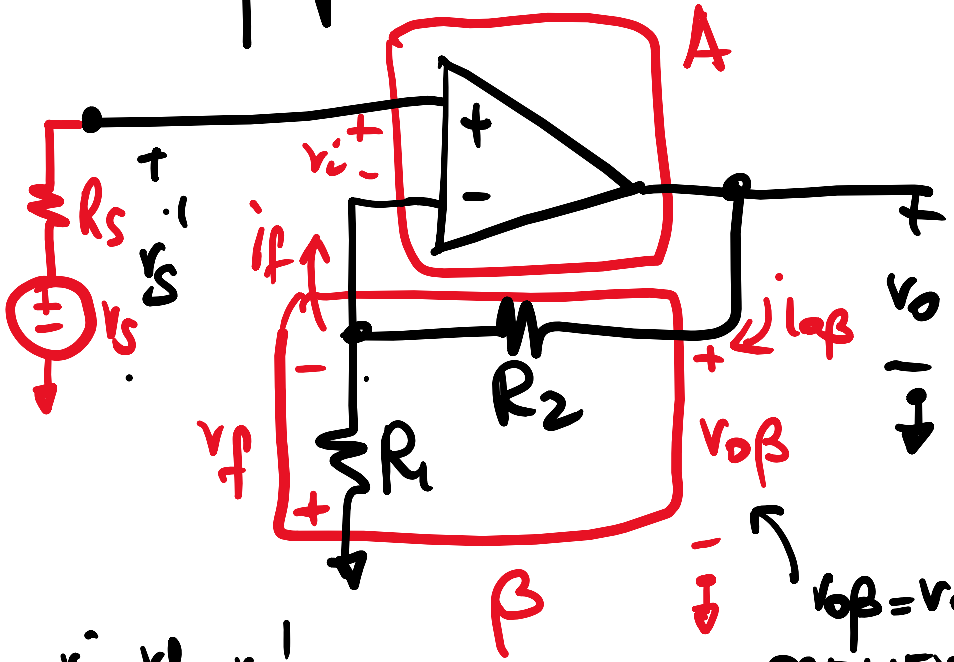
$$R_{OF} = \frac{v_{o\emptyset}}{i_{occ}} = \frac{\lim_{R_L\to\infty}\left[\frac{R_L i_o}{i_S}\right]}{\lim_{R_L\to 0}\left[\frac{i_o}{i_S}\right]} = \frac{\lim_{R_L\to\infty}\left[\frac{R_L A_e}{1-\cancel{\beta A_e}}\right]}{\lim_{R_L\to 0}\left[\frac{A_e}{1-\beta A_e}\right]} = \frac{\lim_{R_L\to\infty}\left[\frac{R_L \cdot}{R_o+R_L+R_{i\beta}}\right]}{\left(\frac{1}{R_o+R_{i\beta}}\right)\frac{1}{1-\beta A_e|_{R_L\to 0}}} =$$

$$\boxed{R_{OF} = (R_o + R_{i\beta})\left(1 - \beta A_e|_{R_L\to 0}\right)}$$

	PRELIEVO	INSERZIONE	x_o	x_s	$A_F = \frac{x_o}{x_s}$	R_{iF}	R_{oF}
1	V PARALLELO	V SERIE	v_o	v_s	$\frac{A_e}{1-\beta A_e}$	$(R_i + R_{o\beta})(1-\beta A_e)\big\|_{\beta=0}$	$\frac{R_o // R_{i\beta}}{1-\beta A_e\big\|_{R_L \to \infty}}$
2	I SERIE	V SERIE	i_o	v_s			$(R_o + R_{i\beta})(1-\beta A_e\big\|_{R_L \to 0})$
3	V PARALLELO	I PARALLELO	v_o	i_s		$\frac{R_i // R_{o\beta}}{(1-\beta A_e\big\|_{R_s \to \infty})}$	$\frac{R_o // R_{i\beta}}{1-\beta A_e\big\|_{R_L \to \infty}}$
4	I SERIE	I PARALLELO	i_o	i_s			$(R_o + R_{i\beta})(1-\beta A_e\big\|_{R_L = 0})$

