DC-DC Converters

DC-DC Converters

Typical uses:

- DC Power supplies
- DC Motor drive

Types of converters

- **Step-down (buck)**
- **Step-up (boost)**
- Buck-boost
- Cuk
- Full-Bridge

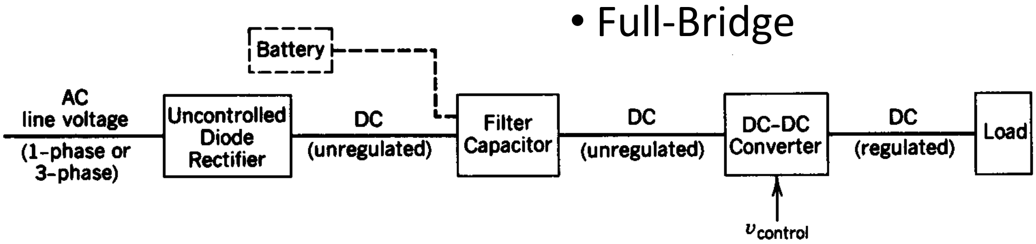
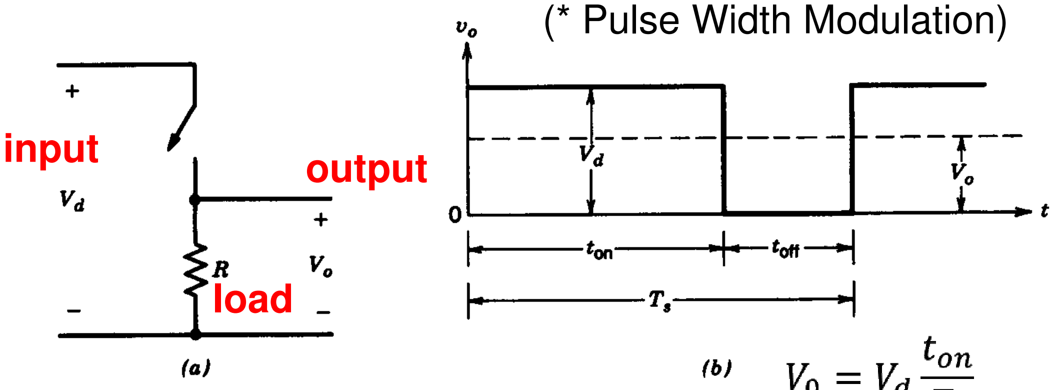


Figure 7-1 A dc–dc converter system.

Ideal concept of step-down converter with PWM* switching

(* Pulse Width Modulation)



$$V_0 = V_d \frac{t_{on}}{T_s}$$

Figure 7-2 Switch-mode dc–dc conversion.

Assumptions: Switches, L, C are lossless, DC input has zero internal impedance, load is an equivalent R

This cannot work: 1. Load is inductive and can destroy switch by dissipating all stored energy, 2. output voltage must be continuous

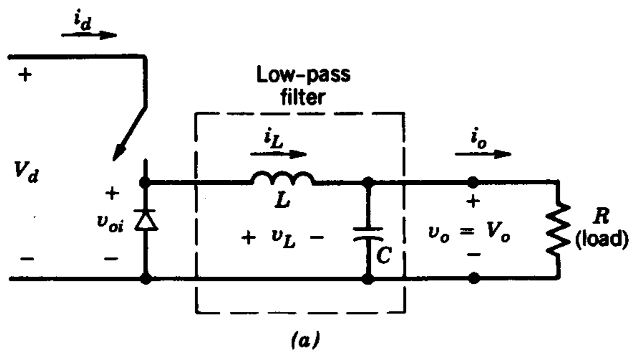
Step-down (buck) converter

DC power supplies, DC motor drives -- $V_o < V_d$

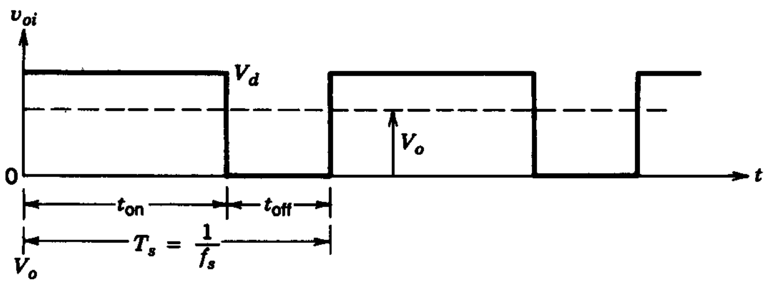
Low-pass filter keeps output voltage constant

Note: 2nd order non dissipative filter

$$f_c = \frac{1}{2\pi}\frac{1}{\sqrt{LC}} \ll f_s$$



Diode avoids voltage spike on switch (when switch is off, diode provides current to L)



Continuous-conduction mode

Current in L is always > 0

$T_s = t_{on} + t_{off}$

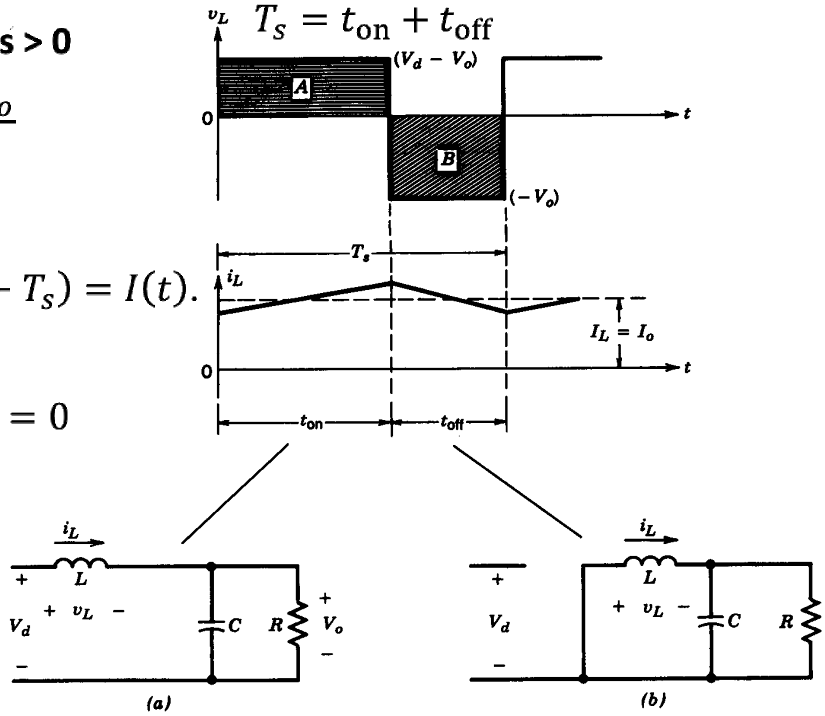
- t_{on}: $\frac{dI}{dt} = \frac{V_d - V_o}{L}$
- t_{off}: $\frac{dI}{dt} = -\frac{V_o}{L}$

At steady state: $I(t + T_s) = I(t)$.

