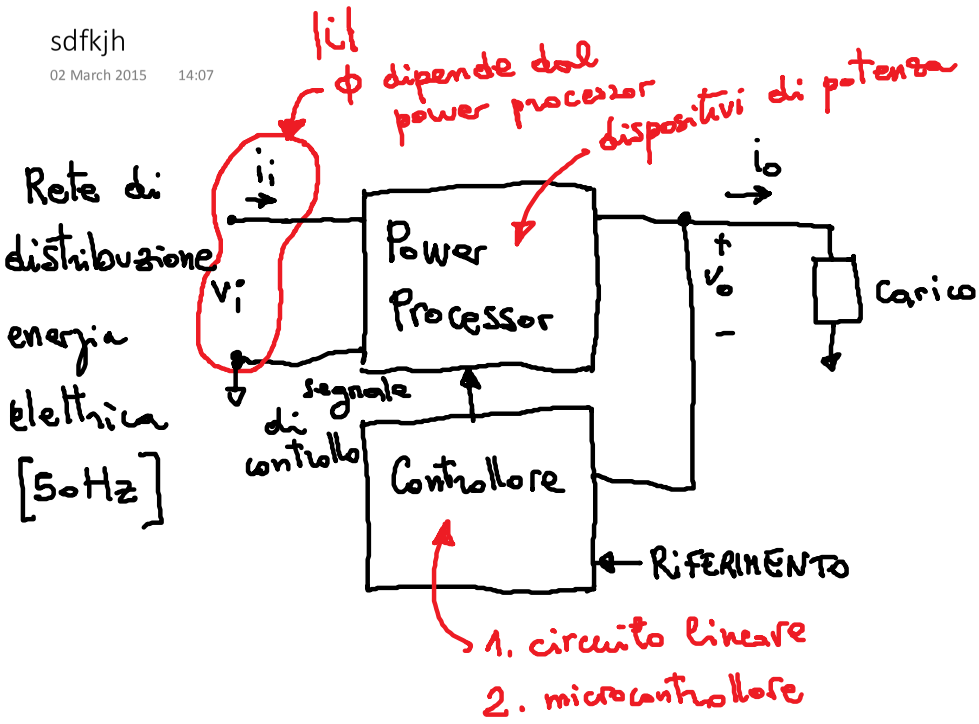
sdfkjh

02 March 2015 14:07



Introduzione

02 March 2015 13:24

Elettronica di Potenza

ha il compito di PROCESSARE e CONTROLLARE

il flusso di ENERGIA ELETTRICA

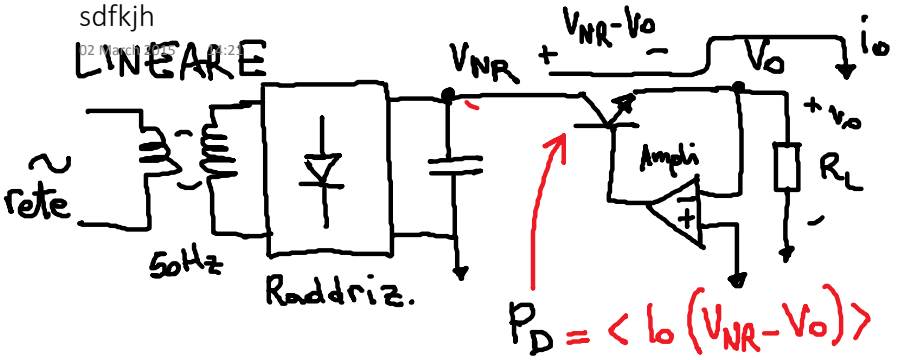
fornendo TENSIONI E CORRENTI

in FORMA OTTIMIZZATA

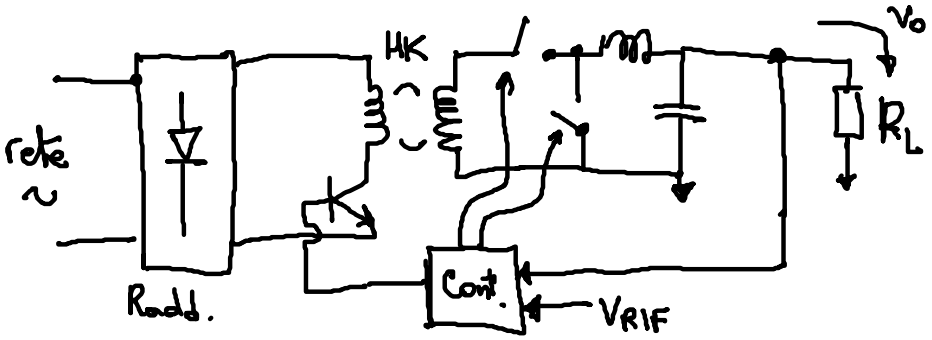
per il carico utente

sdfkjh
02 March 2015 10:25

LINEARE



REGOLATORE SWITCHING



Criteri di merito per un sistema di elettronica di potenza
02 March 2015 14:13

- CONSUMI
  - costo dell'energia
  - raffreddamento e ingombro
  - affidabilità
- COSTO DEL SISTEMA
- PESO E INGOMBRO

→ Non utilizzare sistemi in funzionamento lineare

04 March 2015 08:45

Dispositivi Elettronici di Potenza

- GRADO DI CONTROLLABILITÀ

1) ON dipende dal circuito
   OFF dipende dal circuito → DIODI

2) ON segnale di controllo
   OFF dipende dal circuito → TIRISTORI [SCR (1952) Silicon Controlled Rectifier]

3) ON segnale di controllo
   OFF segnale di controllo → INTERRUTTORI CONTROLLABILI (CONTROLLED SWITCH)
   [MOSFET, BJT, GTO, MCT, IGBT (1982)]

02 March 2015 14:34

Applicazioni tipiche

- DC Power Supply
- PMIC
- UPS Uninterruptable Power Supply
- Controllo di processi industriali
- Trasporti
- Applicazioni per distribuzione energia elettrica
  - interconnessione solare ed eolico
  - linee HVDC
- Applicazioni termoelettrica

DIODO SENZA PUNCHTHROUGH

04 March 2015 09:10

- la zona di svuotamento NON raggiunge la regione n^+
- Approx: tutto il pot. cade nella regione di deriva.

W spessore della zona di svuotamento

$$\mathcal{E}_{max} = \frac{qN_D W}{\varepsilon_0 \varepsilon_r} \leftrightarrow \phi = \frac{1}{2}\frac{qN_D W^2}{\varepsilon_0 \varepsilon_r}$$

$$\boxed{\phi = \frac{1}{2}\mathcal{E}_{max} W} \rightarrow \boxed{V_{BD} = \frac{1}{2}\mathcal{E}_{BD} W}$$

al BREAK DOWN

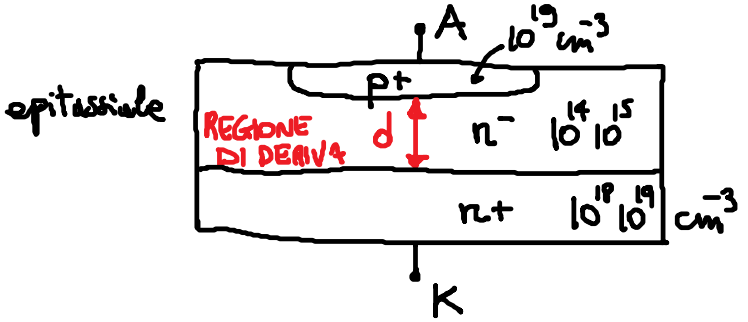
$(W \leq d)$ ← spessore della regione di deriva

Si $\mathcal{E}_{BD} = 2 \cdot 10^5 \, V/cm$ Voglio una $V_{BD} = 1000V$

$W = 100 \, \mu m$ $\boxed{d \geq 100 \mu m}$ $N_D = \frac{\mathcal{E}_{max} \cdot \varepsilon_0 \cdot \varepsilon_r}{qW} = 1{,}3 \, 10^{15} \, cm^{-3}$

DIODO DI POTENZA

04 March 2015 08:57

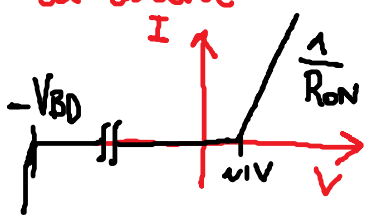


- VERTICALE
- REGIONE DI DERIVA (drift region)

Regione di deriva
- ALTÀ RESISTIVITÀ

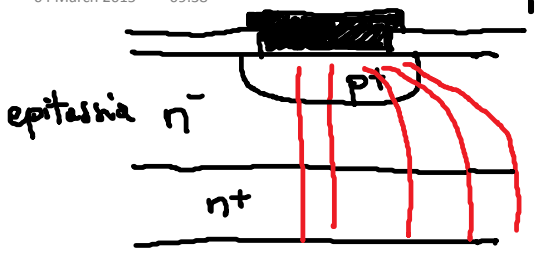
Effetti
- → Aumentare la V_{BD} (Break Down)
- → Aumenta la R_{ON}

↓ tutta la V inversa cade sulla regione di deriva



CURVATURA della diffusione p^+

04 March 2015 09:38



campo + alto in presenza delle regioni curve

↓

RIDUZIONE DELLA V_{BD}

) PER RIDURRE solo del 10% la V_{BD} è necessario

$R_{curvature} > 6\,W$

aumenterebbe troppo lo spessore della regione epitassiale

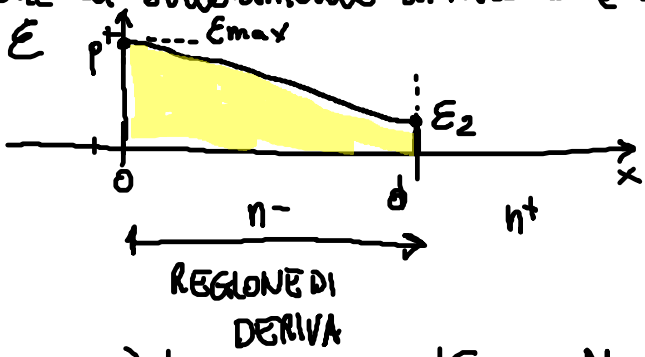
↳ si fa solo per basse V_{BD} (<100V)

altrimenti ➡ ELETTRODI FLOTTANTI o ANELLI DI GUARDIA

DIODI CON PUNCH THROUGH

04 March 2015 09:28

) La regione di svuotamento arriva alla zona n^+



$$\phi = \frac{(\mathcal{E}_{max} + \mathcal{E}_2)\,d}{2} \qquad \frac{d\mathcal{E}}{dx} = -\frac{qN_D}{\varepsilon_0 \varepsilon_r}$$

$$\mathcal{E}_2 = \mathcal{E}_{max} - \frac{qN_D}{\varepsilon\varepsilon_r}\,d$$

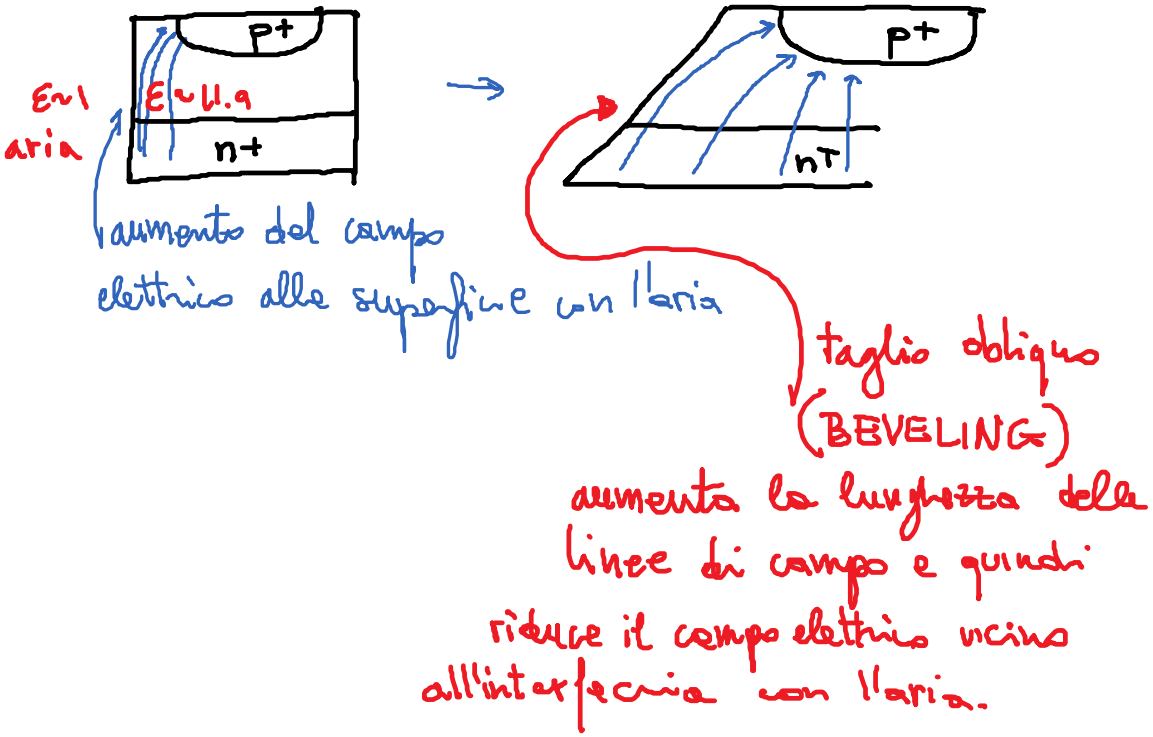
) caso limite ($N_D \to 0 \to \mathcal{E}_2 \sim \mathcal{E}_{max}$)

$\phi = \mathcal{E}_{max}\,d$ → al BREAK DOWN. $\boxed{V_{BD} = \mathcal{E}_{BD}\,d}$

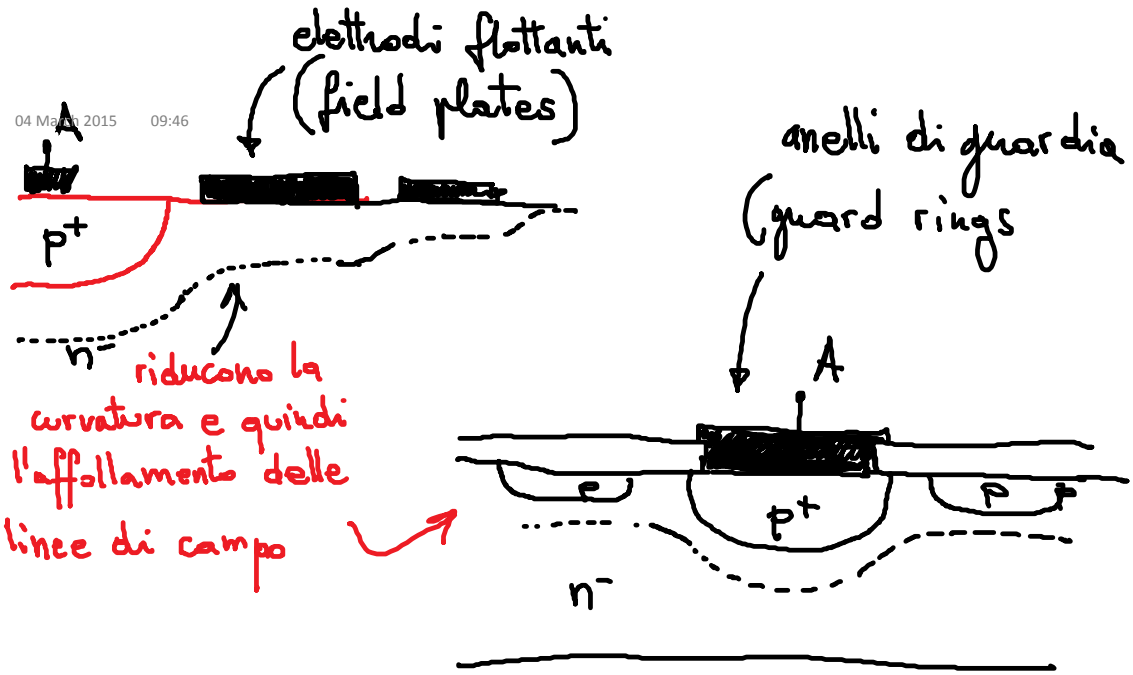
A PARITÀ di d e $\mathcal{E}_{BD}$ abbiamo $V_{BD} = 2\times$ il caso precedente

