DC-DC Converters

Typical uses:
• DC Power supplies
• DC Motor drive

Types of converters
• Step-down (buck)
• Step-up (boost)
• Buck-boost
• Cuk
• Full-Bridge

Figure 7-1 A dc–dc converter system.
Ideal concept of step-down converter with PWM* switching
(* Pulse Width Modulation)

**Assumptions**: Switches, L, C are lossless, DC input has zero internal impedance, load is an equivalent R

**This cannot work**: 1. Load is inductive and can destroy switch by dissipating all stored energy, 2. output voltage must be continuous

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**Step-down (buck) converter**

DC power supplies, DC motor drives -- $V_o < V_d$

**Low-pass filter** keeps output voltage constant

*Note: 2nd order non dissipative filter*

$$f_c = \frac{1}{2\pi \sqrt{LC}} \ll f_s$$

**Diode avoids voltage spike** on switch (when switch is off, diode provides current to L)
Continuous-conduction mode

Current in $L$ is always $> 0$

- $t_{on}: \frac{dl}{dt} = \frac{V_d - V_o}{L}$
- $t_{off}: \frac{dl}{dt} = -\frac{V_o}{L}$

At steady state: $I(t + T_s) = I(t)$.

Therefore

$$\frac{V_d - V_o}{L} t_{on} = \frac{V_o}{L} t_{off} = 0$$

$$\frac{V_o}{V_d} = \frac{t_{on}}{T_s} = D$$

Limit of continuous conduction

If the ripple amplitude $I_{LB} \equiv \frac{I_{peak}}{2} = I_o$, the converter is at the limit of continuous conduction (i.e. $\min\{I_L\} = 0$)

$$I_{LB} \equiv \frac{I_{peak}}{2} = \frac{t_{on}(V_d - V_o)}{2L} = \frac{DT_s V_d (1 - D)}{2L} = I_{LB,max} 4D (1 - D)$$

Figure 7-6  Current at the boundary of continuous–discontinuous conduction: (a) current waveform; (b) $I_{LB}$ versus $D$ keeping $V_d$ constant.
Discontinuous-conduction mode with constant $V_d$

$$I_{\text{peak}} = \frac{(V_d - V_o)DT_S}{L} = \frac{V_o \Delta_1 T_S}{L}$$

$$I_{\text{peak}} = \frac{V_d T_S}{L} \frac{D \Delta_1}{D + \Delta_1}$$

$$I_{\text{peak}} = 4I_{LB\text{max}} \frac{D \Delta_1}{D + \Delta_1}$$

$$I_o T_S = \frac{I_{\text{peak}}(D + \Delta_1)T_S}{2}$$

$$I_o = 4I_{LB\text{max}} D \Delta_1$$

$$\frac{V_o}{V_d} = \frac{D}{D^2 + I_o/(4I_{LB\text{max}})}$$

**Motor drives**

**Figure 7-7** Discontinuous conduction in step-down converter.

Limits of continuous-discontinuous conduction (constant $V_d$)

**Continuous**

$$\frac{I_o}{I_{LB\text{max}}} > 4D(1 - D)$$

$$\frac{V_o}{V_d} = D$$

**Discontinuous**

$$\frac{I_o}{I_{LB\text{max}}} < 4D(1 - D)$$

$$\frac{V_o}{V_d} = \frac{D^2}{D^2 + \frac{I_o}{4I_{LB\text{max}}}}$$

**Figure 7-8** Step-down converter characteristics keeping $V_d$ constant.
Discontinuous-conduction with constant Vo

At the limit of continuous conduction

\[ I_{LB} = \frac{V_0 T_S (1 - D)}{2L} = I_{LB\text{max}} (1 - D) \]

We can write D explicitly from:

\[ I_{\text{peak}} = \frac{V_0 \Delta_1 T_S}{L} = 2 I_{LB\text{max}} \Delta_1 \]

\[ I_o = \frac{I_{\text{peak}} (D + \Delta_1)}{2} = I_{LB\text{max}} \Delta_1 (D + \Delta_1) \]

\[ \frac{V_d}{V_o} = \frac{D + \Delta_1}{D} \]

\[ \frac{I_o}{I_{LB\text{max}}} = D^2 \frac{V_d}{V_o} \left(1 - \frac{V_d}{V_o}\right) \]

\[ D = \left[ \frac{V_o}{V_d I_{LB\text{max}}} \left(1 - \frac{V_d}{V_o}\right)^{-1}\right]^{\frac{1}{2}} \]