Therefore

$$\frac{V_d - V_o}{L} t_{on} - \frac{V_0}{L} t_{off} = 0$$

$$\frac{V_o}{V_d} = \frac{t_{on}}{T_s} = D$$



Limit of continuous conduction

If the ripple amplitude $I_{LB} \equiv \frac{I_{peak}}{2} = I_o$, the converter is at the limit of continuous conduction (i.e. $min\{I_L\} = 0$)

$$I_{LB} \equiv \frac{I_{peak}}{2} = \frac{t_{on}(V_d - V_o)}{2L} = \frac{DT_sV_d(1-D)}{2L} = I_{LBmax}4D(1-D)$$

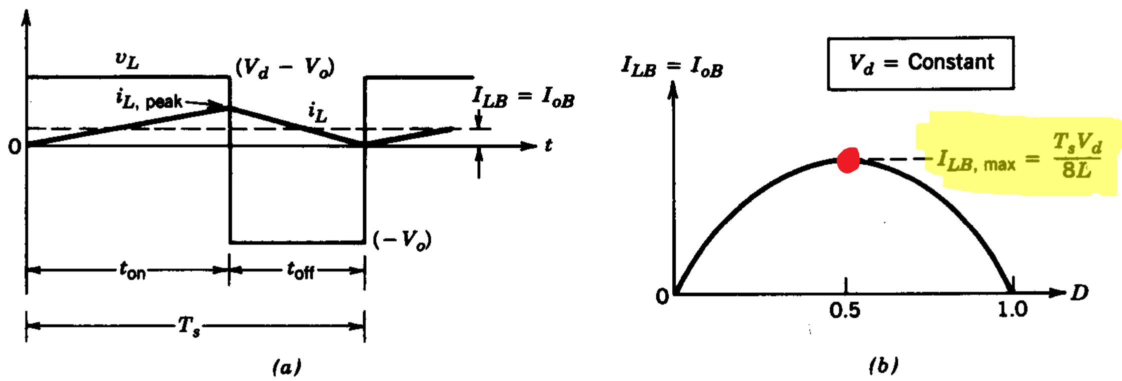


Figure 7-6 Current at the boundary of continuous–discontinuous conduction: (a) current waveform; (b) I_{LB} versus D keeping V_d constant.

Discontinuous-conduction mode with constant V_d

Motor drives

$$I_{peak} = \frac{(V_d - V_o)DT_s}{L} = \frac{V_o \Delta_1 T_s}{L} \quad \Rightarrow \quad \frac{V_o}{V_d} = \frac{D}{(D+\Delta_1)}$$

$$I_{peak} = \frac{V_d T_s}{L} \frac{D\Delta_1}{D+\Delta_1}$$

$$I_{peak} = 4 I_{LBmax} \frac{D\Delta_1}{D+\Delta_1}$$

$$I_o T_s = \frac{I_{peak}(D+\Delta_1)T_s}{2}$$

$$I_o = 4 I_{LBmax} D \Delta_1$$

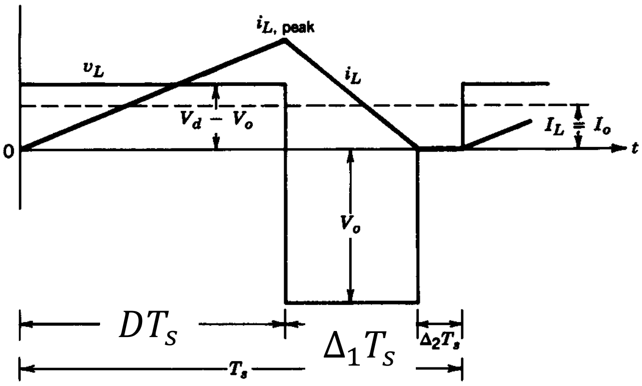


Figure 7-7 Discontinuous conduction in step-down converter.

$$\frac{V_o}{V_d} = \frac{D^2}{D^2 + I_o/(4I_{LBmax})}$$

Limits of continuous-discontinuous conduction (constant Vd)

Continuous

$$\frac{I_o}{I_{LBmax}} > 4D(1-D)$$

$$\frac{V_o}{V_d} = D$$

Discontinuous

$$\frac{I_o}{I_{LBmax}} < 4D(1-D)$$

$$\frac{V_o}{V_d} = \frac{D^2}{D^2 + \frac{I_o}{4I_{LBmax}}}$$

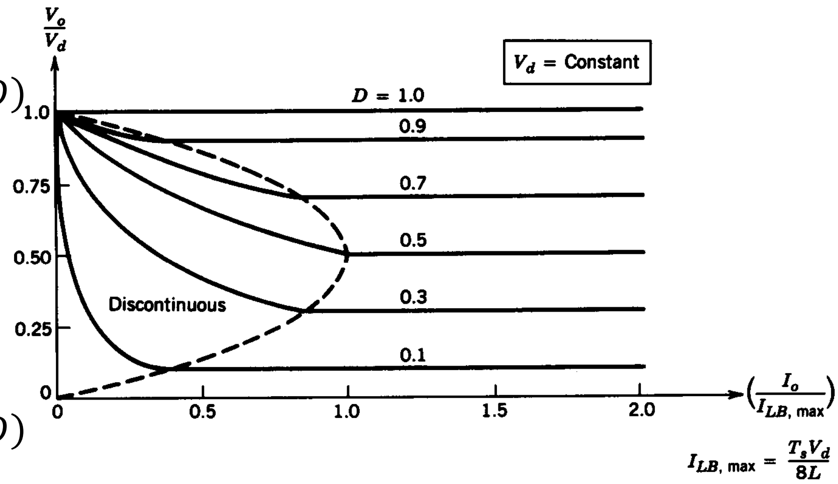


Figure 7-8 Step-down converter characteristics keeping V_d constant.