GESTIONE DELLA SUPERFICIE

04 March 2015 09:54

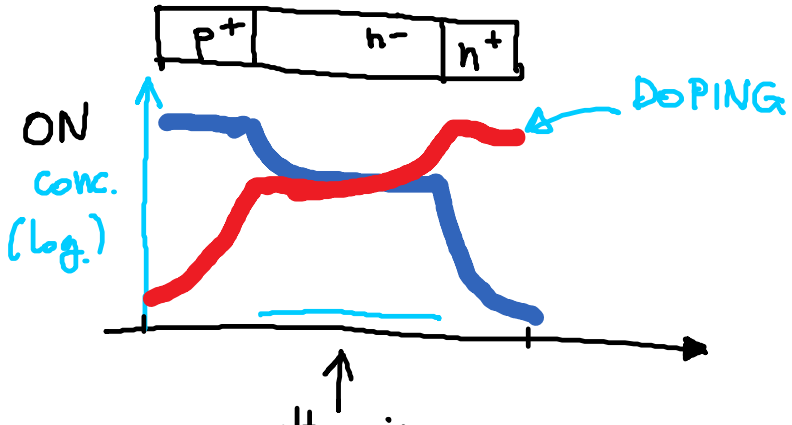


04 March 2015 09:46



MODULAZIONE DELLA CONDUTTIVITÀ

04 March 2015 09:59



alta iniezione $p, n \gg N_D^-$

doppia iniezione in regione di deriva

→ $p \sim n \sim n_a \gg N_D^-$

→ PLASMA DI CARICHE ←

IN conduzione AUMENTA MOLTO LA CONDUTTIVITÀ DELLA REG. DI DERIVA

04 March 2015 10:19

RECAP EFFETTO SPESSORE REGIONE DI DERIVA

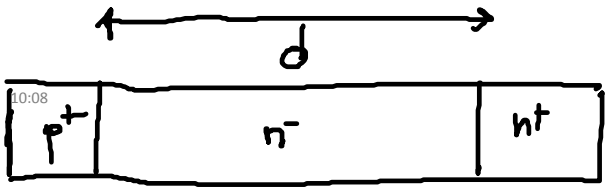
d

$V_{BD} \propto d$

$V_{ON} = V_d + V_j$ $\qquad$ $V_d \propto d^2$

- $V_d \sim$ non dipende dal drogaggio per la modulazione della conduttività

04 March 2015 10:08



Caduta di tensione nella regione di deriva V_d

corrente nella regione di deriva

Corrente di deriva:

$$I = Aq\mu_n n\left(\frac{V_d}{d}\right) + Aq\mu_p p\left(\frac{V_d}{d}\right) =$$

$$I = q(\mu_n + \mu_p) n_a \left(\frac{V_d}{d}\right) A$$

NON DIPENDE DA N_a

$V_d \propto d^2$

ricombinazione

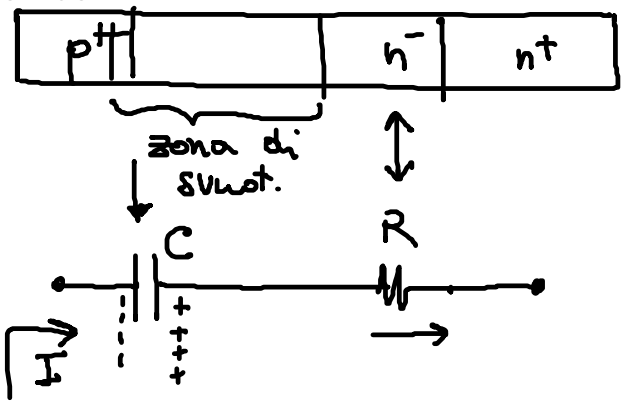
$$I = \frac{Q}{\tau} = \frac{q n_a d A}{\tau} \Rightarrow \boxed{V_d = \frac{d^2}{\tau(\mu_n + \mu_p)}}$$

tempo di vita $\rightarrow \tau$

(dipendenza da n_a)

Accensione

04 March 2015 10:29

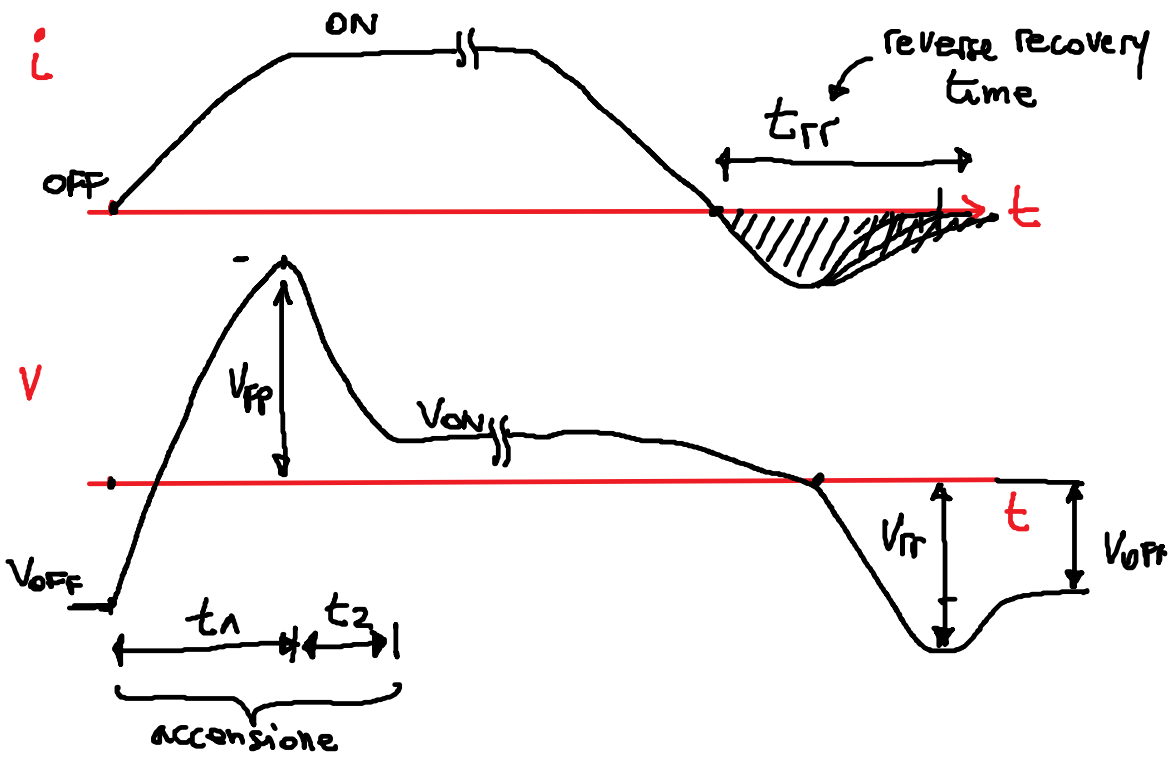


t_1 (~ns) eliminazione della zona di svuotamento
alta resistività della regione di deriva

t_2 (~μs) formazione del plasma
riduzione della resistività della regione di deriva

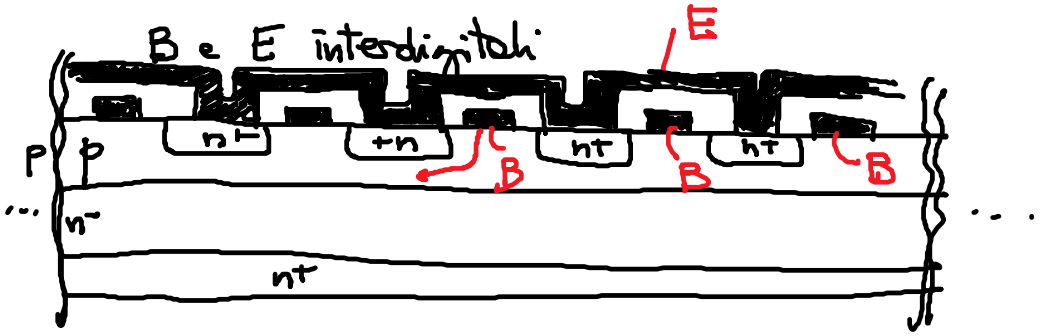
COMMUTAZIONE (SWITCHING)

04 March 2015 10:23



Geometria a Multi emettitori

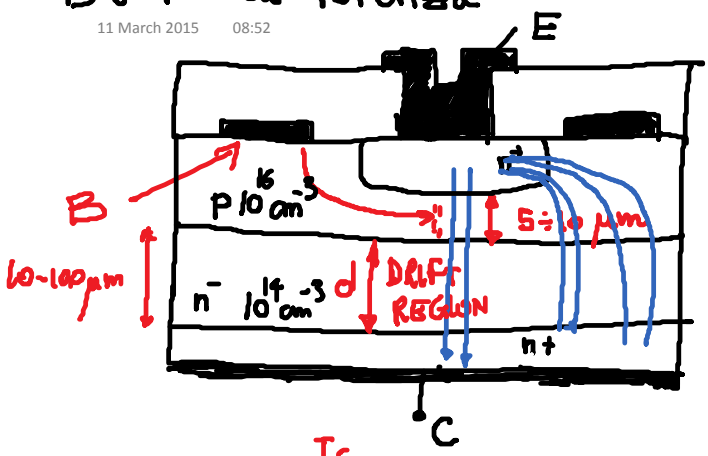
11 March 2015 09:14



- Riduce la r_b del transistore
- Riduce il fenomeno di "CURRENT CROWDING"
  - ↳ mitiga il rischio di FUGA TERMICA
    - ↳ che dà luogo al BREAKDOWN SECONDARIO

BJT di Potenza

11 March 2015 08:52

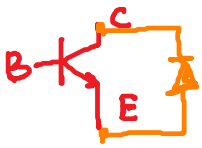


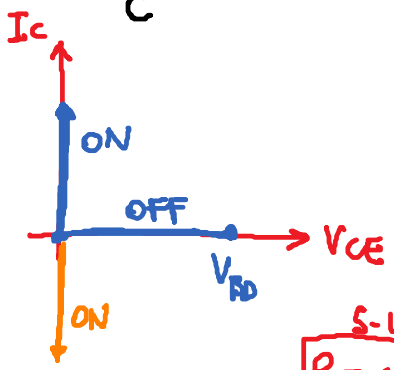
$n^+ \sim 10^{19}$ cm^{-3}

STRUTTURA VERTICALE

- Alta sezione di conduzione
- Bassa resistenza serie
- Bassa resistenza termica

$V_{BD} \sim d E_{BD}$



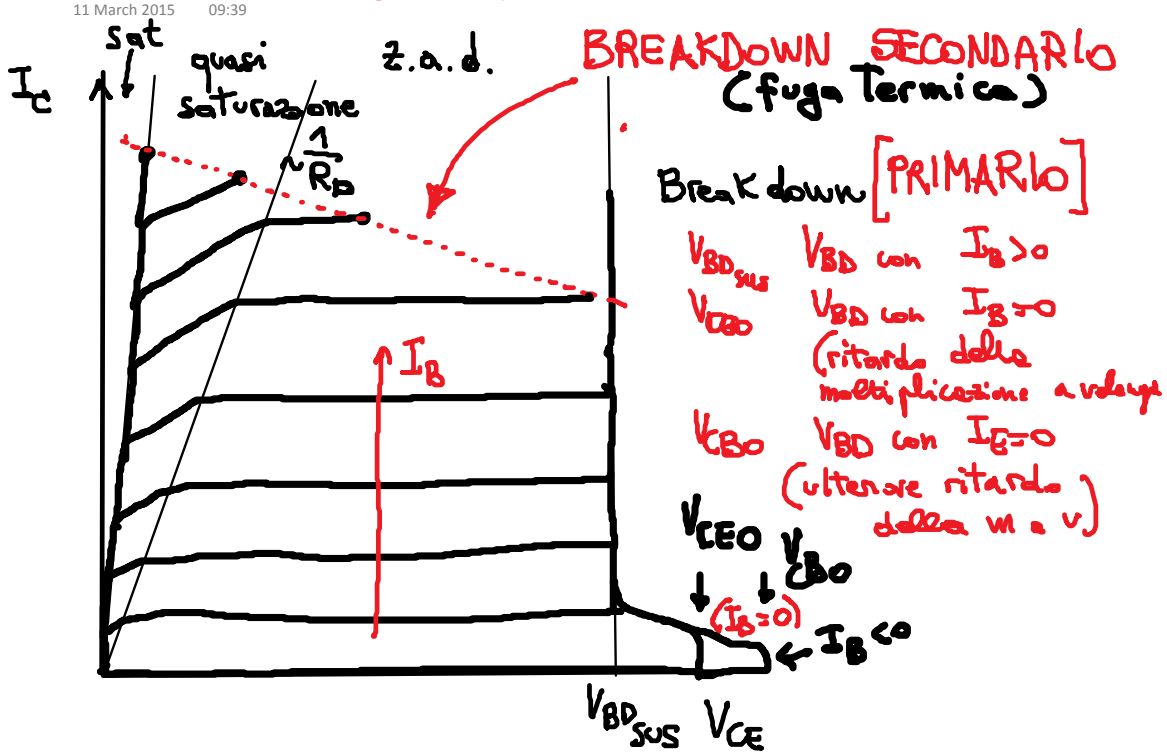


- SPESSORE della base 5-10 μm per evitare PUNCH THROUGH (REACH THROUGH)

→ $\boxed{\beta_F < 10}$

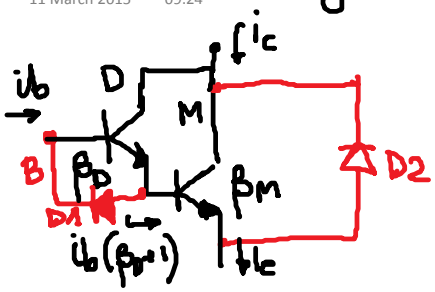
Caratteristiche di uscita

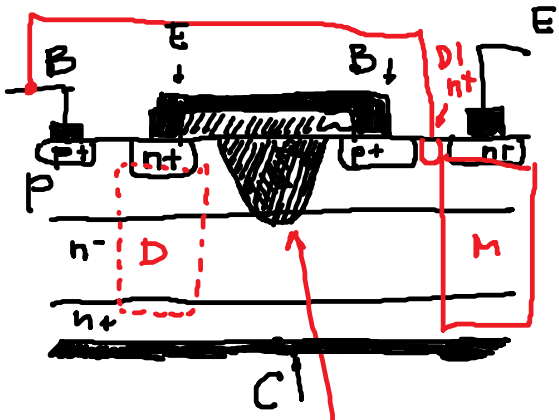
11 March 2015 09:39



BJT Darlington

11 March 2015 09:24





$$i_c = \beta_D i_d + \beta_M(\beta_D+1)\, i_b$$

$$\frac{i_c}{i_b} = \beta_{eq} = [\beta_M\beta_D + \beta_D + \beta_M] \sim 100$$

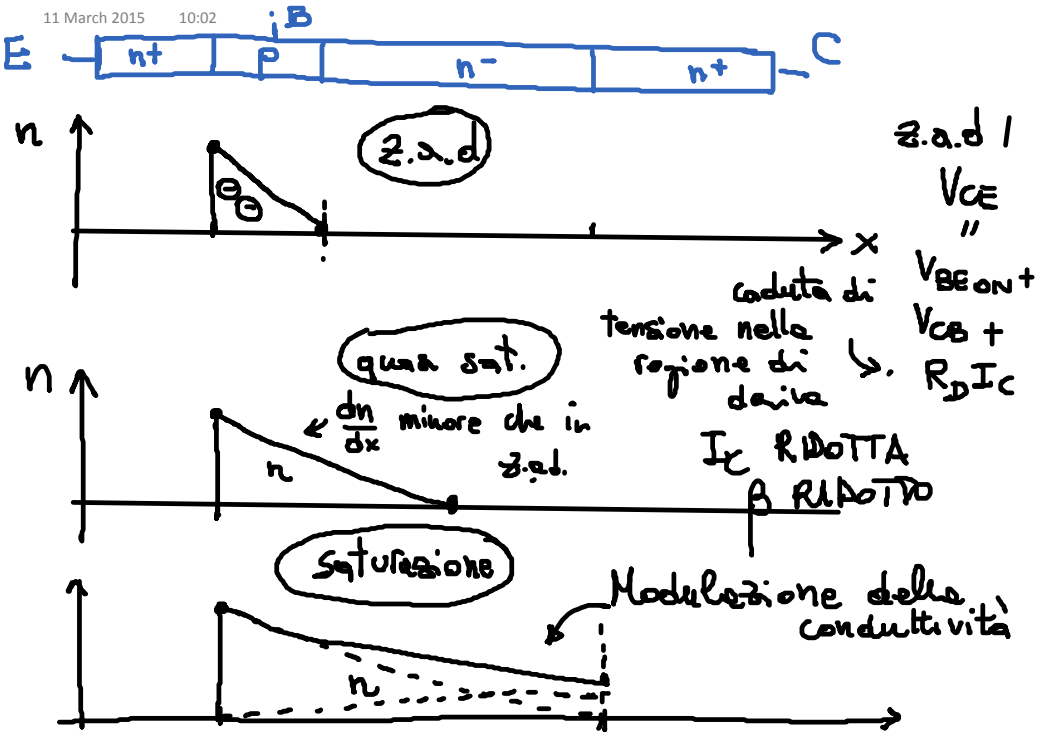
Protrusione dell'ossido

D1: Per applicare $i_B<0$ al Master (riduce il tempo di spegnimento di M)