Discontinuous-conduction with constant Vo

**Continuous:** \( I_o > I_{LB} \)

\[ D > 1 - \frac{I_o}{I_{LB\text{max}}} \]

\[ D = \frac{V_o}{V_d} \]

**Discontinuous:** \( I_o < I_{LB} \)

\[ D < 1 - \frac{I_o}{I_{LB\text{max}}} \]

\[ D = \left[ \frac{V_o}{V_d I_{LB\text{max}}} \left(1 - \frac{V_d}{V_o}\right)^{-1}\right]^{\frac{1}{2}} \]

**Figure 7-9** Step-down converter characteristics keeping \( V_o \) constant.
Output voltage ripple

**First order calculation:**
The average $i_L$ flows in the load, and the ripple component in $C$.

**Additional charge:**
$$\Delta Q = \frac{1}{2} \frac{\Delta I_L}{2} T_S$$

**Current ripple:**
$$\Delta I_L = \left(\frac{V_o}{L}\right) (1 - D) T_S$$

**Voltage ripple:**
$$\Delta V_o = \frac{\Delta Q}{C} = \frac{V_o}{8LC} T_S^2 (1 - D)$$

$$\frac{\Delta V_o}{V_o} = \frac{\pi^2}{2} \left(1 - D\right) \frac{f_c^2}{f_s^2}$$

Step-up (boost) converter

- DC power supplies
- Regenerative breaking of DC motors

Output voltage always larger than the input

**Switch on** → diode off, output isolated, $L$ accumulates energy from input

**Switch off** → diode on, load receives energy from input and from $L$
Continuous-conduction mode

Periodic conditions:
\[ \frac{t_{on} V_d}{L} + \frac{t_{off} (V_d - V_o)}{L} = 0 \]
if \( t_{on} = DT_s \) and
\[ t_{off} = (1 - D)T_s \]

\[ T_s V_d + T_s (1 - D)V_o = 0 \]

\[ \frac{V_o}{V_d} = \frac{1}{1 - D} \]

No losses:
\[ V_o I_o = V_d I_d \]

Continuous-discontinuous boundary

Average current in L = ripple:
\[ I_{LB} = \frac{V_d t_{on}}{2L} = \frac{V_o (1 - D)T_s D}{2L} \]

Average output current at the limit:
\[ I_{oB} = I_{LB} (1 - D) = \frac{V_o T_s (1 - D)^2 D}{2L} \]

\( I_{LB} \) is max if \( D = 0.5 \) \( \rightarrow I_{LBmax} = \frac{V_o T_s}{8L} \)

\( I_{oB} \) is max if \( D = 1/3 \) \( \rightarrow I_{oBmax} = \frac{2V_o T_s}{27L} \) \( \rightarrow I_{oB} = \frac{27}{4} (1 - D)^2 DI_{oBmax} \)
**Discontinuous conduction mode**  
(constant $V_o$)

### Periodic conditions:
\[
\frac{DT_s V_d}{L} + \frac{\Delta_1 T_s (V_d - V_o)}{L} = 0
\]
\[
\frac{V_o}{V_d} = 1 + \frac{L}{\Delta_1} = \frac{I_d}{I_o}
\]

### Average current in L
\[
I_d T_s = \frac{DT_s V_d (D + \Delta_1) T_s}{2}
\]

### Average output current
\[
I_o = I_d \frac{\Delta_1}{D + \Delta_1} = \frac{T_s V_d}{2L} D \Delta_1
\]
\[
= \frac{27}{4} I_{oB_{\text{max}}} D^2 \frac{V_d}{V_o} \frac{V_o}{V_o - V_d}
\]

\[D = \left[ \frac{4}{27} \frac{V_o (V_o - 1)}{V_d} \frac{I_o}{I_{oB_{\text{max}}}} \right]^{\frac{1}{2}}\]

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**Continuous-discontinuous mode**  
(constant $V_o$)

### Continuous mode:
\[
I_o > I_{oB}
\]
\[
= I_{oB_{\text{max}}} \frac{27(1 - D)^2 D}{4}
\]
\[
D = 1 - \frac{V_d}{V_o}
\]