Discontinuous-conduction with constant Vo

DC voltage supply

At the limit of continuous conduction

$$I_{LB} = \frac{V_o T_s (1-D)}{2L} = I_{\text{LBmax}}(1-D)$$

We can write D explicitly from:

$$I_{\text{peak}} = \frac{V_o \Delta_1 T_s}{L} = 2 I_{\text{LBmax}} \Delta_1$$

$$I_o = \frac{I_{\text{peak}}(D+\Delta_1)}{2} = I_{\text{LBmax}} \Delta_1 (D+\Delta_1) \qquad \frac{V_d}{V_o} = \frac{D+\Delta_1}{D}$$

$$\frac{I_o}{I_{\text{LBmax}}} = D^2 \frac{V_d}{V_o}\left(1 - \frac{V_d}{V_o}\right) \Rightarrow D = \left[\frac{V_o}{V_d}\frac{I_o}{I_{LBmax}}\left(1-\frac{V_d}{V_o}\right)^{-1}\right]^{\frac{1}{2}}$$

Discontinuous-conduction with constant Vo

DC voltage supply

Continuous: $I_o > I_{LB}$

$$D > 1 - \frac{I_o}{I_{LBmax}}$$

$$D = \frac{V_o}{V_d}$$

Discontinuous: $I_o < I_{LB}$

$$D < 1 - \frac{I_o}{I_{LBmax}}$$

$$D = \left[\frac{V_o}{V_d}\frac{I_o}{I_{LBmax}}\left(1-\frac{V_d}{V_o}\right)^{-1}\right]^{\frac{1}{2}}$$

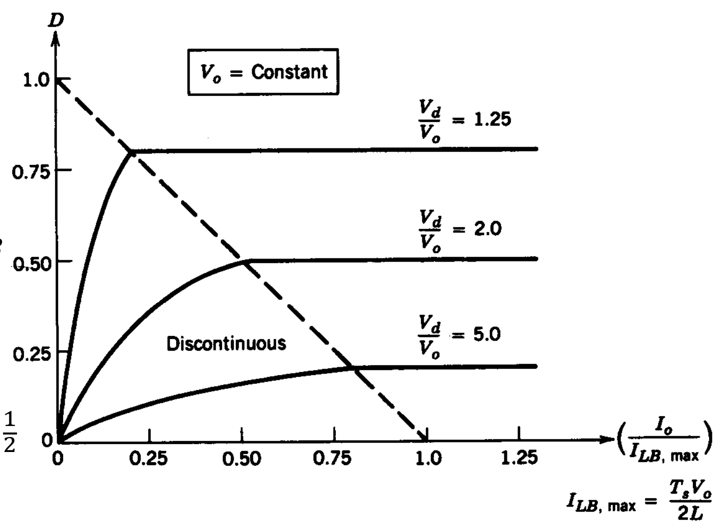
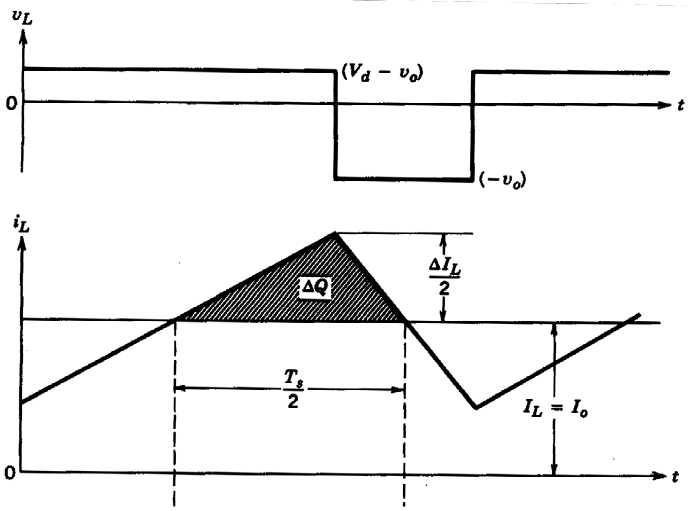


Figure 7-9 Step-down converter characteristics keeping V_o constant.

Output voltage ripple

First order calculation:

The average iL flows in the load, and the ripple component in C.



Additional charge:

$$\Delta Q = \frac{1}{2}\frac{\Delta I_L}{2}\frac{T_s}{2}$$

Current ripple:

$$\Delta I_L = (V_o/L)(1-D)T_s$$

Voltage ripple:

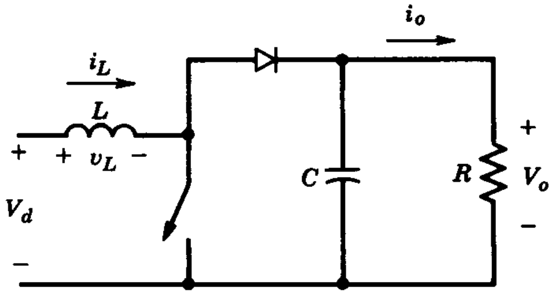
$$\Delta V_o = \frac{\Delta Q}{C} = \frac{V_o}{8LC}T_s^2(1-D)$$

$$\frac{\Delta V_o}{V_o} = \frac{\pi^2}{2}(1-D)\frac{f_c^2}{f_s^2}$$

Step-up (boost) converter

- DC power supplies
- Regenerative breaking of DC motors

Output voltage always larger than the input



Switch on → diode off, output isolated, L accumulates energy from input

Switch off → diode on, load receives energy from input and from L

Continous-conduction mode

Periodic conditions:

$$\frac{t_{\text{on}}V_d}{L} + \frac{t_{\text{off}}(V_d - V_o)}{L} = 0$$

if $t_{\text{on}} = DT_s$ and

$$t_{\text{off}} = (1-D)T_s$$

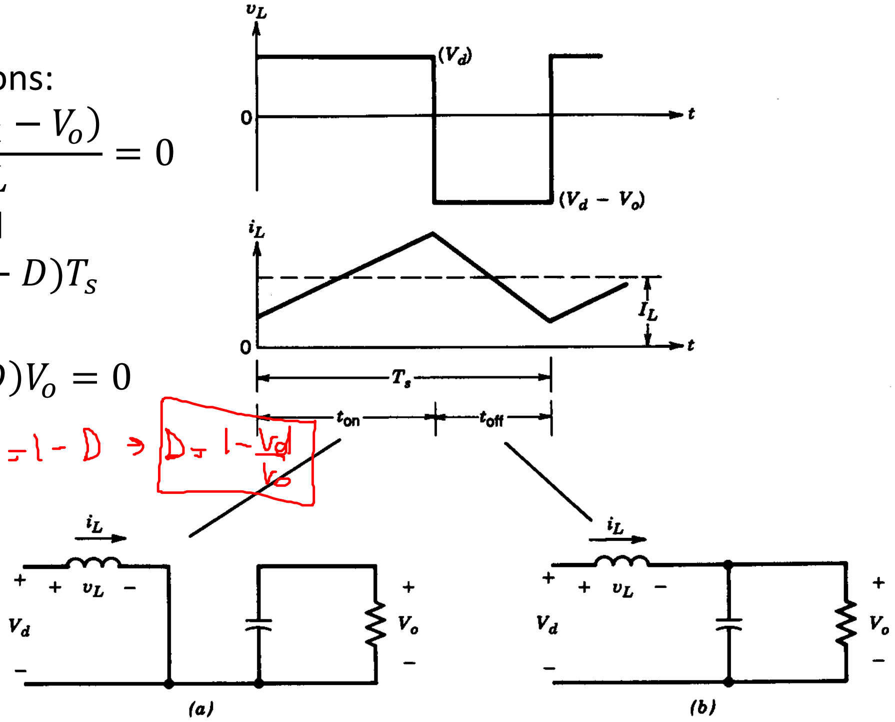
$$T_sV_d + T_s(1-D)V_o = 0$$

$$\frac{V_o}{V_d} = \frac{1}{1-D}$$

$\frac{V_d}{V_o} = 1 - D \rightarrow D = 1 - \frac{V_d}{V_o}$

No losses:

$$V_oI_o = V_dI_d$$



Continuous-discontinuous boundary

Average current in L = ripple :