Amplificatore NON INVERTENTE è un amplificatore in reazione

$$A_F = \frac{v_o}{v_s} = \frac{A_e}{1-\beta A_e}$$

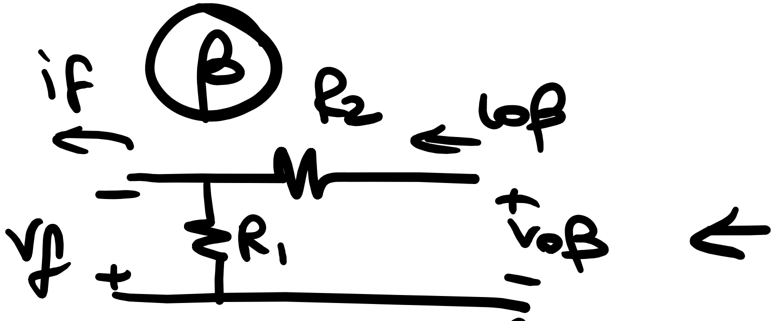


$v_{o\beta} = v_o$

PRELIEVO DI TENSIONE

$v_i = v_f + v_s'$

INSERZIONE DI TENSIONE

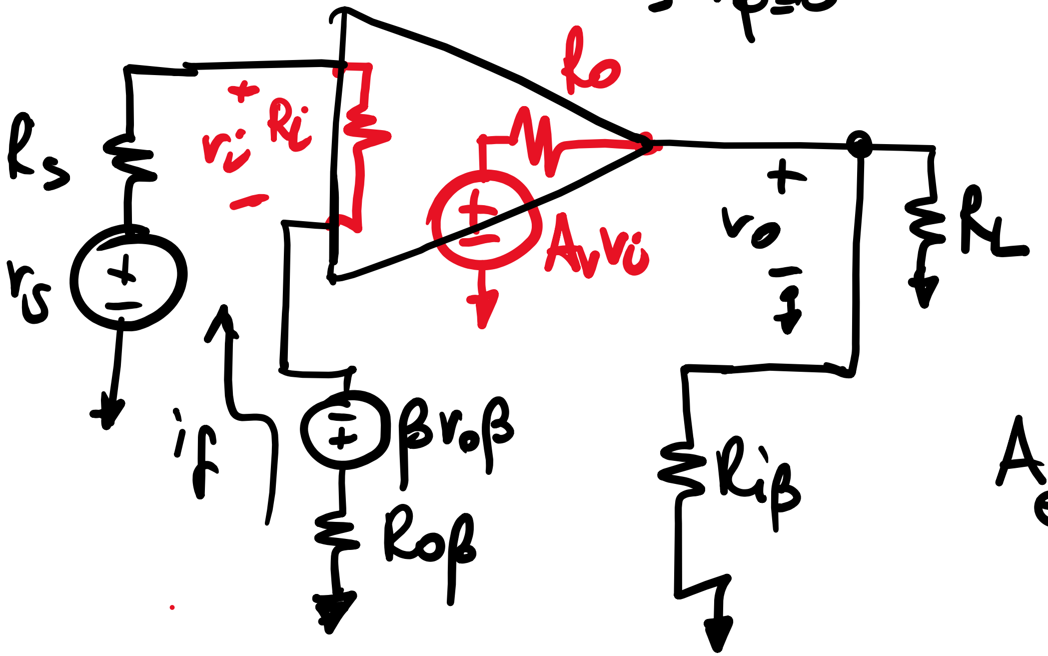


$$\begin{bmatrix} v_f \\ i_{o\beta} \end{bmatrix} = \begin{bmatrix} \beta & R_{o\beta} \\ \frac{1}{R_{i\beta}} & 0 \end{bmatrix} \begin{bmatrix} v_{o\beta} \\ i_f \end{bmatrix}$$

approx

$$\beta = \left.\frac{v_f}{v_{o\beta}}\right|_{i_f=0} = -\frac{R_1}{R_1+R_2} \; ; \; R_{o\beta} = \left.\frac{v_f}{i_f}\right|_{v_{o\beta}=0} = R_1 /\!/ R_2 \; ; \; R_{i\beta} = \left.\frac{v_{o\beta}}{i_{o\beta}}\right|_{i_f=0} = R_1 + R_2$$

Calcolo di $A_e = v_o / v_s \big|_{\beta=0}$



$$v_i = \frac{R_i}{R_i + R_s + R_{o\beta}}$$

$$v_o = \frac{R_L /\!/ R_{i\beta}}{R_L /\!/ R_{i\beta} + R_o} A_v v_i$$

$$A_e = \frac{v_u}{v_s}\bigg|_{\beta=0} = A_v \left(\frac{R_L /\!/ R_{i\beta}}{R_L /\!/ R_{i\beta} + R_o}\right)\left(\frac{R_i}{R_i + R_s + R_{o\beta}}\right)$$

funzione di trasferimento

$\boxed{\text{CASO 1}}$ $A_e \gg 1$

$$A_F = \frac{A_e}{1-\beta A_e} \sim -\frac{1}{\beta} = \frac{R_1+R_2}{R_1} = \boxed{1+\frac{R_2}{R_1}} \leftarrow \text{c.c.v}$$

$\boxed{\text{CASO 2}}$ A_e ha un polo

$$A_e = \frac{A_{e0}}{1-\frac{s}{s_p}} \Rightarrow A_F = \frac{\frac{A_{e0}}{1-s/s_p}}{1-\beta\frac{A_{e0}}{1-s/s_p}} = \frac{\frac{A_{e0}}{\cancel{1-s/s_p}}}{\frac{1-s/s_p-\beta A_{e0}}{\cancel{1-s/s_p}}}$$

$$A_F = \frac{A_{e0}}{\underbrace{1-\beta A_{e0}}_{A_{F0}}} \cdot \frac{1}{1-\frac{s}{\underbrace{s_p(1-\beta A_{e0})}_{S_H}}}$$

A_F ha un polo

PRODOTTO GUADAGNO BANDA COSTANTE

$$A_F = \frac{A_{F0}}{1-\frac{s}{S_H}} \Rightarrow \text{PGB di } A_e = A_{e0}|s_p|$$

$$\text{PGB di } A_F = A_{F0}|S_H| = A_{e0}|s_p|$$