D2: diodo di ricircolo

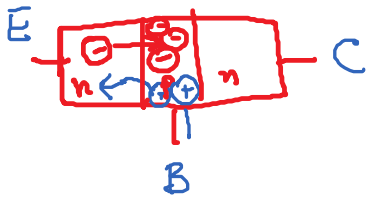
Quasi saturazione

11 March 2015 10:02



Alta iniezione

11 March 2015 09:55

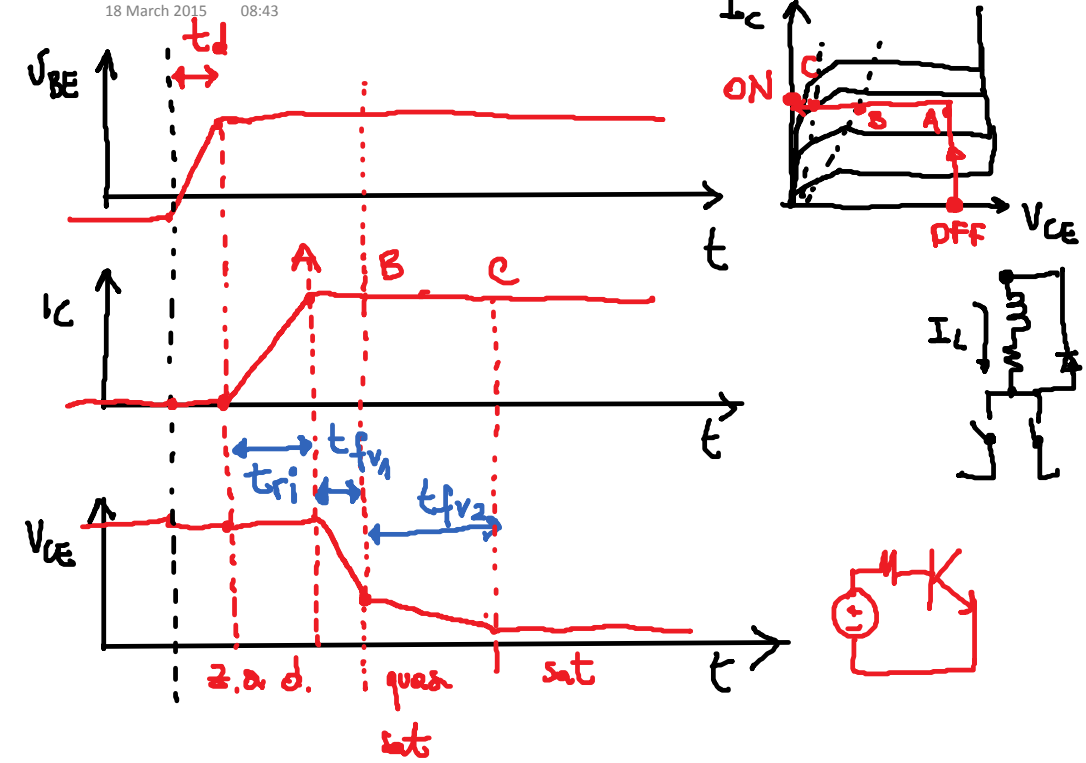


in alta iniezione la concentrazione di elettroni in base è così alta da causare l'aumento dell'iniezione di lacune dal contatto di base

↳ aumenta la corrente di lacune tra base e emettitore (I_B)

Transistorio di accensione

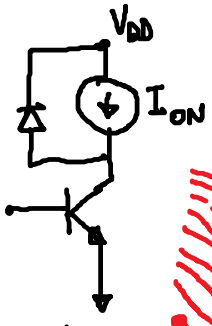
18 March 2015 08:43



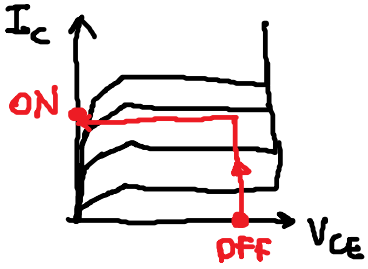
Transistorio di accensione

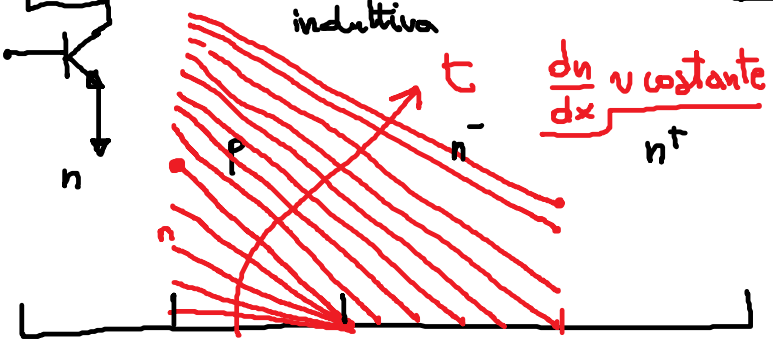
11 March 2015 10:16

[OFF → SAT]



← schematizzazione ideale del carico induttivo





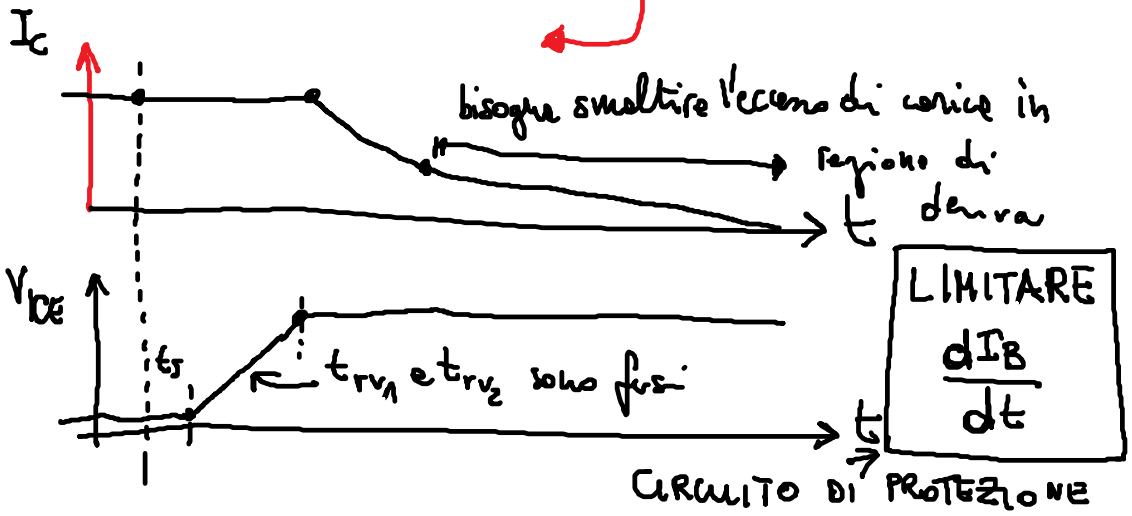
$\frac{dn}{dx}$ v costante

n^+

Spegnimento non controllato

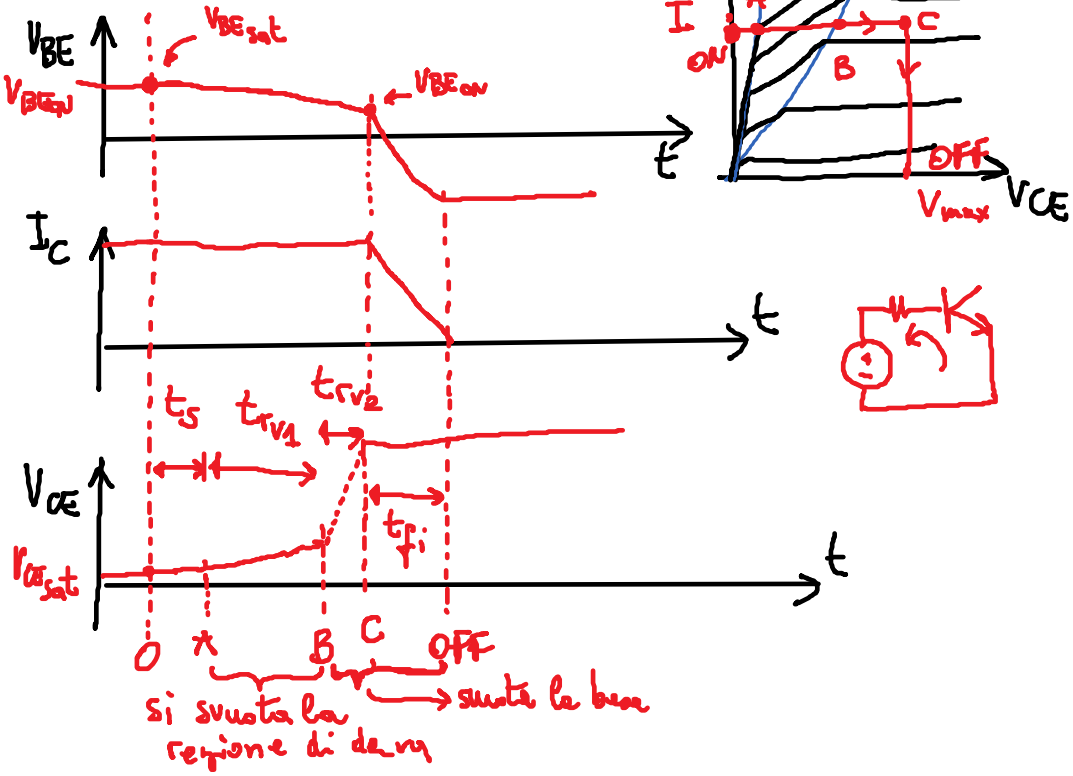
18 March 2015 09:15

concentrazione di elettroni nella base estesa



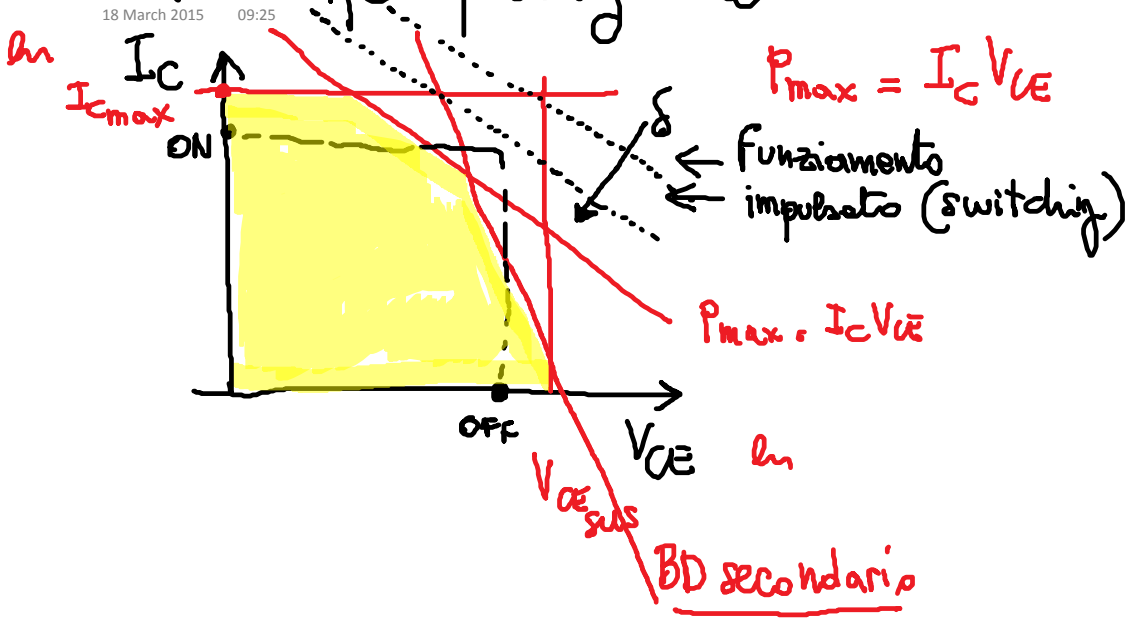
Transitorio di spegnimento

18 March 2015 09:00



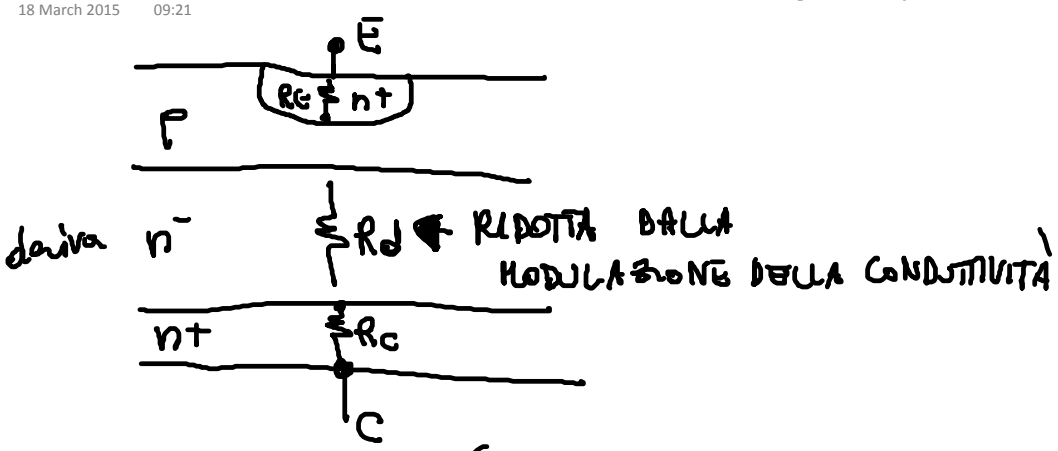
SOA Safe Operating Area

18 March 2015 09:25



V_{ON} caduta di tensione in conduzione

18 March 2015 09:21



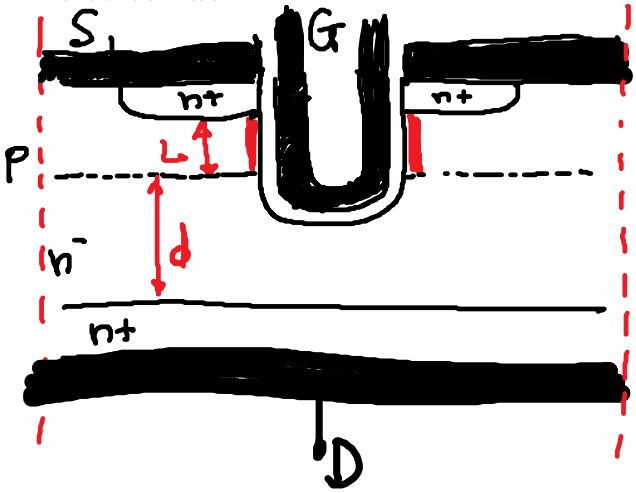
$$V_{ON} = V_{CE_{sat}} + (R_d + \cancel{R_C} + \cancel{R_E}) I_{ON} \sim \underline{1\,V}$$

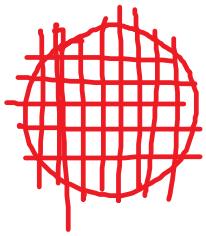
$$V_{CE_{sat}} = V_{BE_{sat}} - V_{BC_{sat}}$$

$\sim 0.1 - 0.2\,V$

U MOSFET

18 March 2015 09:54



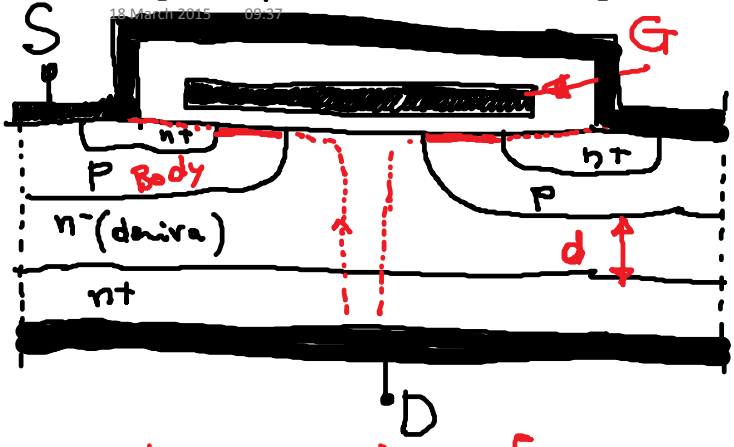


$W_{eff} = W \cdot N$

↑ numero di celle in parallelo

MOSFET di Potenza

18 March 2015 09:57



VD-MOSFET

80°

Vertically Diffused

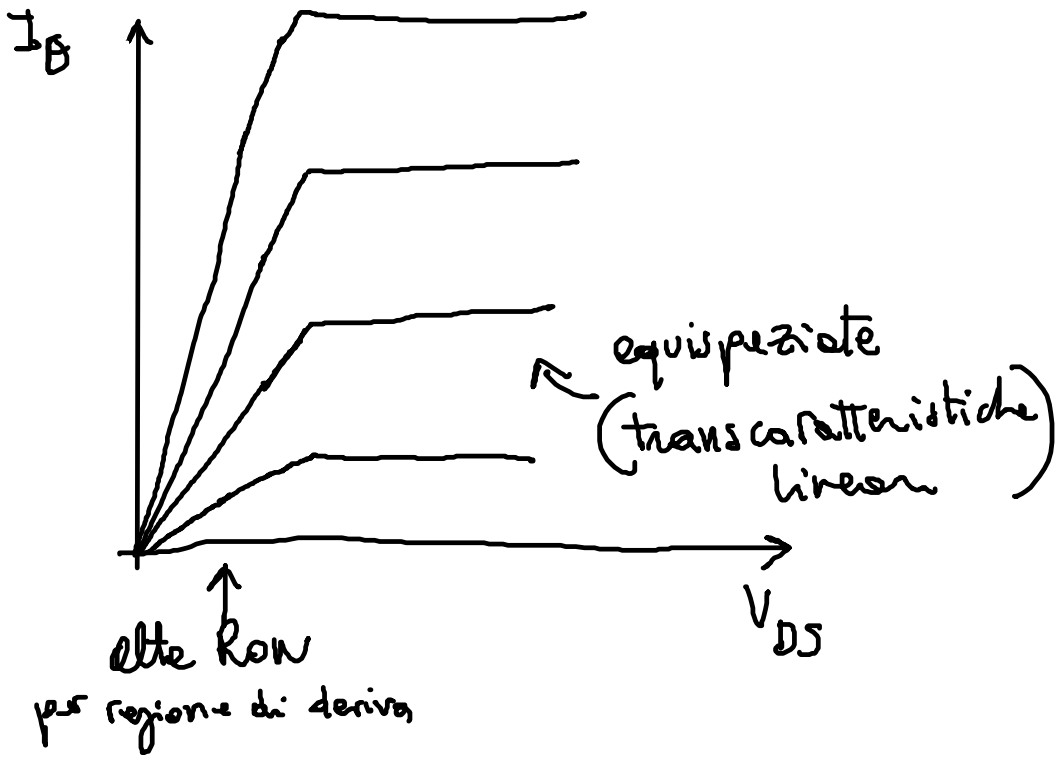
$V_{BD} \simeq d\,\mathcal{E}_{max}$

- Body e source in CC [per spegnere npn parassita]
- Gate su regione di deriva
  - in ON: Riduce resistenze serie
  - in OFF: fa da field plate