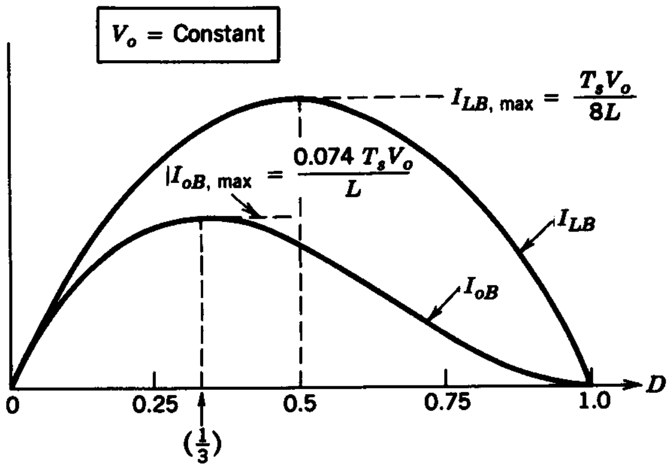
$$I_{LB} = \frac{V_d t_{on}}{2L}$$

$$= \frac{V_o(1-D)T_sD}{2L}$$

Average output current at the limit:

$$I_{oB} = I_{LB}(1-D)$$

$$= \frac{V_oT_s(1-D)^2D}{2L}$$



I_{LB} is max if D=0.5 → $I_{LBmax} = \frac{V_oT_s}{8L}$,

I_{oB} is max if D=1/3 → $I_{oBmax} = \frac{2V_oT_s}{27L}$ → $I_{oB} = \frac{27}{4}(1-D)^2DI_{oBmax}$

Discontinuous conduction mode (constant V_o)

Periodic conditions:

$$\frac{DT_sV_d}{L} + \frac{\Delta_1 T_s(V_d - V_o)}{L} = 0$$

$$\frac{V_o}{V_d} = 1 + \frac{D}{\Delta_1} = \frac{I_d}{I_o}$$

Average current in L

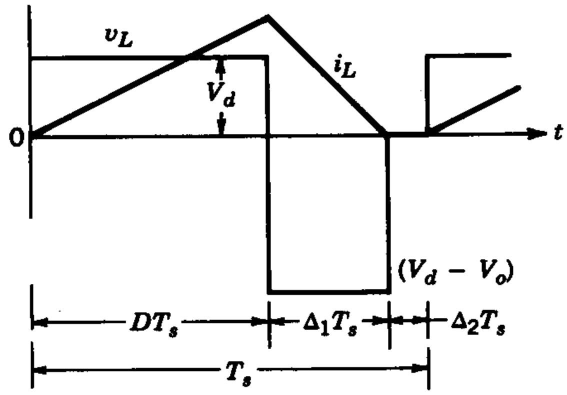
$$I_d T_s = \frac{DT_sV_d}{L}\frac{(D+\Delta_1)T_s}{2}$$

Average output current

$$I_o = I_d\frac{\Delta_1}{D+\Delta_1} = \frac{T_sV_d}{2L}D\Delta_1$$

$$= \frac{27}{4}I_{oBmax}\frac{V_d}{V_o}D^2\frac{V_d}{V_o - V_d}$$

$$D = \left[\frac{4}{27}\frac{V_o}{V_d}\left(\frac{V_o}{V_d} - 1\right)\frac{I_o}{I_{oBmax}}\right]^{\frac{1}{2}}$$



Continuous-discontinuous mode (constant V_o)

Continuous mode:

$$I_o > I_{oB}$$

$$= I_{oBmax}\frac{27(1-D)^2D}{4}$$

$$D = 1 - \frac{V_d}{V_o}$$

Discontinuous mode:

$$I_o < I_{oB}$$

$$D = \left[\frac{4}{27}\frac{V_o}{V_d}\left(\frac{V_o}{V_d} - 1\right)\frac{I_o}{I_{oBmax}}\right]^{\frac{1}{2}}$$

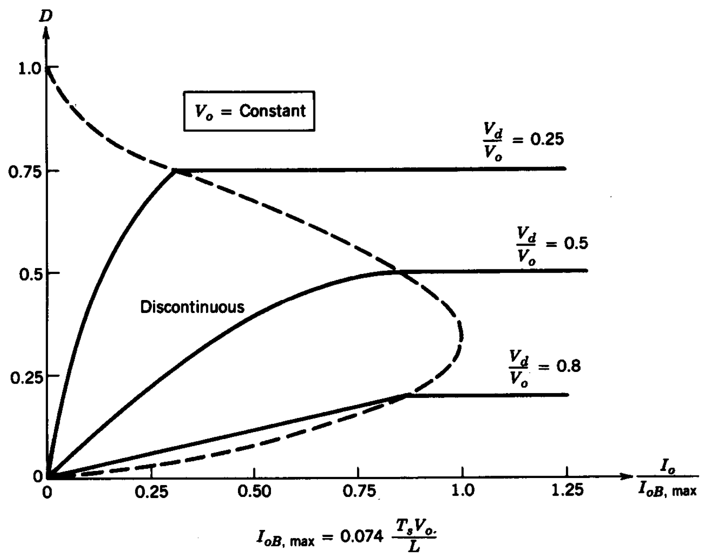


Figure 7-15 Step-up converter characteristics keeping V_o constant.

Losses and ripple

Losses: inductor, capacitor, switch, diode

Ripple: first order assumption: when the switch is on the C is discharged through the load

$$\Delta V_o = \frac{\Delta Q}{C} = \frac{I_o D T_s}{C} = \frac{V_o D T_s}{RC}$$

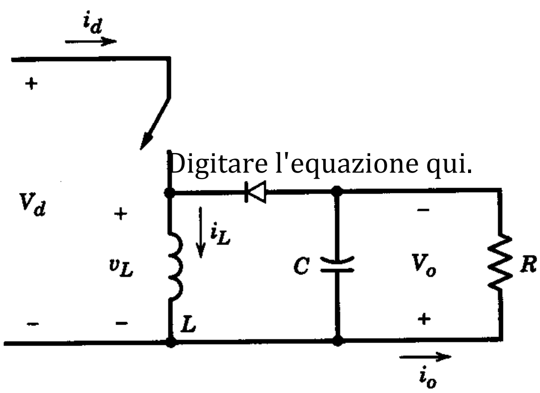
$$\frac{\Delta V_o}{V_o} = D\frac{T_s}{\tau}$$