### Discontinuous mode:
\[
I_o < I_{oB}
\]
\[
D = \left[ \frac{4}{27} \frac{V_o (V_o - 1)}{V_d} \frac{I_o}{I_{oB_{\text{max}}}} \right]^{\frac{1}{2}}\]

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**Figure 7-15** Step-up converter characteristics keeping $V_o$ constant.
Losses and ripple

**Losses**: inductor, capacitor, switch, diode

**Ripple**: first order assumption: when the switch is on the C is discharged through the load

\[
\Delta V_o = \frac{\Delta Q}{C} = \frac{I_o D T_S}{C} = \frac{V_o D T_S}{R C}
\]

\[
\frac{\Delta V_o}{V_o} = D \frac{T_S}{\tau}
\]

Buck-boost converter

**Negative DC power supply**

**Switch on**: inductance accumulates energy, diode off, C supplies the load

**Switch off**: diode on, inductance transfers energy to the capacitance and to the load

**Periodic conditions in continuous conduction mode**:

\[
\frac{D T_S V_d}{L} - \frac{V_o (1 - D) T_S}{L} = 0
\]

\[
\frac{V_o}{V_d} = \frac{D}{1 - D} = \frac{I_d}{I_o}
\]

\[
i_L = I_o + I_d = \frac{I_o}{1 - D}
\]
Continuous-discontinuous boundary

Current in L at the boundary
\[ I_{LB} = \frac{D T_s V_d}{2L} \]

Output current at the boundary:
\[ I_{oB} = I_{LB} (1 - D) = \frac{T_s V_o}{2L} (1 - D)^2 \]

Discontinuous conduction

Periodic conditions:
\[ \frac{D V_d T_s}{L} - \frac{V_o \Delta_1 T_s}{L} = 0 \]
\[ \frac{V_o}{V_d} = \frac{D}{\Delta_1} = \frac{I_d}{I_o} \]

Average current in L:
\[ I_L T_s = \frac{V_d D T_s (D + \Delta_1) T_s}{L} \frac{2}{2} \]

Therefore:
\[ I_L = I_o \left(1 + \frac{D}{\Delta_1}\right) = \frac{V_d T_s}{2L} D (D + \Delta_1) \]
\[ \frac{I_o}{I_{oBmax}} = D \Delta_1 \frac{V_d}{V_o} = D^2 \left(\frac{V_d}{V_o}\right)^2 \rightarrow D = \frac{V_o}{V_d} \sqrt{\frac{l_o}{I_{oBmax}}} \]
Continuous-discontinuous mode

**Continuous operation**
\[ I_o > I_{OB} = I_{OBmax} (1 - D)^2 \]
\[ D = \frac{V_o}{V_d - V_o} \]

**Discontinuous operation**
\[ I_o > I_{OB} \]
\[ D = \frac{V_o}{V_d} \sqrt{\frac{I_o}{I_{OBmax}}} \]

Output voltage ripple

When the switch is ON, C is discharged through the load
\[ \Delta V_o = \frac{\Delta Q}{C} = \frac{DT_S V_o}{RC} \Rightarrow \Delta V_o = \frac{\Delta V_o}{V_o} = D \frac{T_S}{\tau} \]
Cuk DC-DC converter

Negative DC power supply
DC analysis: \( V_{C1} = V_d + V_o \) note: \( V_{C1} > V_d \)

Assumption: Large C1 (Voltage almost constant)

Switch OFF: C1 is charged through L1 and the input, Diode ON, L2 supplies energy to R (currents in L1 and L2 decrease)

Switch ON: L1 receives energy, Diode OFF, C supplies current to R, C1 gives energy to L2 (currents in L1 and L2 increase)

![Cuk converter diagram](image)

Figure 7-25 Cuk converter.