NON SI HA MODULAZIONE DELLA CONDUTTIVITÀ
↑ più veloce ↓ alta R_{ON}

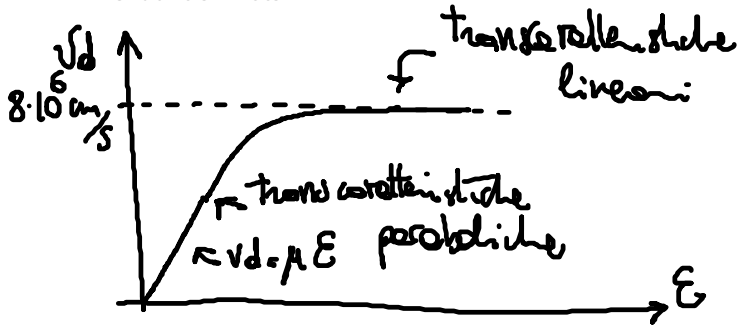
Caratteristiche di uscita

18 March 2015 10:12



Saturazione della velocità → TRANSCARATTERISTICA LINEARE

18 March 2015 10:03



MOSFET DI POTENZA (alta V_{ds})

$I \propto Q v_d$

$Q \propto (V_{GS} - V_{th})$

$v_d \propto \mathcal{E} \sim \frac{V_{GS} - V_{th}}{L}$

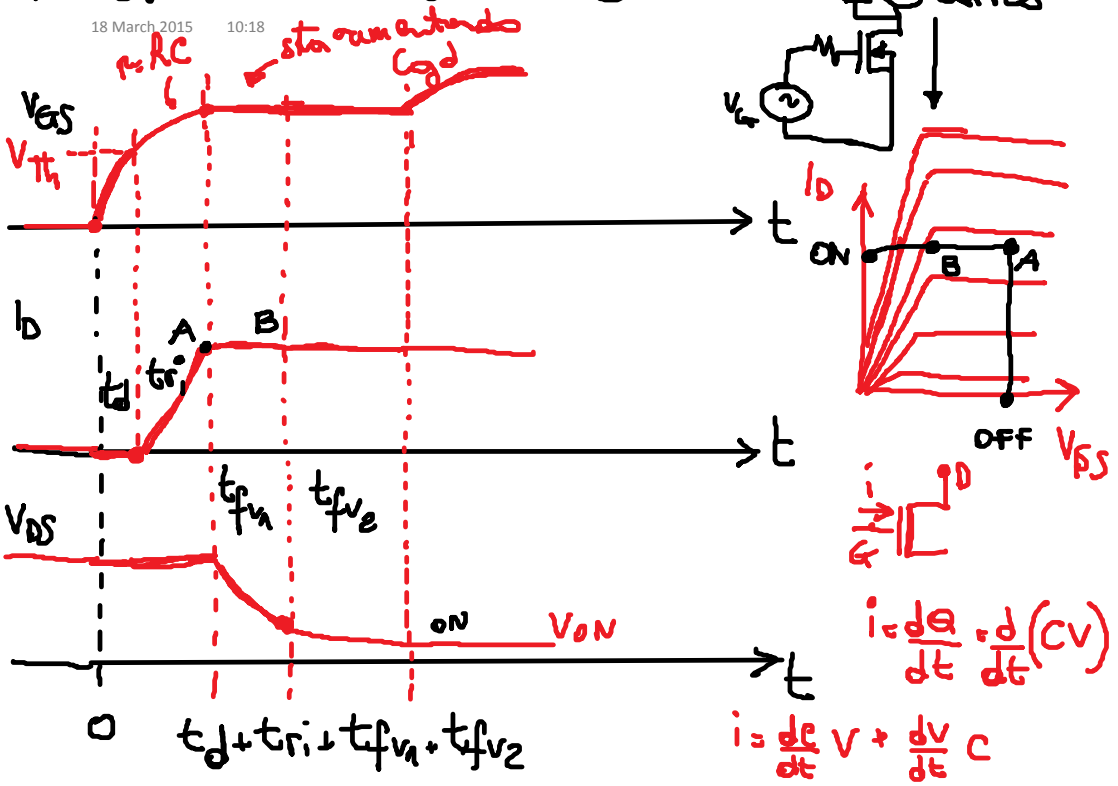
$I \propto (V_{GS} - V_{th})^2$

se $v_d = V_{sat}$

$I \propto (V_{GS} - V_{th})^1$

Transitorio di accensione

18 March 2015 10:18

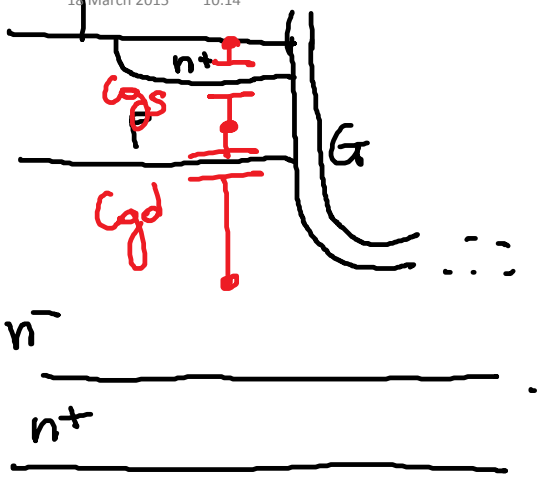


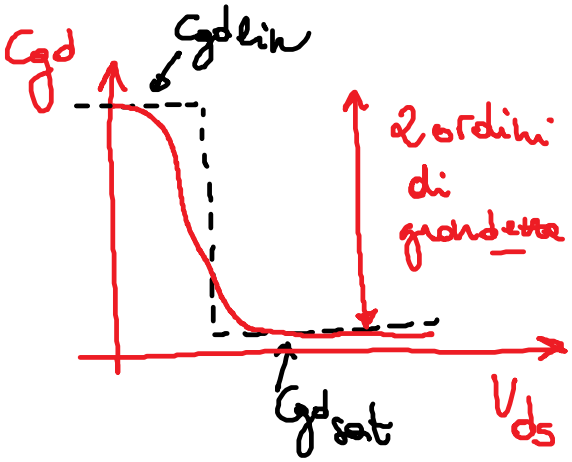
$$i = \frac{dQ}{dt} = \frac{d}{dt}(CV)$$

$$i = \frac{dC}{dt}V + \frac{dV}{dt}C$$

Capacità interne

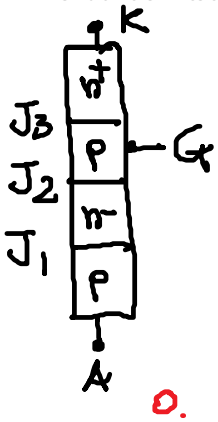
18 March 2015 10:14





Tiristore ideale

25 March 2015 09:09

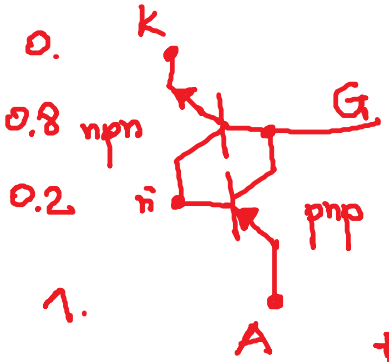


Forward Blocking

$V_{AK} > 0$ J_2 supports V_{AK}

Reverse Blocking

$V_{AK} < 0$ J_1 supports V_{AK}



$\alpha_{npn} \simeq 0.9 \div 1$

$\alpha_{pnp} \sim 0.01 \div 0.1$

→ Two States.

→ OFF BJTs are in cutoff

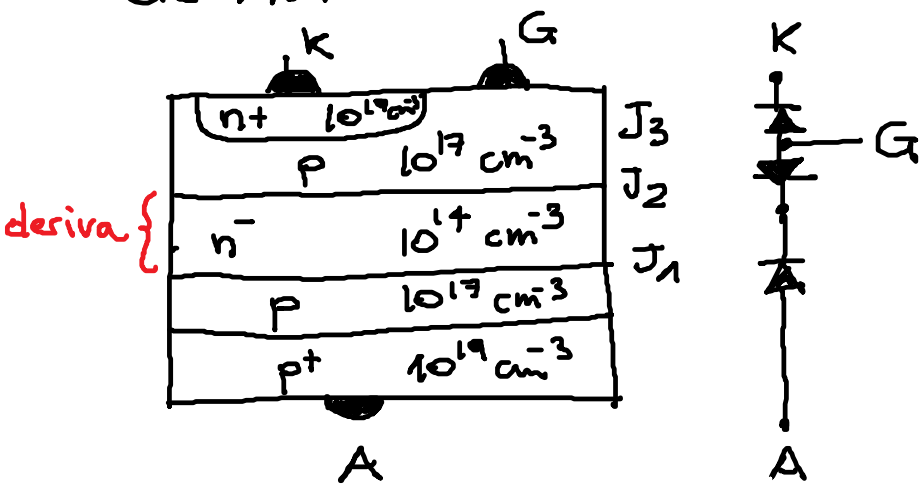
→ ON BJTs are is sat.

Tiristori (Thyristor)

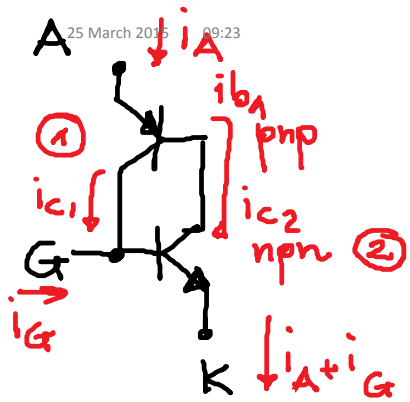
25 March 2015 09:01

SCR Silicon Controlled Rectifier

GE 1957



25 March 2015 09:23



$$i_A = i_{c_1} + i_{c_2}$$

$$i_A = \alpha_{pnp}\, i_A - I_{co_1} + \alpha_{npn}(i_A + i_G) + I_{co_2}$$

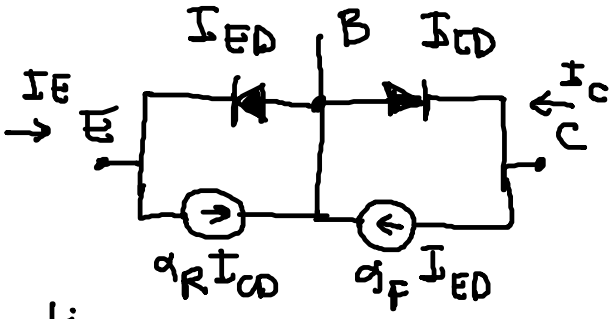
$$i_A = \frac{\alpha_{npn}\, i_G - I_{co_1} + I_{co_2}}{1 - (\alpha_{pnp} + \alpha_{npn})}$$

if $(\alpha_{pnp} + \alpha_{npn}) < 1$ **OFF**

if $(\alpha_{pnp} + \alpha_{npn}) = 1$ **ON**

EBERS-MOLL MODEL of the BJT

25 March 2015 09:18



(npn)

in the active zone:

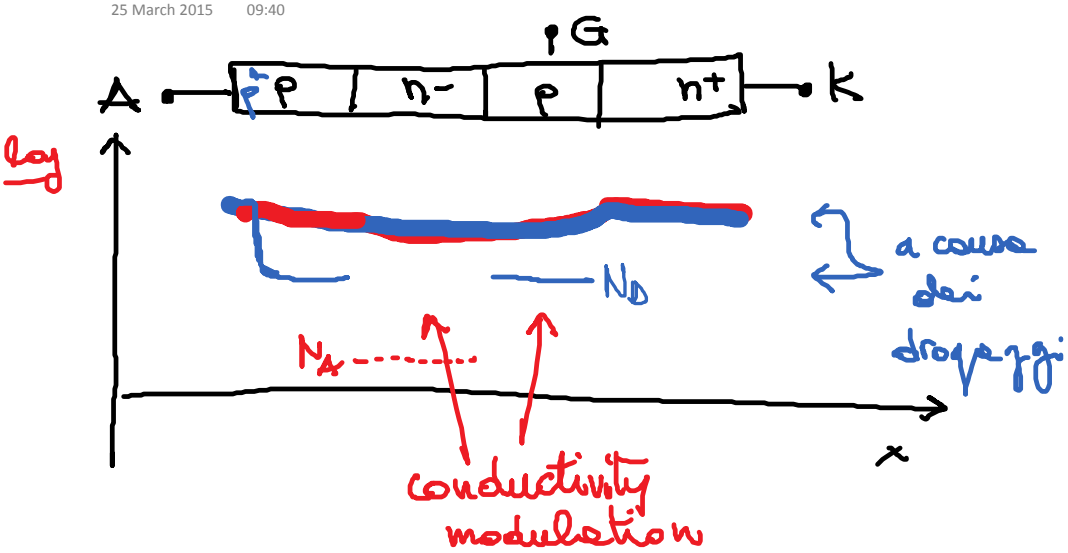
$$I_{CD} = -I_{CS}$$

- $I_E = -I_{ED} - \alpha_R I_{CS}$
- $I_C = \alpha_F I_{ED} + I_{CS} = -\alpha_F I_E + \overbrace{I_{CS}(1 - \alpha_R \alpha_F)}^{I_{CO}}$

$$\boxed{I_C = -\alpha_F I_E + I_{CO}}$$

ON STATE OPERATION

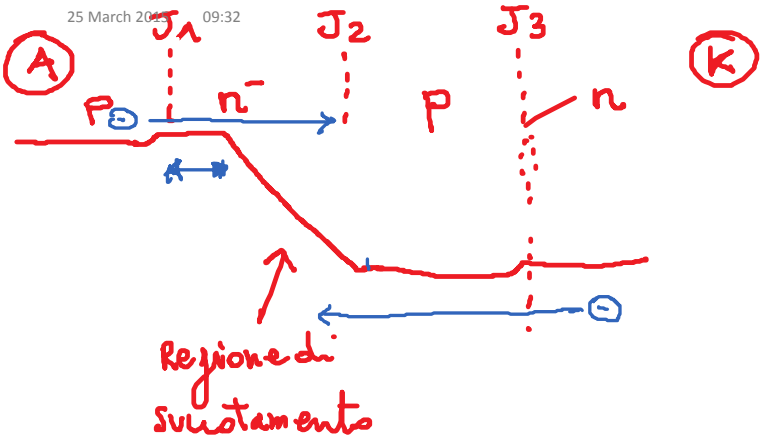
25 March 2015 09:40



$$V_{ON} = V_j + R_{ON} I$$

$V_j \rightarrow 0.7\,V$

25 March 2015 09:32



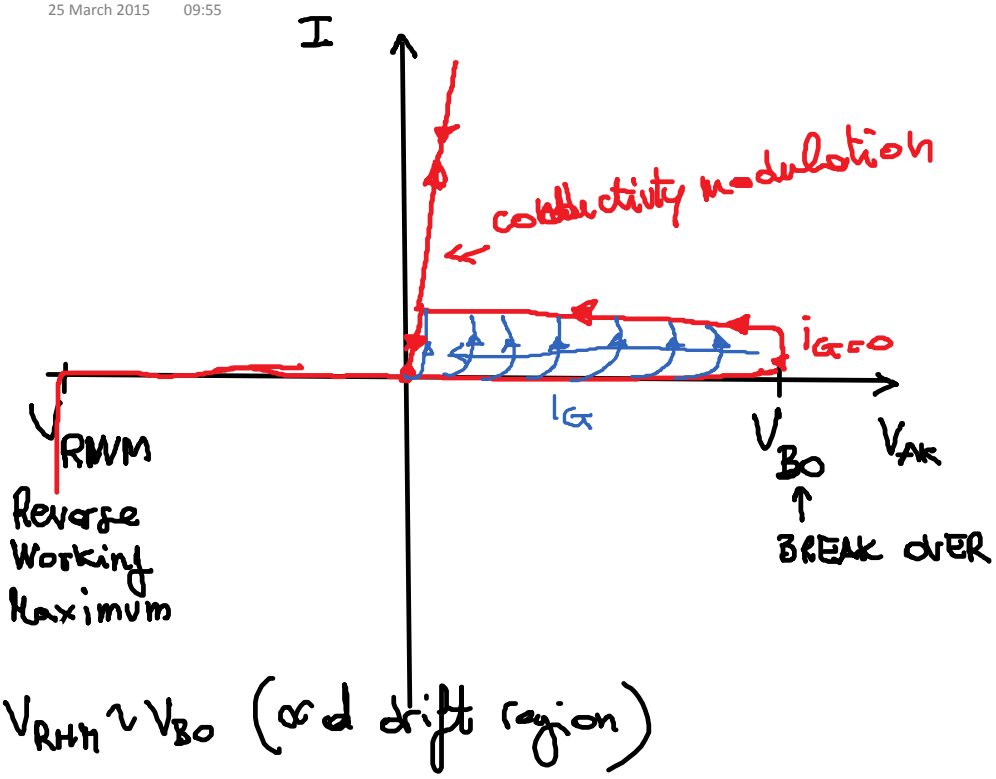
$V_{AK} > 0$

potential

- when $V_{AK}\uparrow$, the effective base of the pnp shrinks $\alpha_{pnp}\uparrow \rightarrow ON\uparrow$
- when $I_G\uparrow$, $\alpha_{npn}\uparrow \rightarrow ON\uparrow$