Buck-boost converter

Negative DC power supply

Switch on: inductance accumulates energy, diode off, C supplies the load

Switch off: diode on, inductance transfers energy to the capacitance and to the load



Periodic conditions in continuous conduction mode:

$$\frac{DT_sV_d}{L} - \frac{V_o(1-D)T_s}{L} = 0$$

$$\frac{V_o}{V_d} = \frac{D}{1-D} = \frac{I_d}{I_o}$$

$$I_L = I_o + I_d = \frac{I_o}{1-D}$$

Continuous-discontinuous boundary

Current in L at the boundary

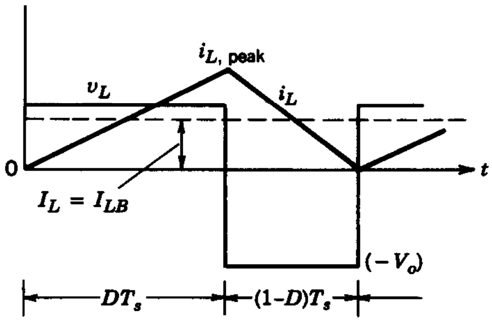
$$I_{LB} = \frac{DT_s V_d}{2L}$$

Output current at the boundary:

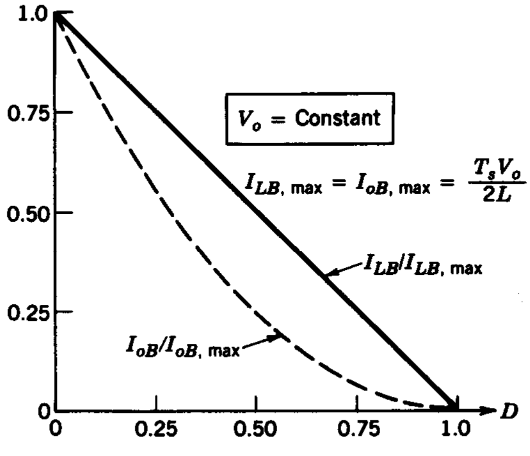
$$I_{oB} = I_{LB}(1-D) = \frac{T_s V_o}{2L}(1-D)^2$$

$$I_{LB} = I_{LBmax}(1-D)$$

$$I_{oB} = I_{oBmax}(1-D)^2$$



(a)



(b)

Discontinuous conduction

Periodic conditions:

$$\frac{DV_d T_s}{L} - \frac{V_o \Delta_1 T_s}{L} = 0$$

$$\frac{V_o}{V_d} = \frac{D}{\Delta_1} = \frac{I_d}{I_o}$$

Average current in L:

$$I_L T_s = \frac{V_d D T_s}{L}\frac{(D+\Delta_1)T_s}{2}$$

Therefore:

$$I_L = I_o\left(1 + \frac{D}{\Delta_1}\right) = \frac{V_d T_s}{2L} D(D+\Delta_1)$$

$$\frac{I_o}{I_{oBmax}} = D\Delta_1 \frac{V_d}{V_o} = D^2\left(\frac{V_d}{V_o}\right)^2 \rightarrow D = \frac{V_o}{V_d}\sqrt{\frac{I_o}{I_{oBmax}}}$$

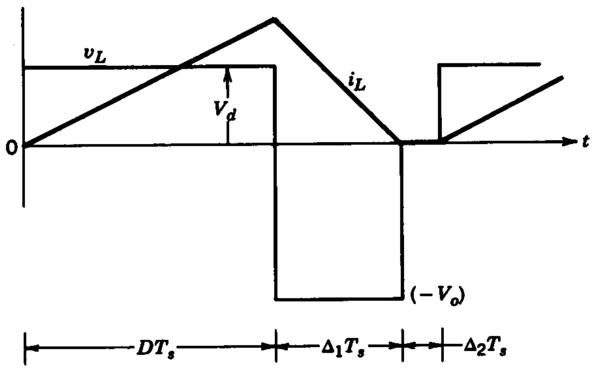


Figure 7-21 Buck–boost converter waveforms in a discontinuous-conduction mode.

Continuous-discontinuous mode

Continuous operation

$I_o > I_{oB} = I_{oBmax}(1 - D)^2$

$$D = \frac{V_o}{V_d - V_o}$$

Discontinuous operation

$I_o > I_{oB}$

$$D = \frac{V_o}{V_d}\sqrt{\frac{I_o}{I_{oBmax}}}$$

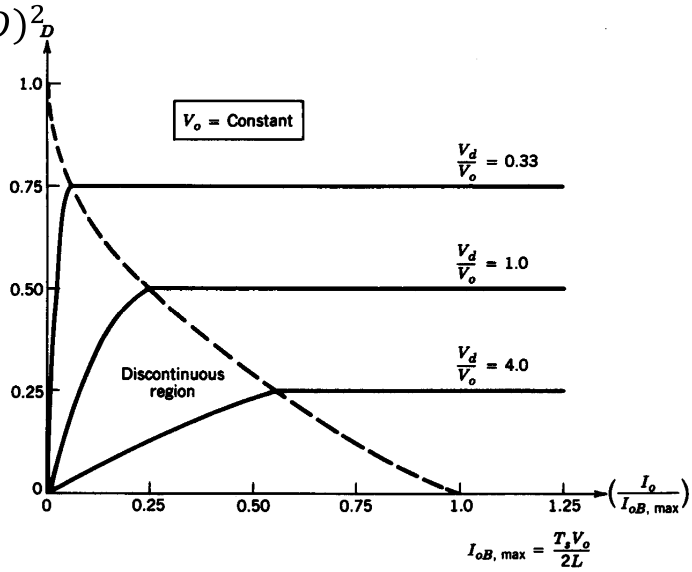


Figure 7-22 Buck–boost converter characteristics keeping V_o constant.

Output voltage ripple

When the switch is ON, C is discharged through the load

$$\Delta V_o = \frac{\Delta Q}{C} = \frac{DT_sV_o}{RC} \rightarrow \frac{\Delta V_o}{V_o} = D\frac{T_s}{\tau}$$

Cuk DC-DC converter

Negative DC power supply

DC analysis: $V_{C1} = V_d + V_o$ note: $(V_{C1} > V_d)$

Assumption: Large C1 (Voltage almost constant)

Switch OFF: C1 is charged through L1 and the input, Diode ON, L2 supplies energy to R (currents in L1 and L2 decrease)

Switch ON: L1 receives energy, Diode OFF, C supplies current to R, C1 gives energy to L2 (currents in L1 and L2 increase)

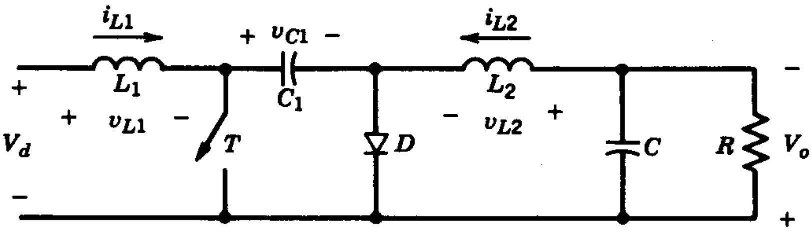


Figure 7-25 Cúk converter.