Cuk

Periodic conditions in L1
\[
V_d DT_s + (1 - D)T_s (V_d - V_{C1}) = 0
\]
\[
V_{C1} = \frac{V_d}{1 - D}
\]

Periodic conditions in L2
\[
(V_{C1} - V_o)DT_s - V_o (1 - D)T_s = 0
\]
\[
V_{C1} = \frac{V_o}{D}
\]

Therefore
\[
\frac{V_o}{V_d} = \frac{D}{1 - D}
\]

Pro: currents in L1 and L2 ripple free

Con: C1 must be large

![Cuk converter waveforms](image)

Figure 7-26 Cuk converter waveforms: (a) switch off; (b) switch on.
**Full bridge DC-DC converter**

When switch TA+ is on:
- $i_o > 0$: $i_o$ through TA+
- $i_o < 0$: $i_o$ through DA+
$V_{AN} = V_d \text{dutycycle}(TA^+)$

When switch TB+ is on:
- $i_o < 0$: $i_o$ through TB+
- $i_o > 0$: $i_o$ through DB+
$V_{BN} = V_d \text{dutycycle}(TB^+)$

$$V_o = V_{AN} - V_{BN}$$

Four quadrant operation on $V_o, I_o$

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**PWM with bipolar voltage switching**

When $v_{control} > v_{tri}$:
- TA+ and TB- are ON
Duty cycle
$$D_1 = \frac{1}{2} + \frac{v_{control}}{\frac{v_{tri}}{2}}$$

When $v_{control} < v_{tri}$:
- TA- and TB+ are ON
$$D_2 = 1 - D_1$$
$$V_o = V_{AN} - V_{BN} = D_1 V_d - D_2 V_d$$
$$= (2D_1 - 1)V_d$$
$$= \frac{V_d}{v_{tri}} v_{control}$$
PWM with unipolar voltage switching

When $v_{\text{control}} > v_{\text{tri}}^{(a)}$, TA+ and TB- are ON

Duty cycle

$$D_1 = \frac{1}{2} + \frac{v_{\text{control}}}{V_{\text{tri}}}$$

When $-v_{\text{control}} < v_{\text{tri}}^{(b)}$, TA- and TB+ are ON

$$D_2 = 1 - D_1$$

$$V_o = V_{AN} - V_{BN} = D_1 V_d - D_2 V_d = (2D_1 - 1)V_d = \frac{V_d}{V_{\text{tri}}} v_{\text{control}}$$

Less ripple w.r.t. the bipolar case because frequency of $V_o$ is double

PWM signal generation

$V_o$ (desired) $\rightarrow$ Amplifier $\rightarrow$ Comparator

$V_o$ (actual) $\rightarrow$ Comparator

$\Rightarrow$ Switch control signal

$\Rightarrow$ Repetitive waveform

$\Rightarrow$ Repetitive waveform
Full Bridge DC-DC Converter

\[ T_A^+ \quad T_B^+ \quad T_A^- \quad T_B^- \]

\[ V_{AN} + V_{BN} \]

\[ i_A \]

\[ V_o \]

\[ V_{AN} = D_1 V_d \]

\[ V_{BN} = D_2 V_d \]

\[ V_o = V_{AN} - V_{BN} = (D_1 - D_2) V_d \]

Control: PWM

Bipolar voltage

Unipolar voltage
PWM with bipolar voltage

\[ V_{\text{control}} \]

\[ V_{\text{tri}} \]

\[ V_{\text{max}} \]

\[ V_0 \]

\[ V_d \]

\[ T_A \]

\[ T_B \]

\[ T_s \]

\[ D_1 = \frac{1}{2} + \frac{1}{2} \frac{V_{\text{control}}}{V_{\text{max}}} \]

\[ D_2 = (1 - D_1) \]

\[ V_0 = (D_1 - D_2) V_d = (2D_1 - 1) V_d = \frac{V_{\text{control}} V_d}{V_{\text{max}}} \]

PWM with unipolar voltage

\[ V_{\text{control}} \]

\[ V_{\text{max}} \]

\[ V_0 = V_{\text{AN}} - V_{\text{BN}} \]

\[ V_0 \]

\[ V_d \]

\[ T_A \]

\[ T_B \]

\[ T_s \]

\[ D_1 = \frac{1}{2} + \frac{1}{2} \frac{V_{\text{control}}}{V_{\text{max}}} \]

\[ D_2 = \frac{1}{2} - \frac{1}{2} \frac{V_{\text{control}}}{V_{\text{max}}} \]

\[ D_2 = 1 - D_1 \]

\[ V_0 = \frac{V_{\text{control}} V_d}{V_{\text{max}}} \]

Variation of \( V_d \) from 0 to \( V_d = -V_d \) (ripple inferior to the bipolar case).