DC characteristics

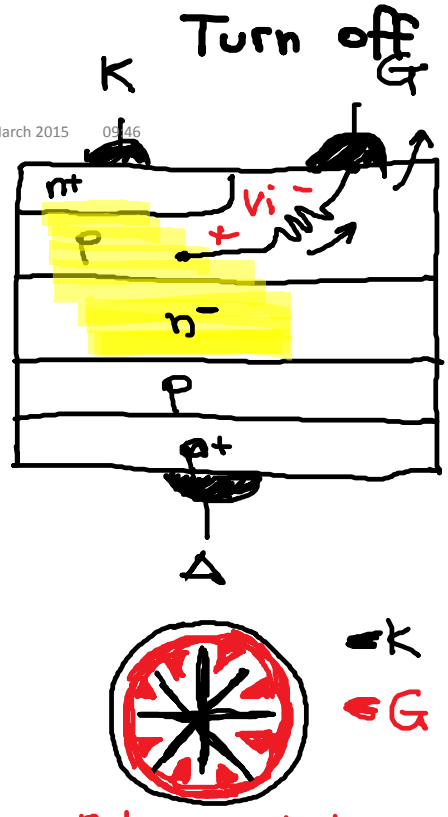
25 March 2015 09:55



$V_{RWM} \sim V_{BO}$ (of drift region)

Turn off

25 March 2015 09:46



Interdigitated cathode

$i_G < 0$

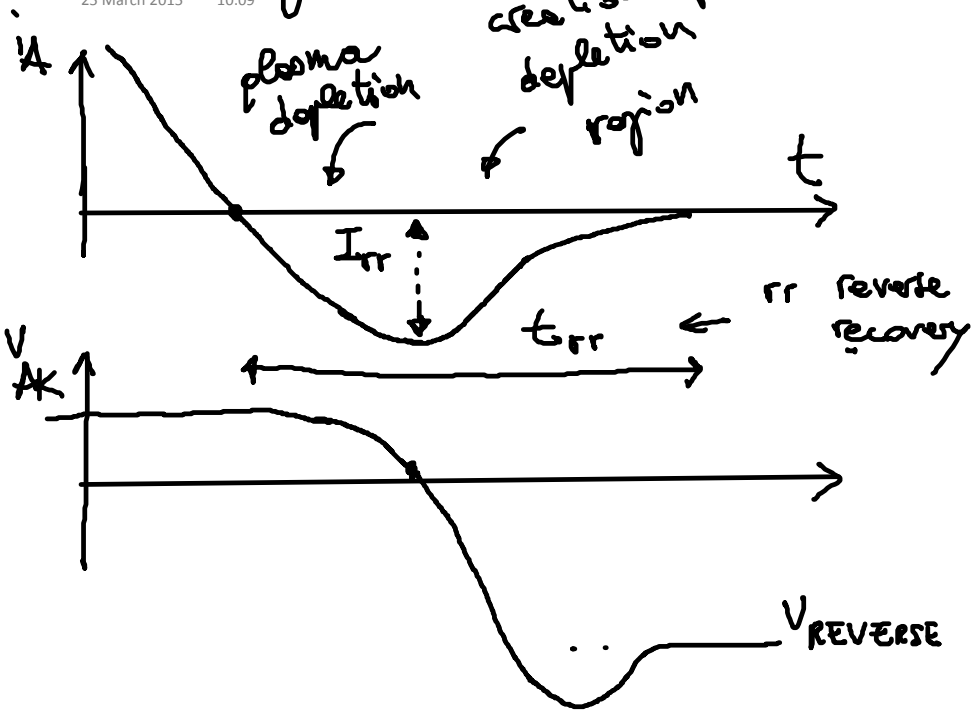
cannot deplete the p and n− regions.

Too high voltage drop V_i

SCR is turned off with $V_{AK} < 0$

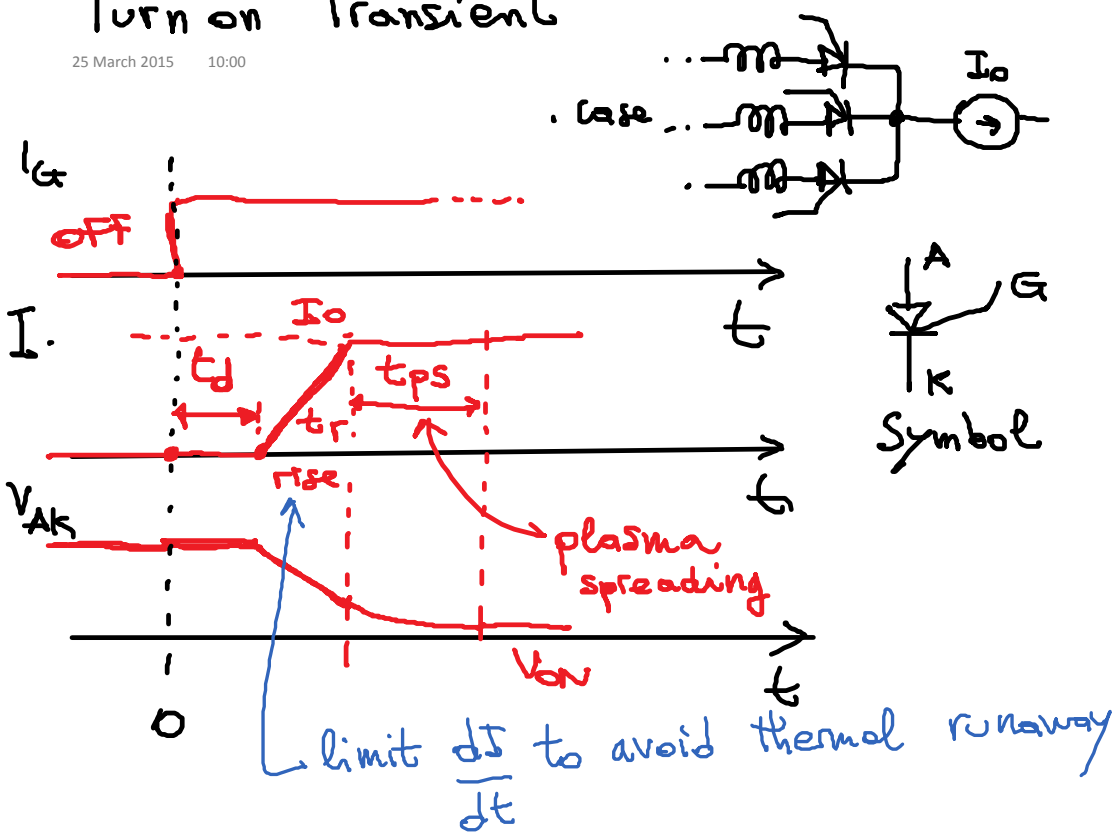
Turn off

25 March 2015 10:09

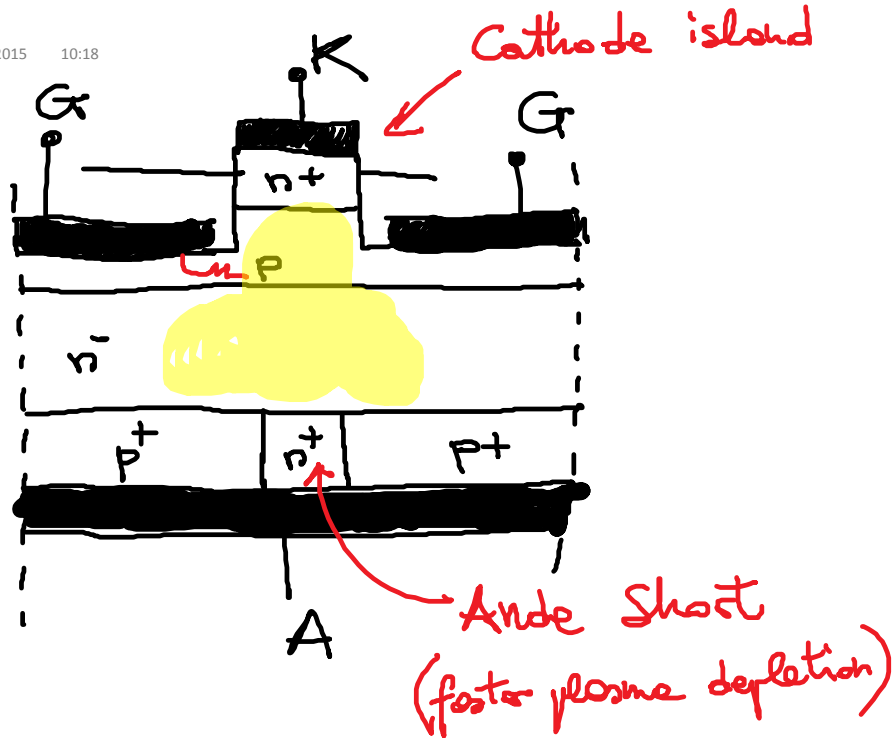


Turn on Transient

25 March 2015 10:00



25 March 2015 10:18



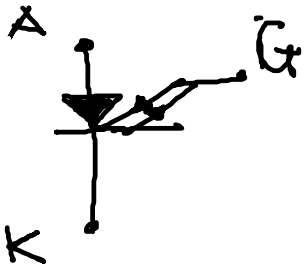
GTO Thyristor

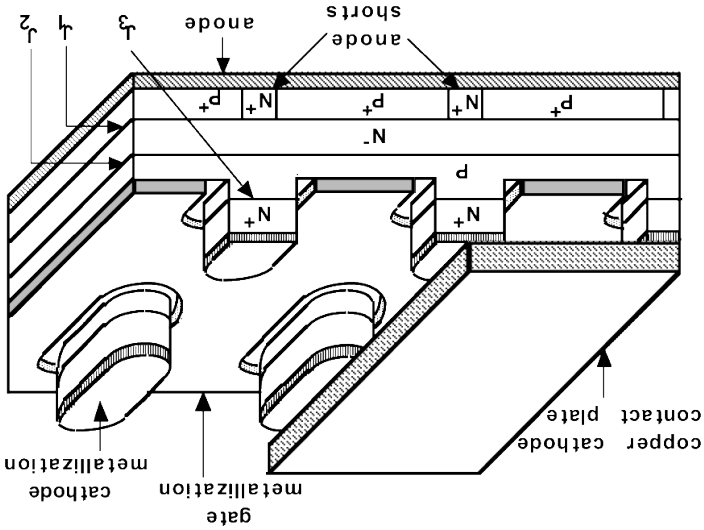
25 March 2015 10:15

Gate Turnoff

turnoff : i_{GCO}

1. Highly interdigitated structure (1k cell)
2. Cathode islands
3. Anode short



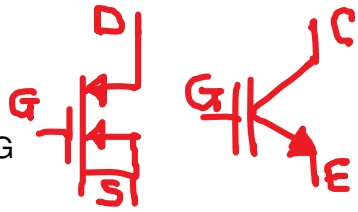


IGBT
Insulated-Gate Bipolar Transistor

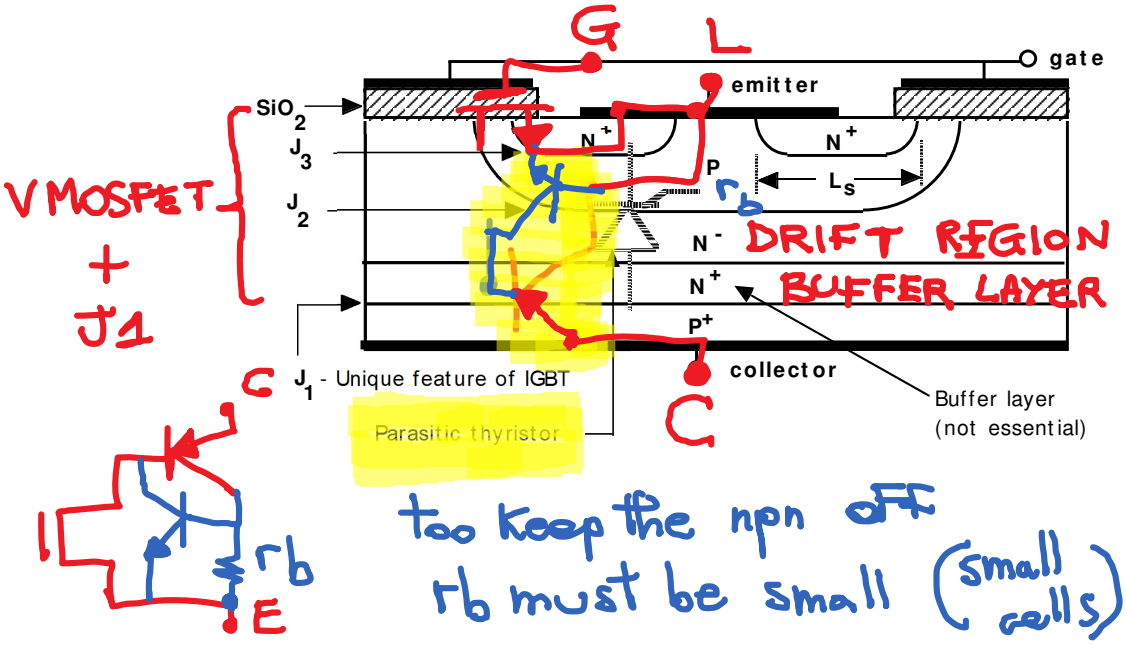
	PRO	CON
MOSFET	Fast Turn Off	High V_{ON} (no conductance modulation)
BJT	Low V_{ON} (conductance modulation)	Slow Turn Off

IGBT

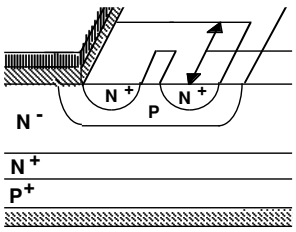
MOSFET with added junction between D and G to introduce conductance modulation



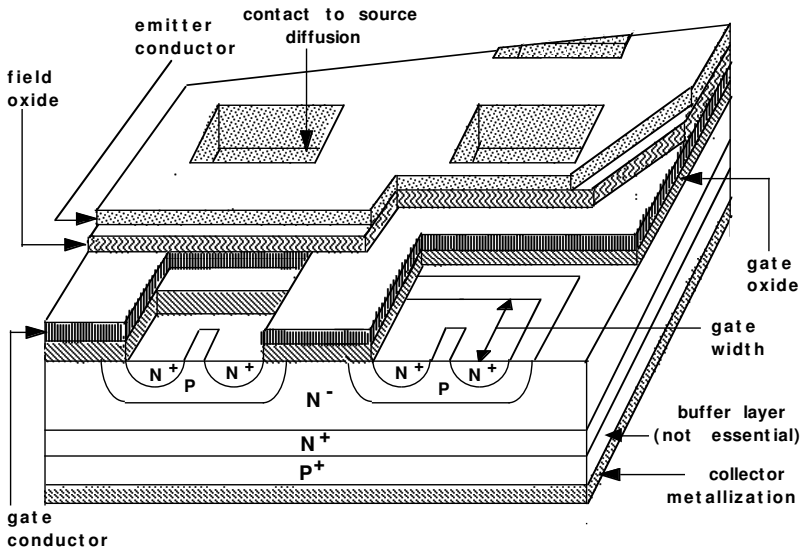
IGBT



IGBT
Insulated-Gate Bipolar Transistor



IGBT
Insulated-Gate Bipolar Transistor

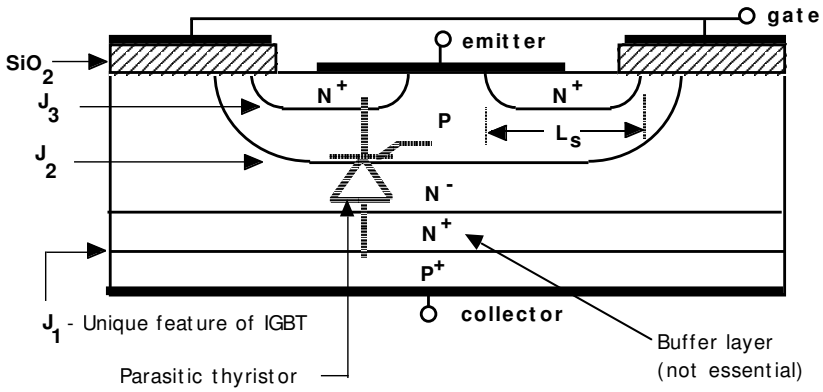


Blocking state

MOSFET is **OFF**

If $V_{CE} > 0$
Voltage is sustained by J_2

If $V_{CE} < 0$
Voltage is sustained by J_1 and J_3



The buffer layer allows punchthrough:

- If buffer layer is present → V_{RB} only 10-20 V

Asymmetric IGBT (punchthrough IGBT)

- If buffer layer is **not** present → $V_{RB} = V_{BD}$

Symmetric IGBT (non punchthrough IGBT9

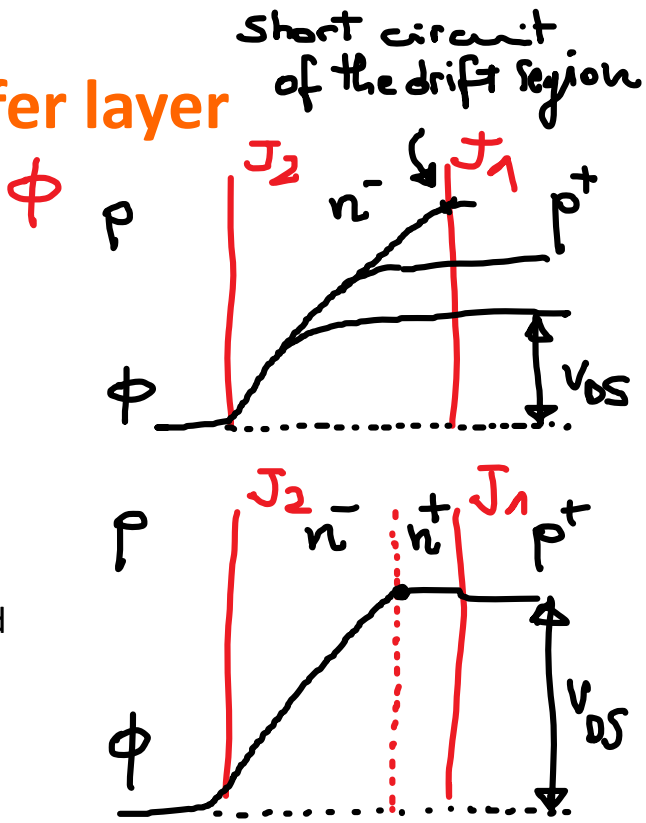
Buffer layer

No buffer layer

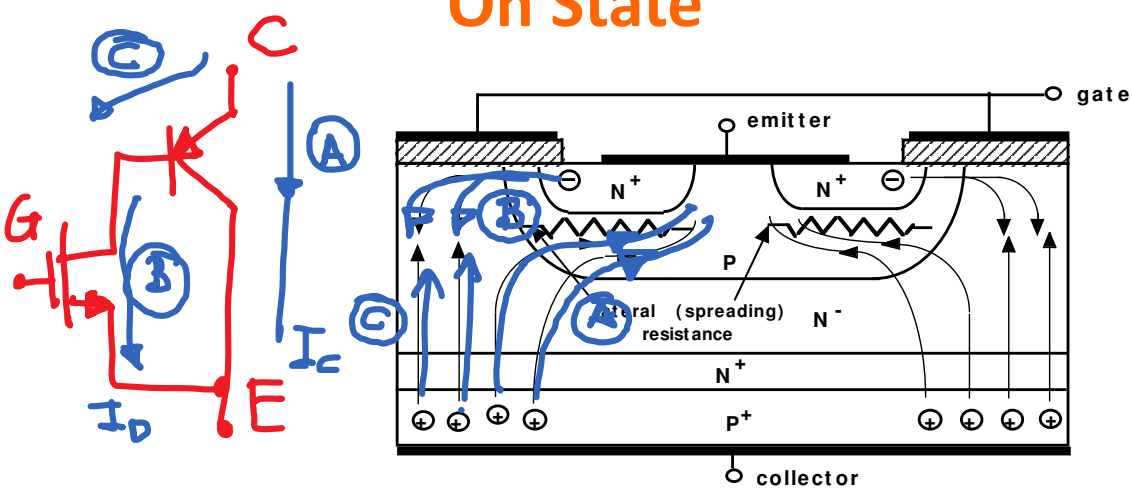
- Drift region cannot be completely depleted (punchthrough implies reachthrough)

Buffer layer

- The buffer layer can totally screen the electric field and allows punchthrough.
- (same V_{BD}, lower V_{ON}, but low V_{RB})

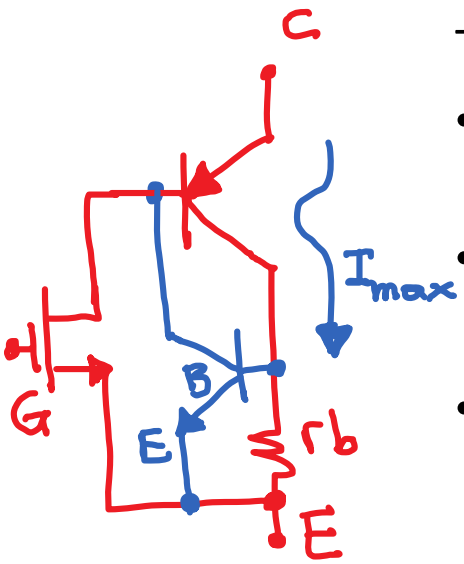


On State



- Typically $I_D > I_C$
- Lower V_{ON} than the MOSFET because "C" component causes conductivity modulation in the drift region.
- $V_{ON} = V_{J1} + R_{ON} I = 1 \div 2\ \mathrm{V} < V_{ON}$ of a MOSFET

Latch up



The npn BJT must be in cut off

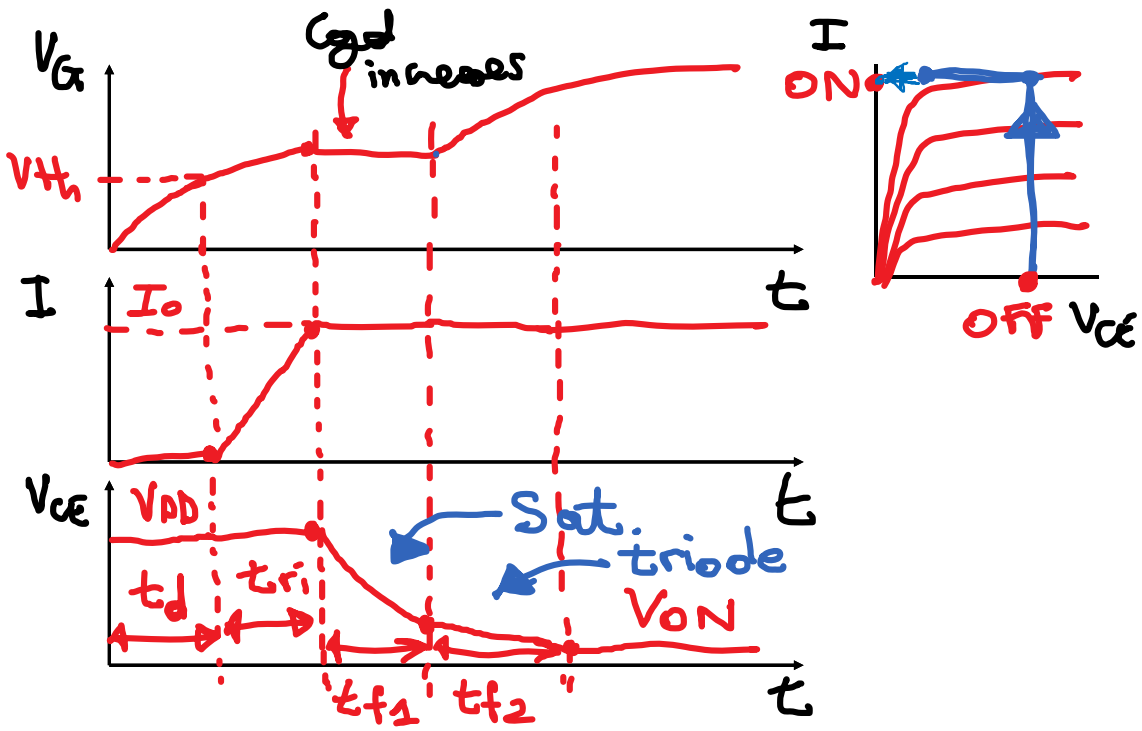
- V_{BE} depends on voltage drop on r_b.
- **Static Latch up**

  I_{MAX_Static}
- **Dynamic Latch up (during IGBT turn off*)**

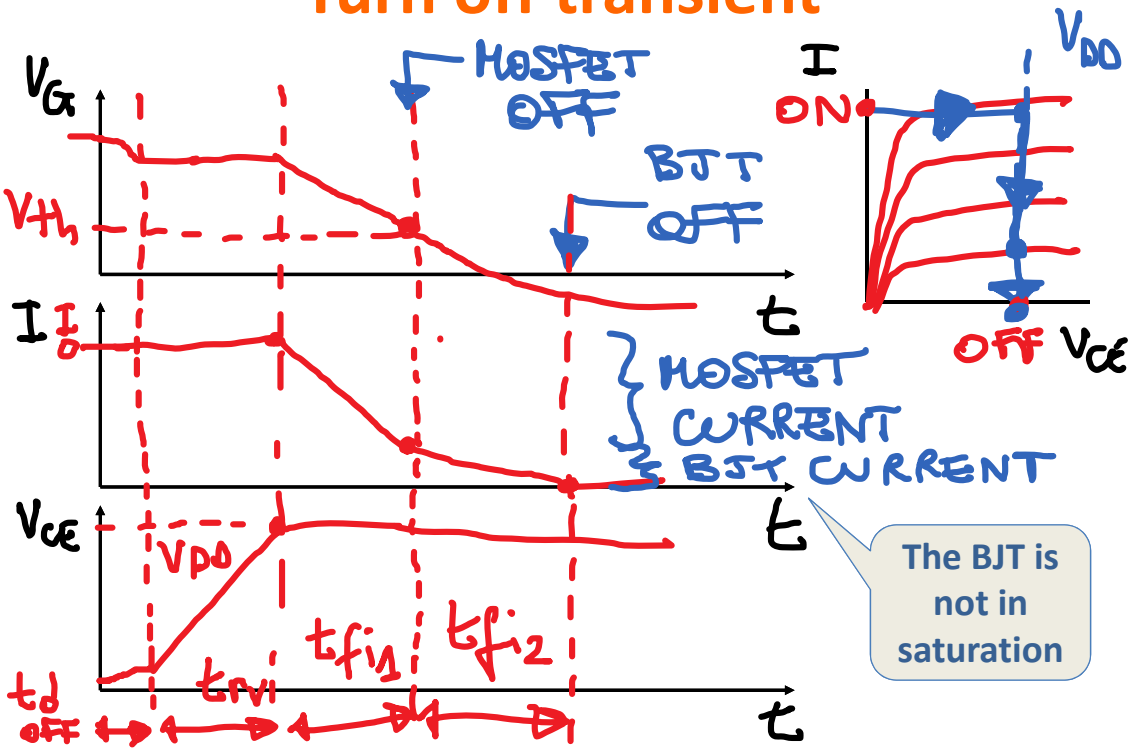
  $I_{MAX\_Dynamic} < I_{MAX\_Static}/2$

* During turn off the MOSFET turns off first → all IGBT current + CB displacement current can pass through rb

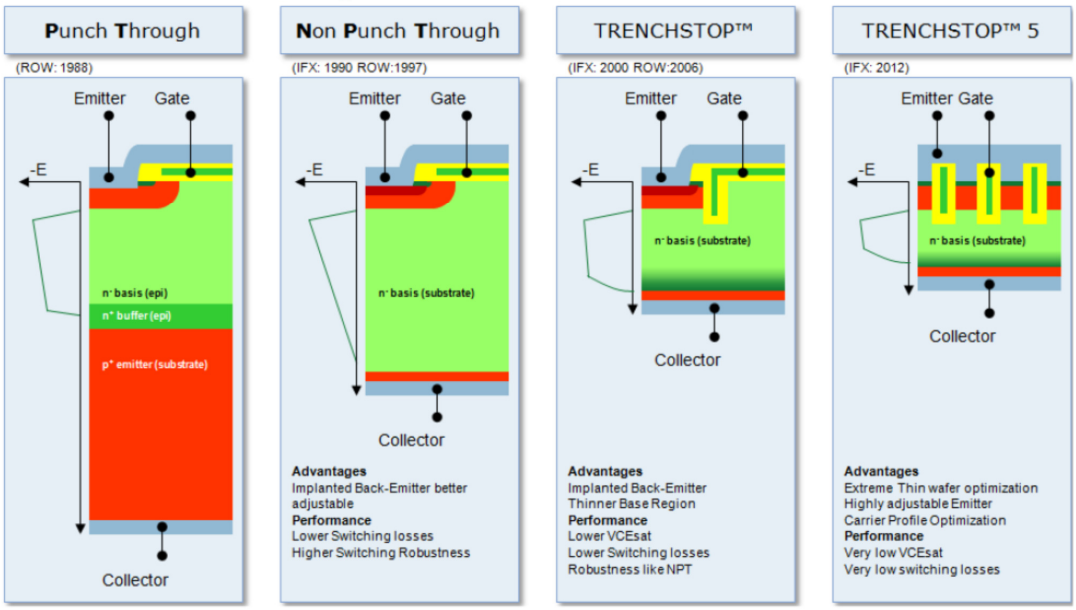
Turn on transient



Turn off transient



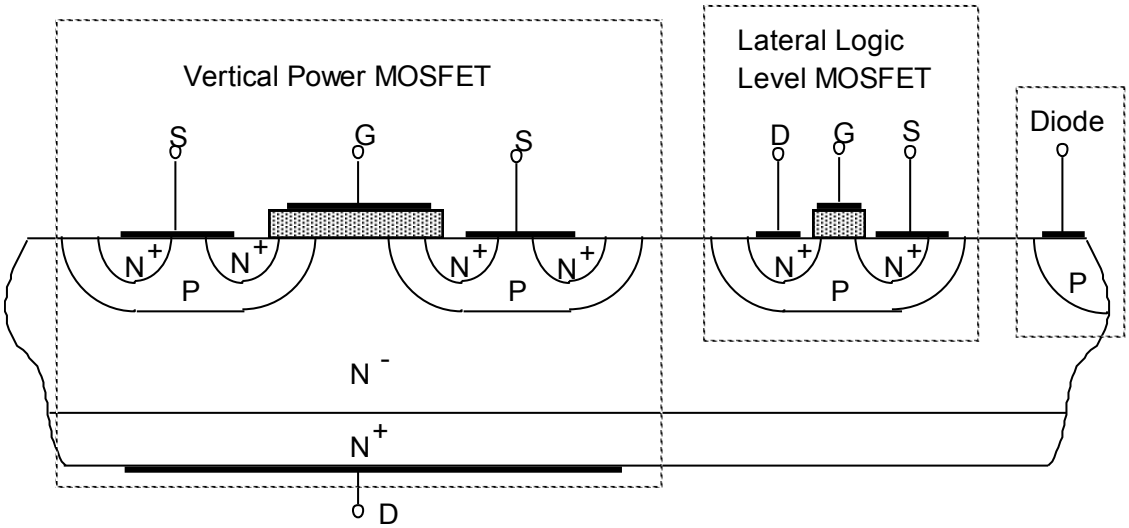
Example: Infineon IGBTs



Trenchstop 5: I_{max} 80 A, V_{BD} 650 V, V_{ON} 1.6-2 V,

Smart Power / Smart Switches (I < 50-100 A, V < 1KV):

Vertical Power devices + Lateral Devices for (some) logic

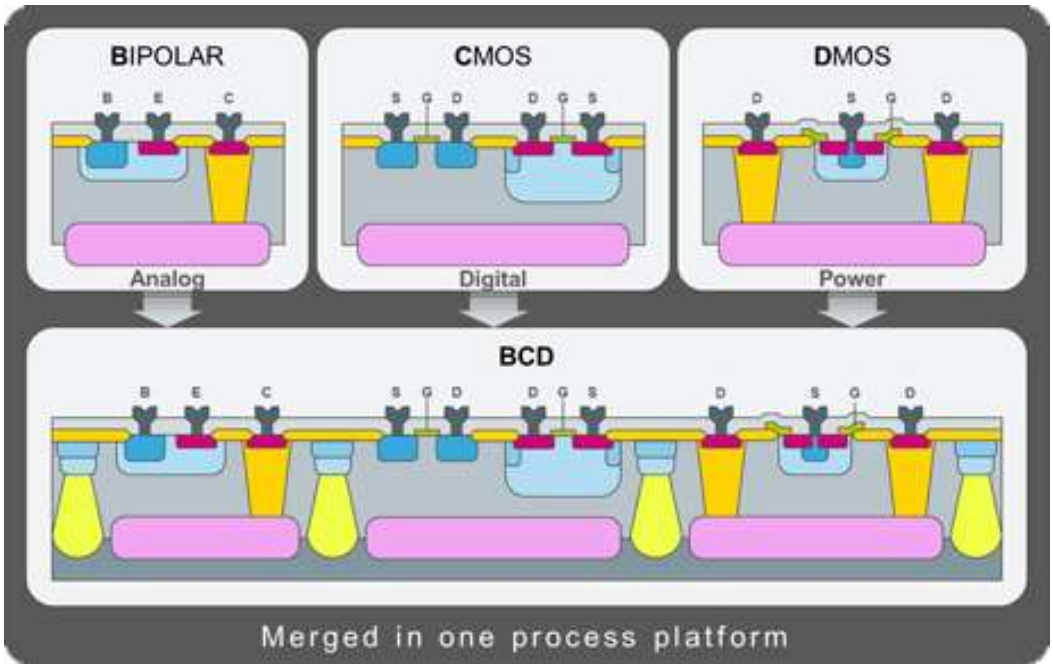


If Drain of Power MOSFET at positive voltage → devices are insulated by the reversed biased p-body - n-drift region junction