Cuk

Periodic conditions in L1

$$V_d DT_s + (1-D)T_s(V_d - V_{C1}) = 0$$

$$V_{C1} = \frac{V_d}{1-D}$$

Periodic conditions in L2

$$(V_{C1} - V_o)DT_s - V_o(1-D)T_s = 0$$

$$V_{C1} = \frac{V_o}{D}$$

Therefore

$$\frac{V_o}{V_d} = \frac{D}{1-D}$$

Pro: currents

in L1 and L2 ripple free

Con: C1 must be large

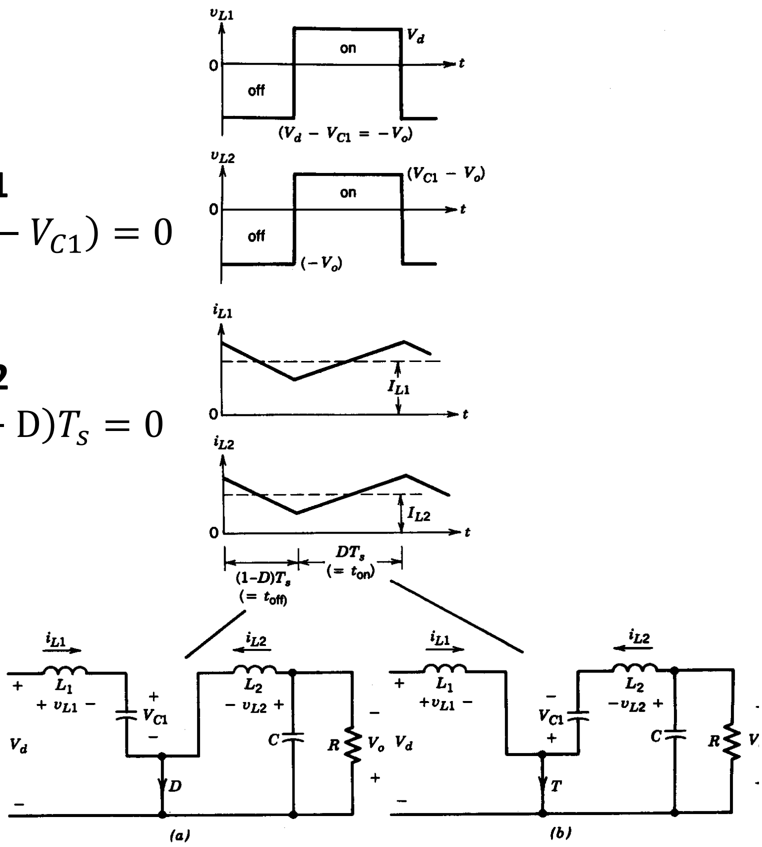


Figure 7-26 Cúk converter waveforms: (*a*) switch off; (*b*) switch on.

Full bridge DC-DC converter

When switch TA+ is on:

$i_o > 0$: i_o through TA+

$i_o < 0$: i_o through DA+

$V_{AN} = V_d \text{dutycycle}(TA^+)$

When switch TB+ is on:

$i_o < 0$: i_o through TB+

$i_o > 0$: i_o through DB+

$V_{BN} = V_d \text{dutycycle}(TB^+)$

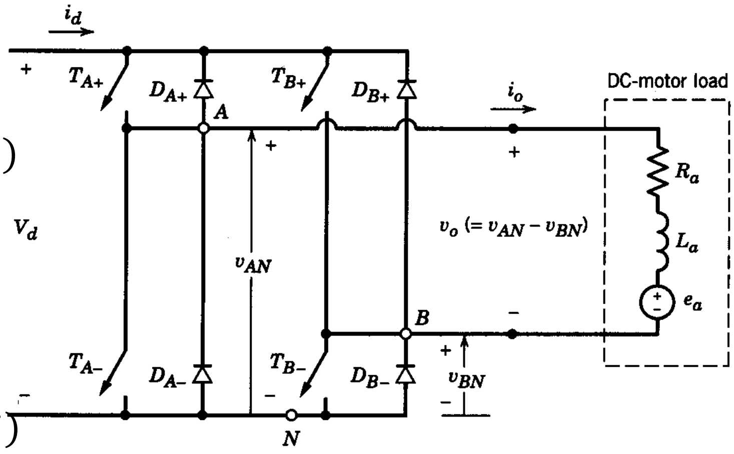
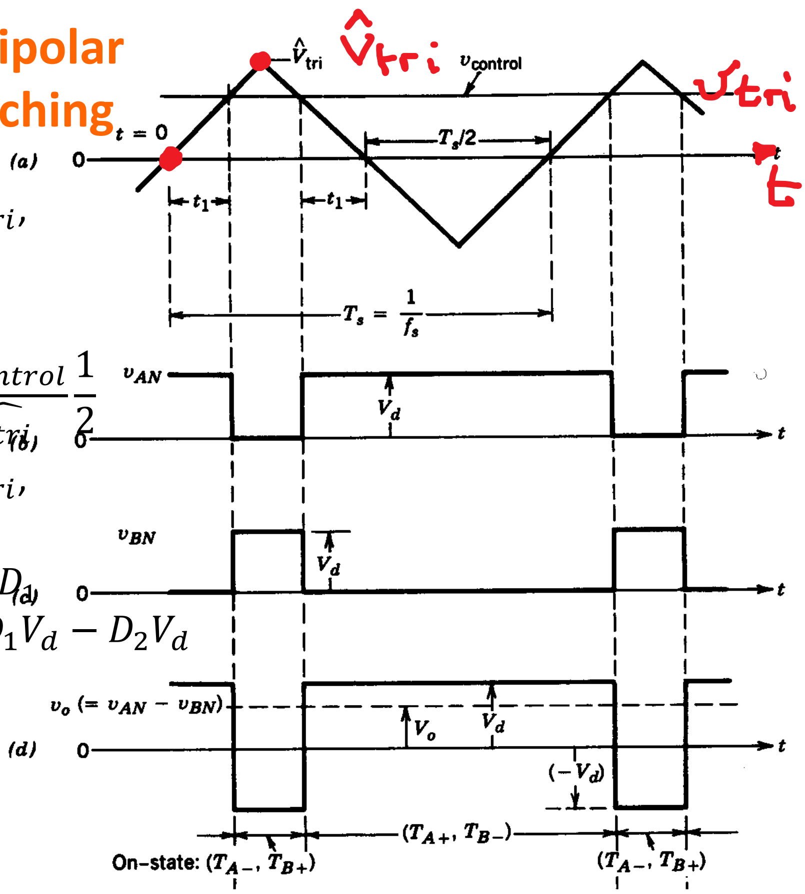