Power Integrated Circuits

1. Discrete Modules (Higher I-V range)
2. Smart Power / Smart Switches (I < 50-100 A, V < 1KV):
3. High-Voltage Integrated Circuits (e.g. BCD process - I < 50-100 A, V < 1KV)
4. High-density Power Management IC (e.g. high density BCD process – V < 100 V)

STM BCD Process



STM BCD Process

Three process technologies on a single chip

- Bipolar for precise analog circuits (e.g. bandgap)
- CMOS for digital design
- DMOS for power and high voltage

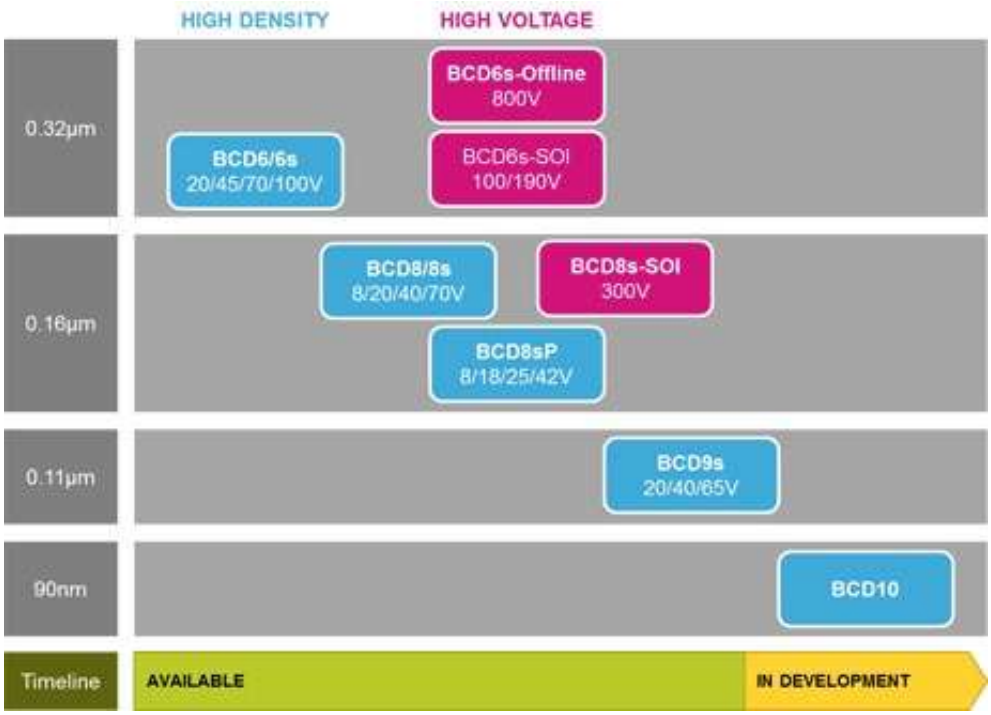
Pros:

- Improved reliability (no bonding, no complex packaging)
- Reduced EMI
- Smaller chip area (improved integration)

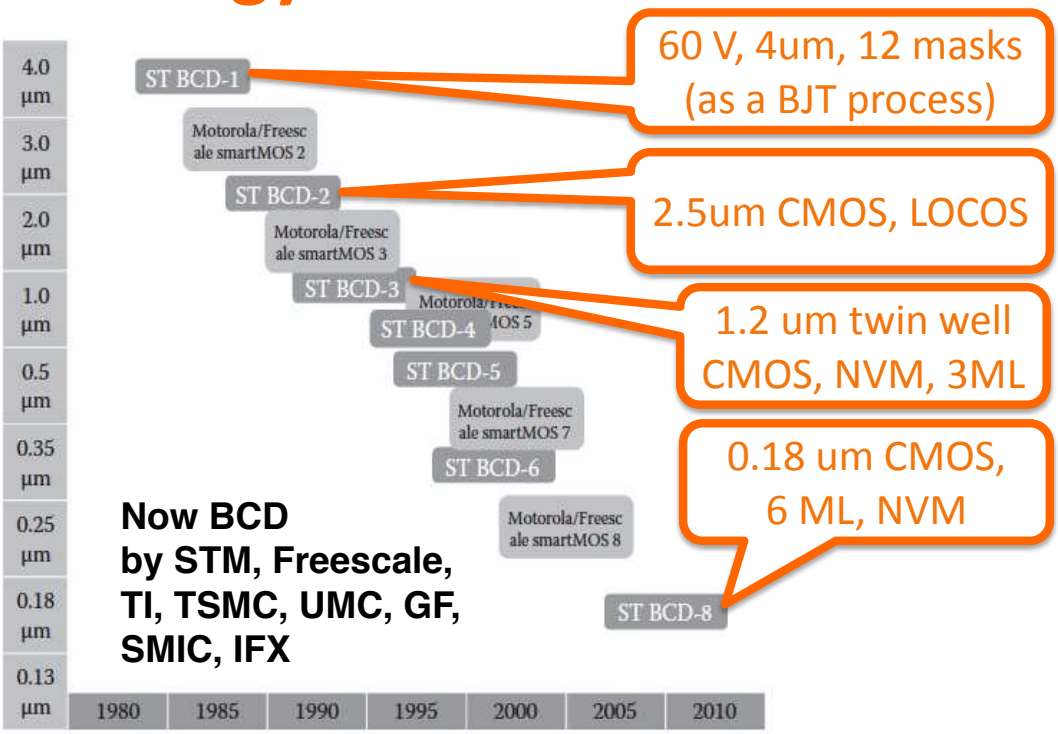
Cons

- No component is optimized (e.g. digital is not optimized (long channel lengths and thick oxides))

STM BCD process family

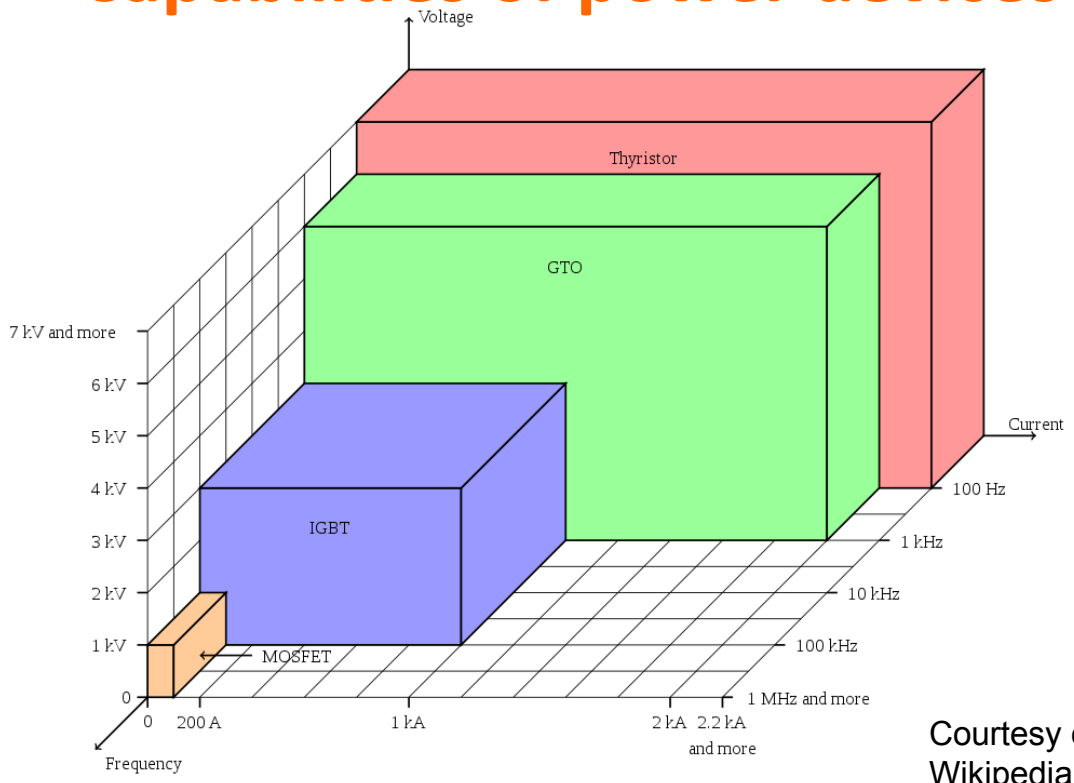


Chronology of BCD Processes



Chronology of BCD process taken from Fig. 4.4 of Y. Fu et al. CRC Press, 2014

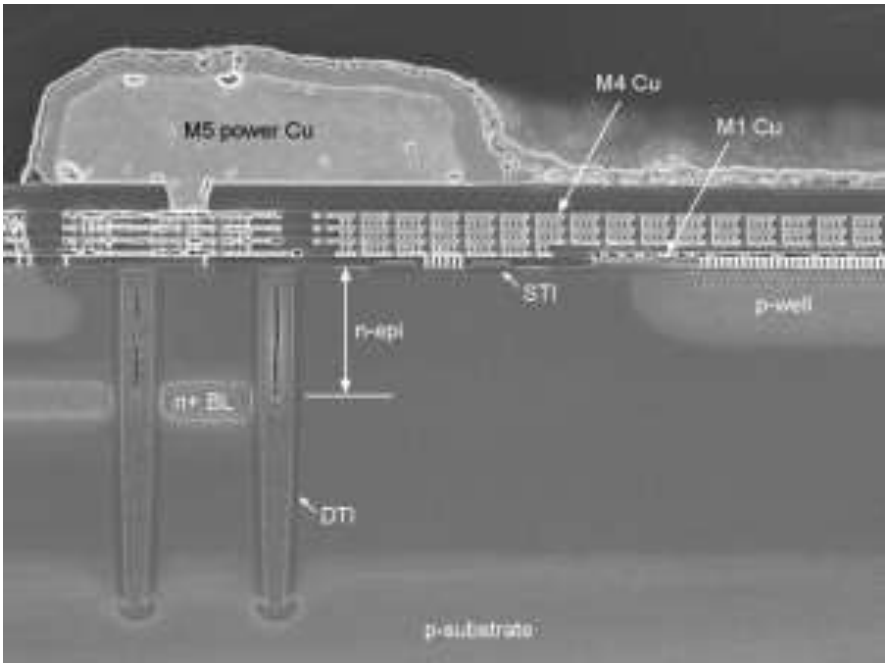
Capabilities of power devices



Courtesy of Wikipedia (Cyril

The domain of MOSFETs and IGBTs is increasing

Infineon 130 nm BCD



5 metal layer

STI

Buried layer

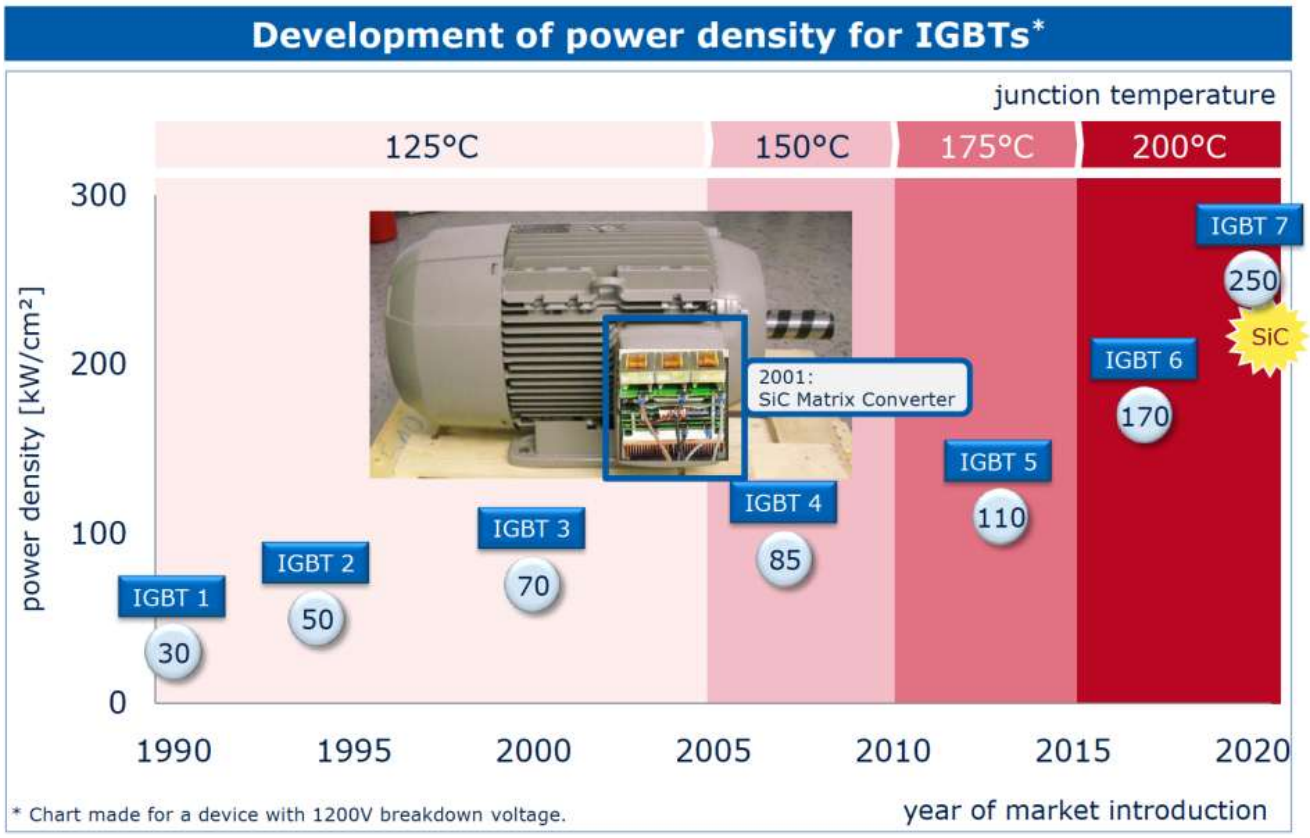
DTI

Evolution of power semiconductor devices

Active devices are a large fraction of the total system cost → actual design try to minimize the number of active devices used and their maximum ratings (cost)

Progress in IGBTs

Courtesy of Infineon 2011



Evolution of power semiconductor devices

Active devices are a large fraction of the total system cost → actual design try to minimize the number of active devices used and their maximum ratings (cost)

Progress in Power devices DRIVE changes in circuit choices and market adoption.

Examples:

- power MOSFETs —> switched-mode power supplies
- IGBT -> Energy efficient motor drives with inverters

Next

New materials: SiC, GaN -> Class D audio amplifier, inverter for motion control – AC-DC and DC-DC power supply

Evolution of power semiconductor devices

Active devices are a large fraction of the total system cost → actual design try to minimize the number of active devices used and their maximum ratings (cost)

Progress in Power devices DRIVE changes in circuit choices and market adoption.