Figure 7-27 Full-bridge dc–dc converter.

$$V_o = V_{AN} - V_{BN}$$

Four quadrant operation on V_o, I_o

PWM with bipolar voltage switching



When $v_{control} > v_{tri}$,

TA+ and TB- are **ON**

Duty cycle

$$D_1 = \frac{1}{2} + \frac{v_{control}}{\widehat{V_{tri}}}\frac{1}{2}$$

When $v_{control} < v_{tri}$,

TA- and TB+ are **ON**

$$D_2 = 1 - D_1$$

$$V_o = V_{AN} - V_{BN} = D_1 V_d - D_2 V_d$$

$$= (2D_1 - 1)V_d$$

$$= \frac{V_d}{\widehat{V_{tri}}} v_{control}$$

PWM with unipolar voltage switching

When $v_{control} > v_{tri}$,

TA+ and TB- are **ON**

Duty cycle

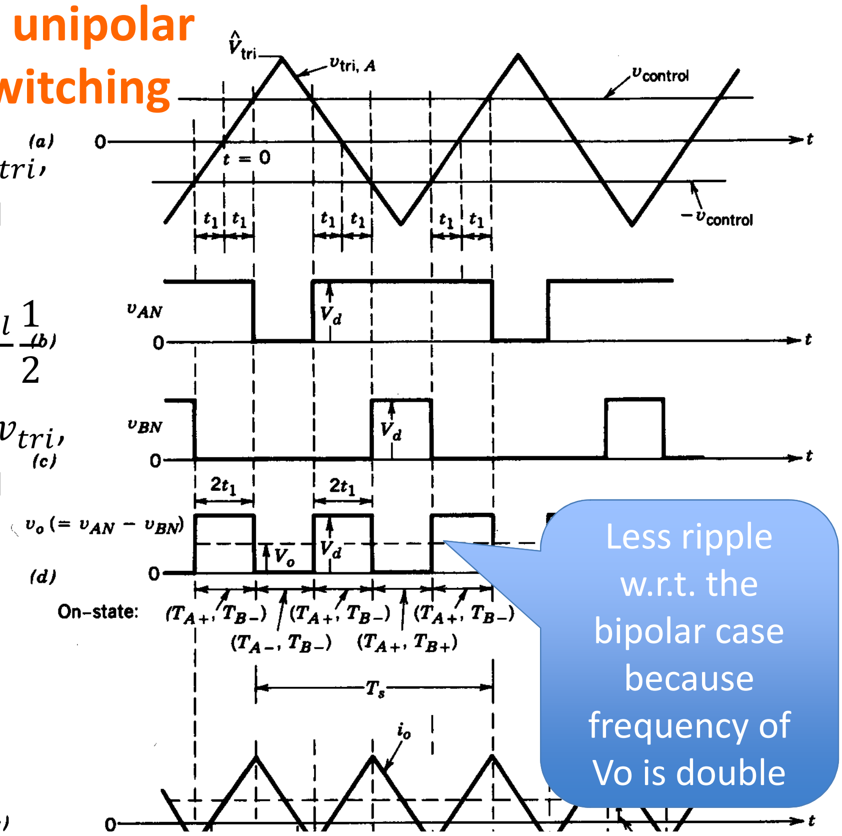
$$D_1 = \frac{1}{2} + \frac{v_{control}}{\widehat{V_{tri}}}\frac{1}{2}$$

When $-v_{control} < v_{tri}$,

TA- and TB+ are **ON**

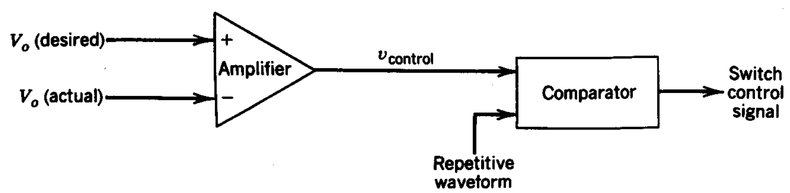
$$D_2 = 1 - D_1$$

$$V_o = V_{AN} - V_{BN}$$
$$= D_1 V_d - D_2 V_d$$
$$= (2D_1 - 1)V_d$$
$$= \frac{V_d}{\widehat{V_{tri}}} v_{control}$$

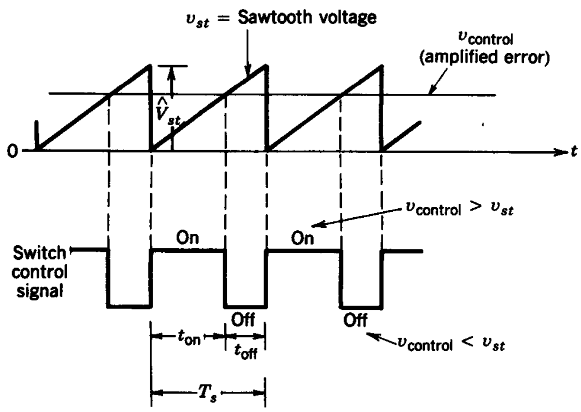


Less ripple w.r.t. the bipolar case because frequency of Vo is double

PWM signal generation

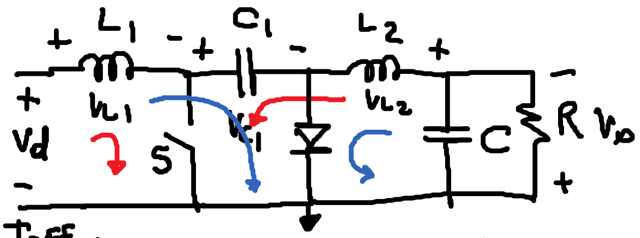


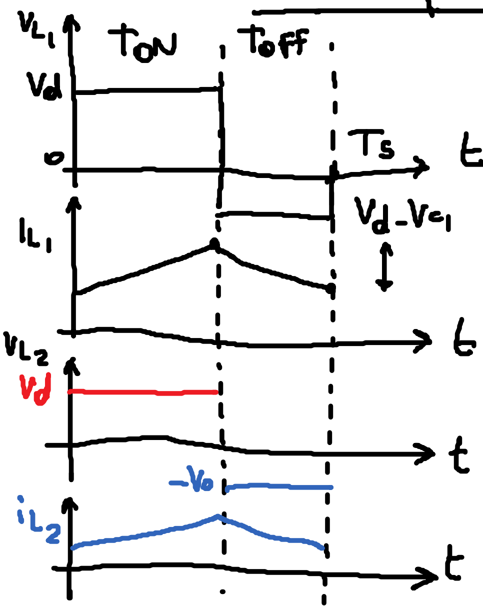
(a)



(switching frequency $f_s = \frac{1}{T_s}$)

Cúk





$V_{C_1} = V_d + V_o$

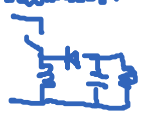
$t_{on} = DT_s$

$\Delta I_{L_1} = \frac{t_{ON} V_d}{L_1} = t_{off}\frac{V_{C_1} - V_d}{L_1} = \frac{t_{off} V_o}{L_1}$

$DV_d = (1-D)V_o$

$V_o = \frac{D}{1-D} V_D$

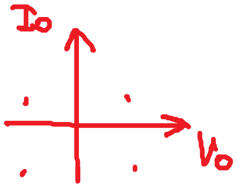
(buck-boost

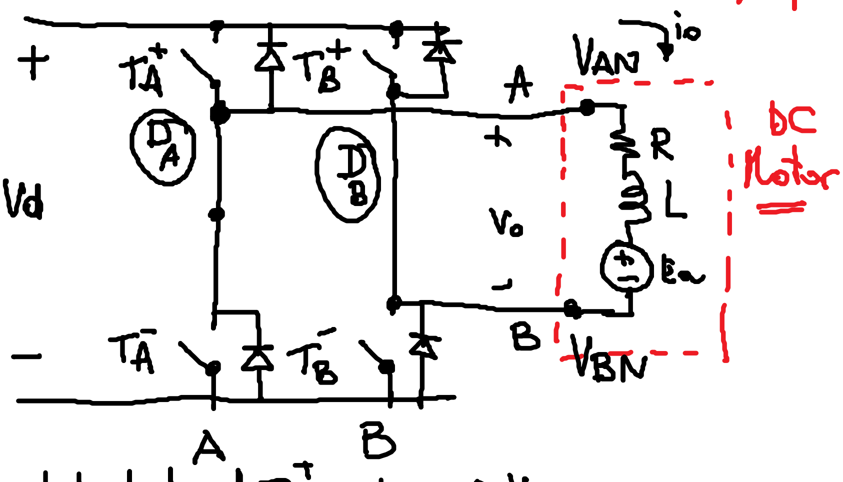


PRO: corrente continua dall'ingresso

CONTRO: C_1 grande

Full Bridge DCDC Converter (4Q)





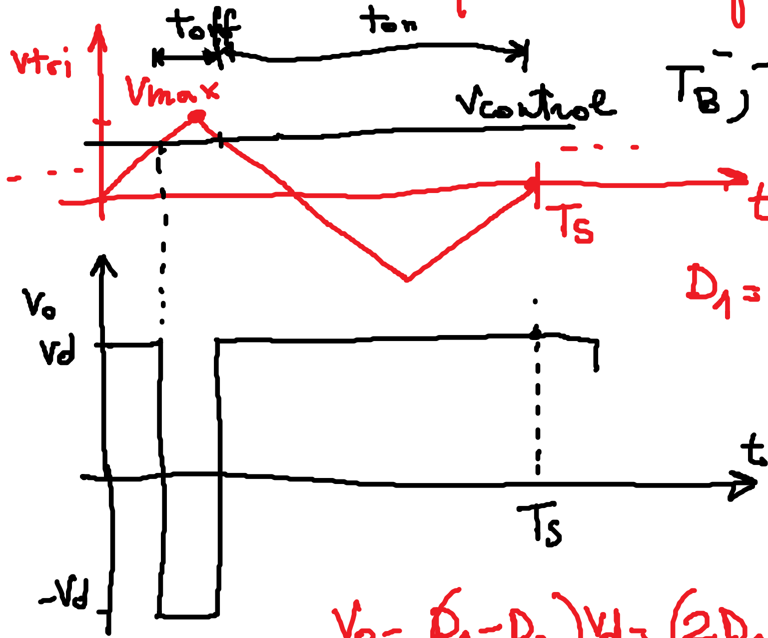
D_A: duty cycle of T_A^+ $\quad V_{AN} = D_1 V_d$

D_B: duty cycle of T_B^+ $\quad V_{BN} = D_2 V_d$

$\Big\} \; V_o = V_{AN} - V_{BN} = (D_1 - D_2) V_d$

Control: PWM → bipolar voltage
→ unipolar voltage

PWM with bipolar voltage



T_B^-, T_A^+ ON if

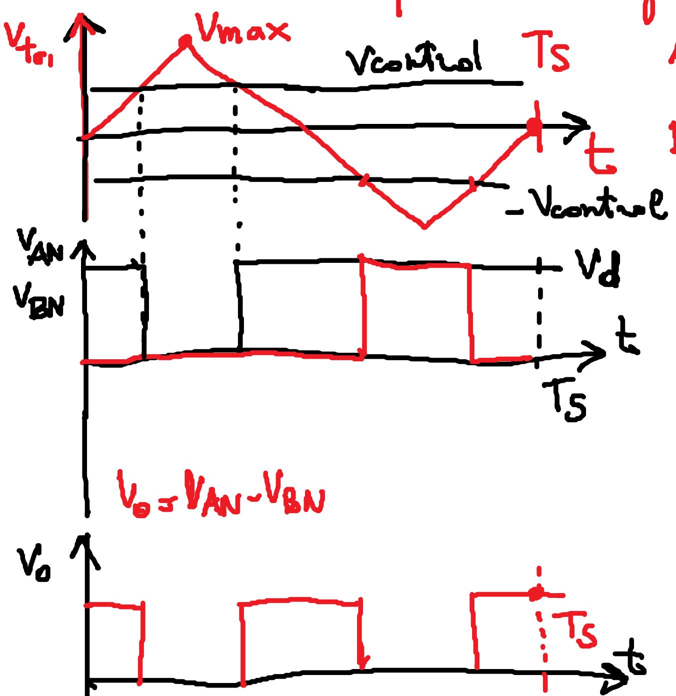
$V_{control} > V_{tri}$

$$D_1 = \frac{1}{2} + \frac{1}{2}\frac{V_{control}}{V_{max}}$$

$$D_2 = (1 - D_1)$$

$$V_o = (D_1 - D_2)V_d = (2D_1 - 1)V_d = V_{control} \cdot \frac{V_d}{V_{max}}$$

PWM with unipolar voltage



A: T_A^+ ON if $V_{control} > V_{tri}$

B. T_B^+ ON if $-V_{control} > V_{tri}$

$$D_1 = \frac{1}{2} + \frac{1}{2}\frac{V_{control}}{V_{max}}$$

$$D_2 = \frac{1}{2} - \frac{1}{2}\frac{V_{control}}{V_{max}}$$

$$D_2 = 1 - D_1$$

$$V_o = V_{control}\frac{V_d}{V_{max}}$$

↑ varia da Vd a 0 (o da 0 a -Vd)
ripple inferiore rispetto al caso bipolare