Examples:

- power MOSFETs —> switched-mode power supplies
- IGBT -> Energy efficient motor drives with inverters

Resistance in the ON state R_{ON}

if we put the breakdown field E_{BD} in the place of E, and the breakdown voltage V_{BD} in the place of V:

- $2V_{BD} = WE_{BD} \rightarrow W = \frac{2V_{BD}}{E_{BD}}$
- $2V_{BD}\frac{qN_D}{\epsilon} = E_{BD}^2 \rightarrow qN_D = \frac{\epsilon E_{BD}^2}{2V_{BD}}$

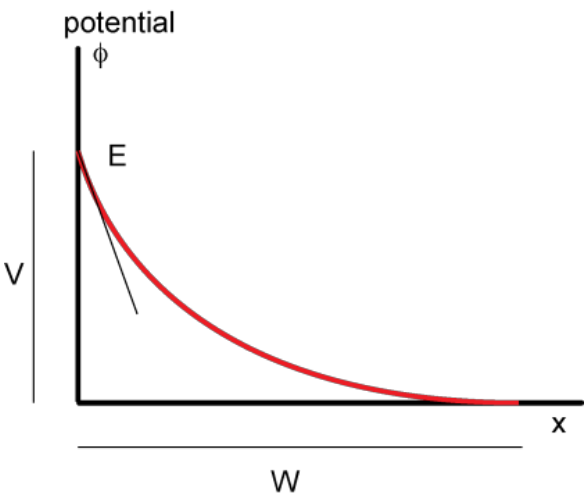
R_{ON} is due to transport in the drift region. In the case of no conductivity modulation n=N_D (MOSFETs and Schottky diodes):

$$R_{ON} = \frac{W}{A}\frac{1}{q\mu n} = \frac{W}{A}\frac{1}{q\mu N_D}$$

$$R_{ON}A = \frac{2V_{BD}}{E_{BD}}\frac{1}{\mu}\frac{2V_{BD}}{\epsilon E_{BD}^2} = \frac{4}{\mu\varepsilon}\frac{V_{BD}^2}{E_{BD}^3}$$

Comparison between different materials for power FETs

Let us consider a PN junction with NO punchthrough:



- W is the width of the depletion region (contained in the drift region)
- Electric field at the junction:

$$E = \frac{q\,N_D}{\varepsilon}W$$

- Voltage drop V in W:

$$V = \frac{1}{2}\frac{q\,N_D}{\varepsilon}W^2 = \frac{WE}{2}$$

- We also have $2V\frac{qN_D}{\epsilon} = E^2$

Thermal properties of alternative semiconductors

	Si	GaAs	SiC	GaN
Bandgap at Room T (eV)	1.12	1.43	2.2-3	3.4
Thermal conductivity (W/(cm K))	1.5	0.5	5	1.3
Max Operating Temp. (C)	150	300	600-1000	400
Saturation velocity (cm/s)	1e7	2e7	2.5e7	2.5e7

Higher bandgap → Harder impact ionization → Higher E_{BD}

Higher bandgap → Lower intrinsic carrier density n_i at a given T
→ Lower leakage currents at given T
→ Higher Max operating Temp

FOM of alternative materials (to Si)

$$R_{ON}A = \frac{4}{\mu\varepsilon}\frac{V_{BD}^2}{E_{BD}^3}$$

The breakdown voltage is a system specification
→ For the same V_{BD}, different materials give different R_{ON}

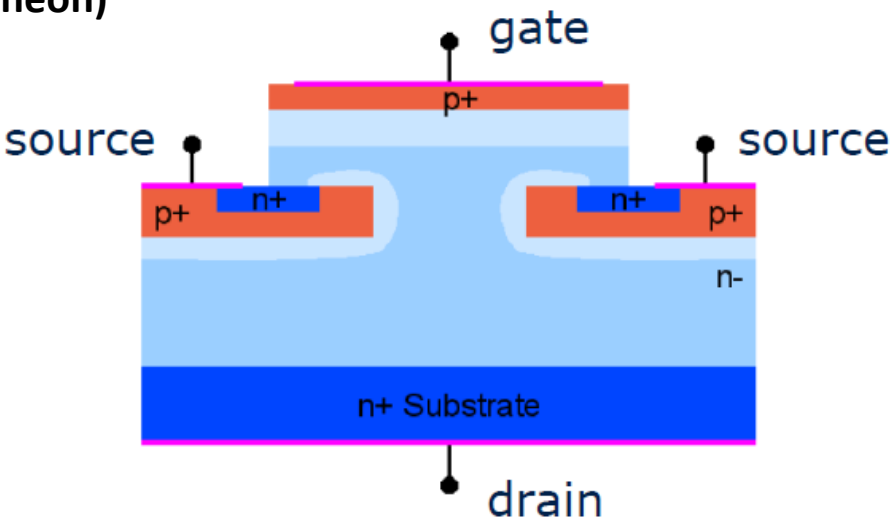
Baliga proposed a Figure of Merit for materials normalized to Si:

$$FOM = \mu\varepsilon E_{BD}^3$$

	Si	GaAs	SiC	GaN
Breakdown Electric Field (MV/cm)	0.3	0.4	2.4	3.0
Electron mobility (cm^2/Vs) at 300K	1350	8500	370	900
Relative dielectric constant	11.8	13.1	10	9.5
BFOM = $1/(\mu\varepsilon E_{BD})$ normalized to Si	1	17	119	537

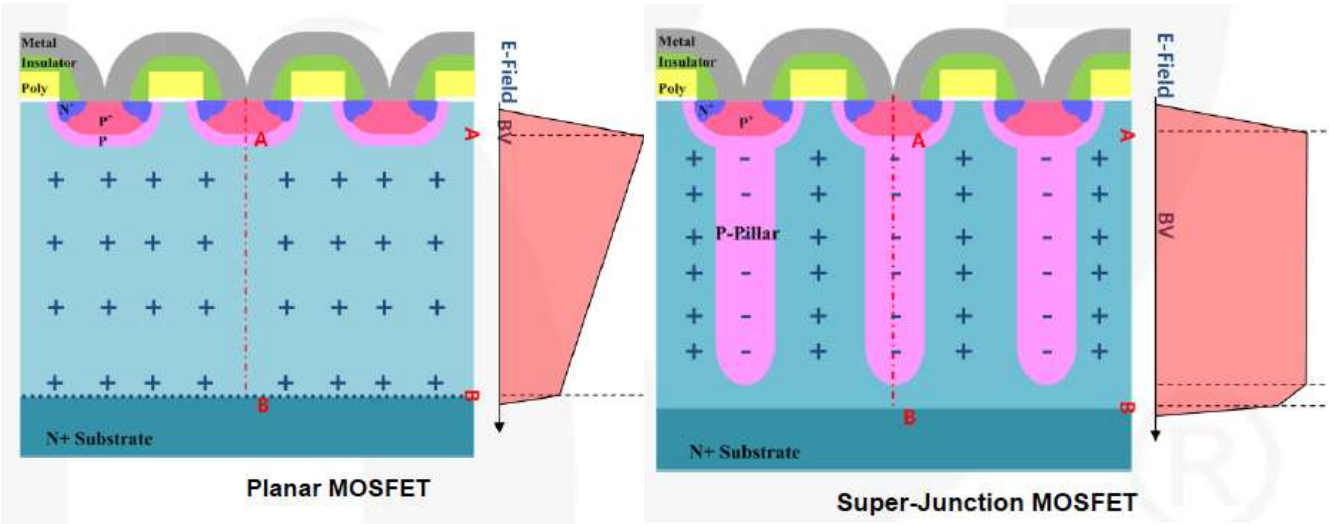
SiC devices

- SiC diodes, SiC JFETs, SiC MOSFETs
- **SiC JFET (Infineon)**



Siemens 1999,
STM 2000

Superjunction MOSFET

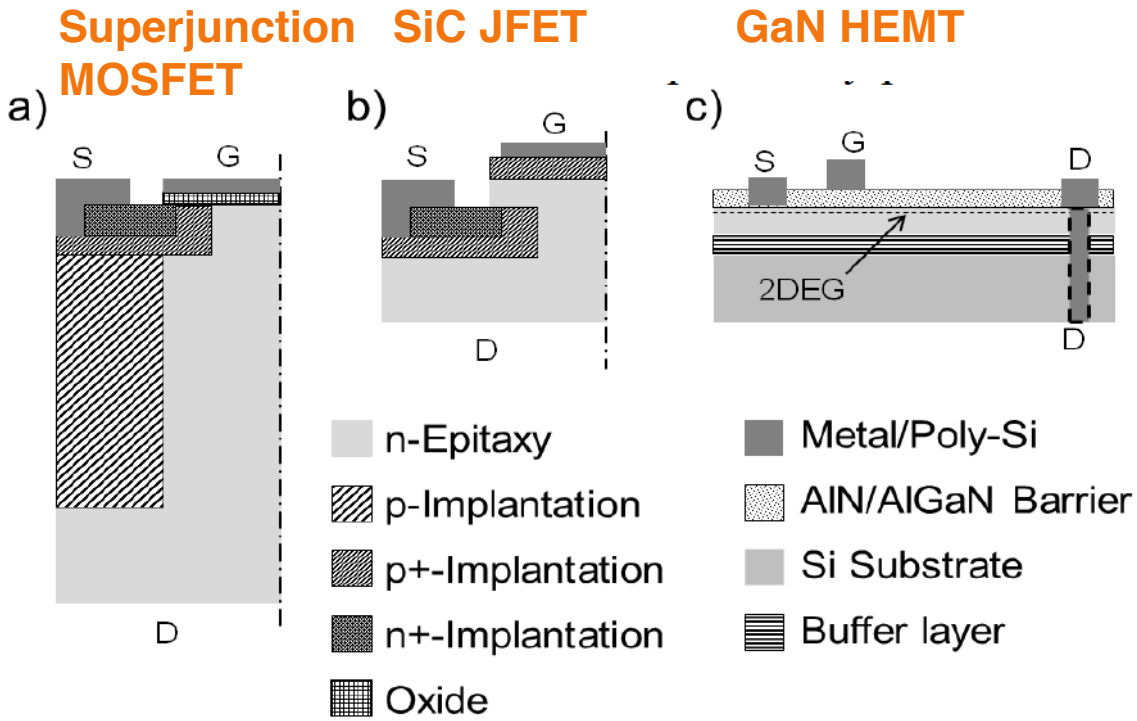


P-pillar introduces a charge sharing mechanism that enables to increase drift region doping (10x) for the same V_{BD} and drift region thickness

For V_{BD} = 600 V → 5x reduction in R_{ON} wrt MOSFET

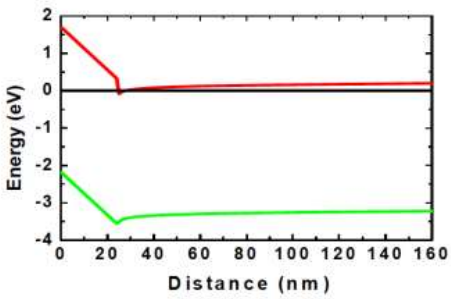
Source:
Fairchild AN5232

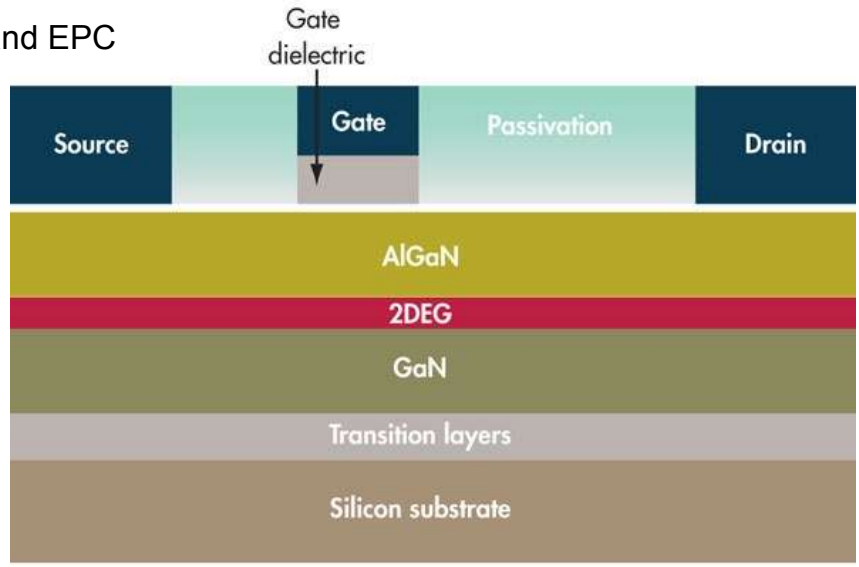
Comparison between different technologies



GaN-AlGaN MIS-HEMT

Only 2 producers now: IRF and EPC

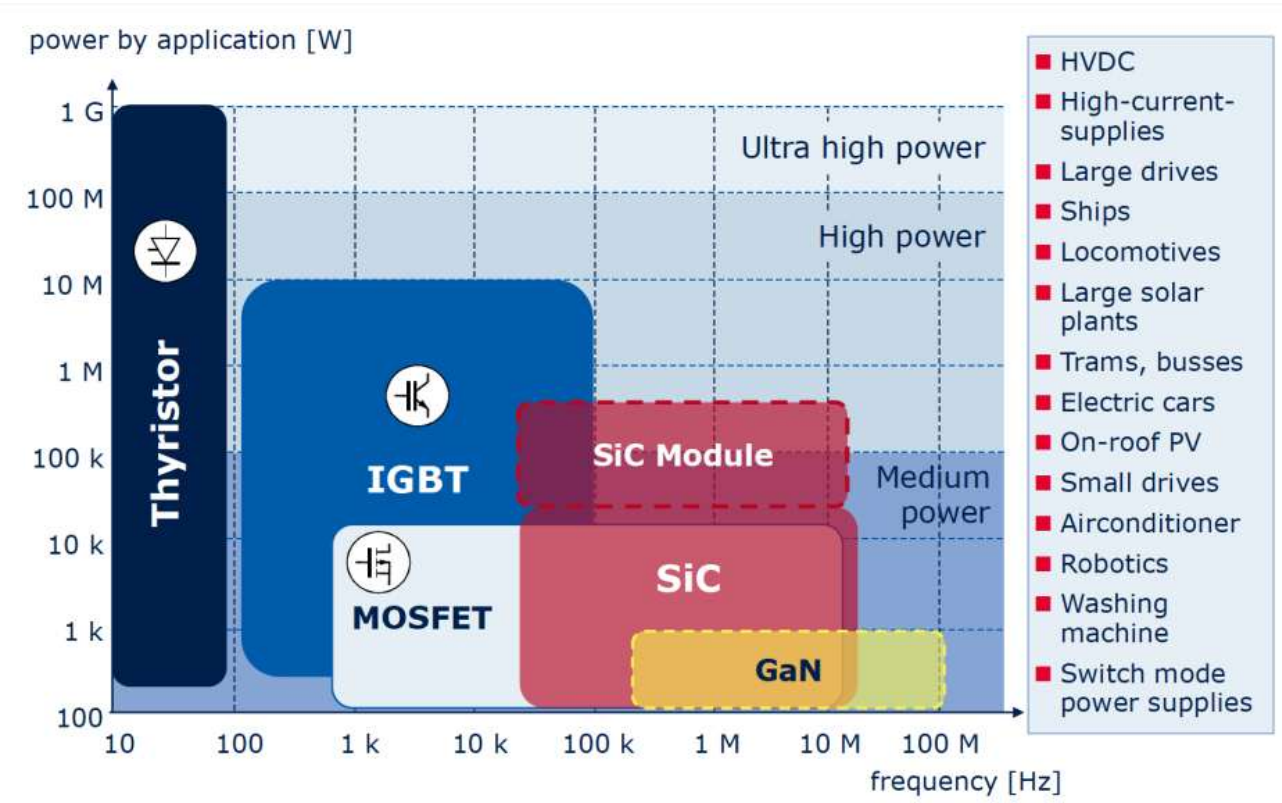




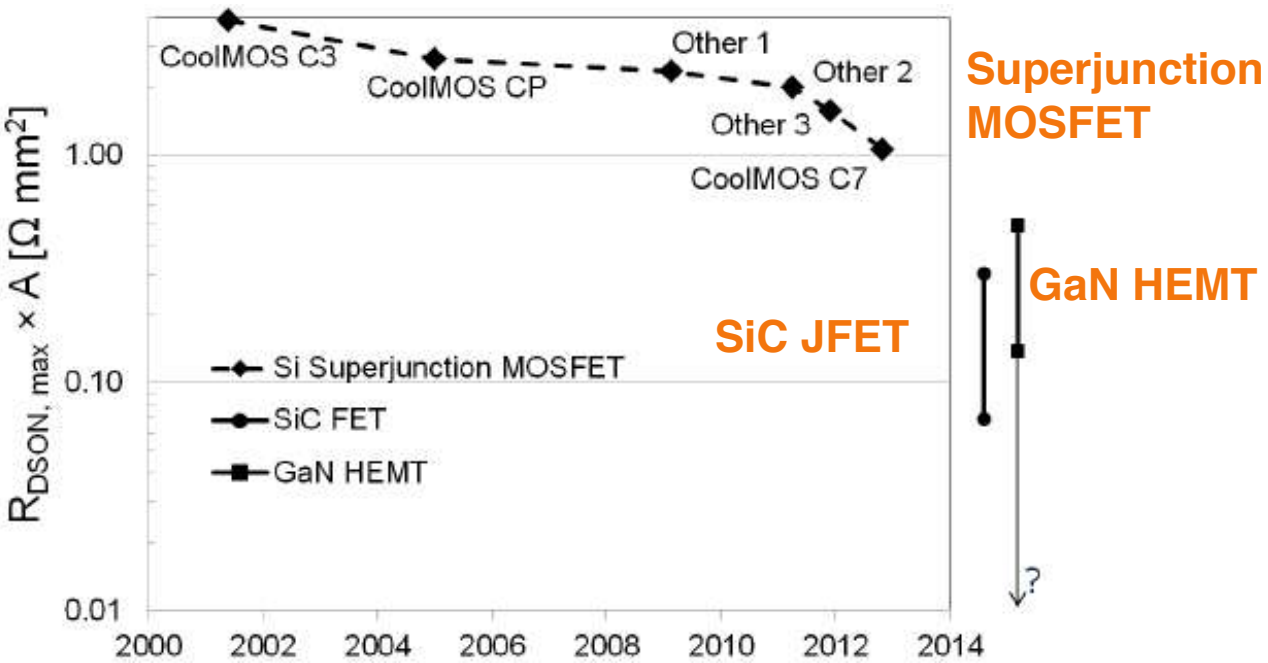
- No pn junctions (only majority carriers)
- Lateral device (reduced capacitances, high fields in the upper layers)
- Normally ON

Power versus frequency

Courtesy of Infineon 2011



R_{ON} comparison



GaN also has lower output switching charge, enabling higher frequency

Problems of alternative materials

1. Silicon has enormous accumulated past investments. Money spent on other materials is small in comparison
2. GaAs
   - Small wafer size (→ higher cost)
   - Unwanted impurities → reduce EBD and carrier lifetime
   - No oxide (is it really a problem?)
3. SiC
   - Even smaller wafer size and more impurities
   - SiC-SiO_2 interface not perfect
4. GaN
   - Reliability issues (impurities)

Ideal limit of SiC and GaN have not been reached yet

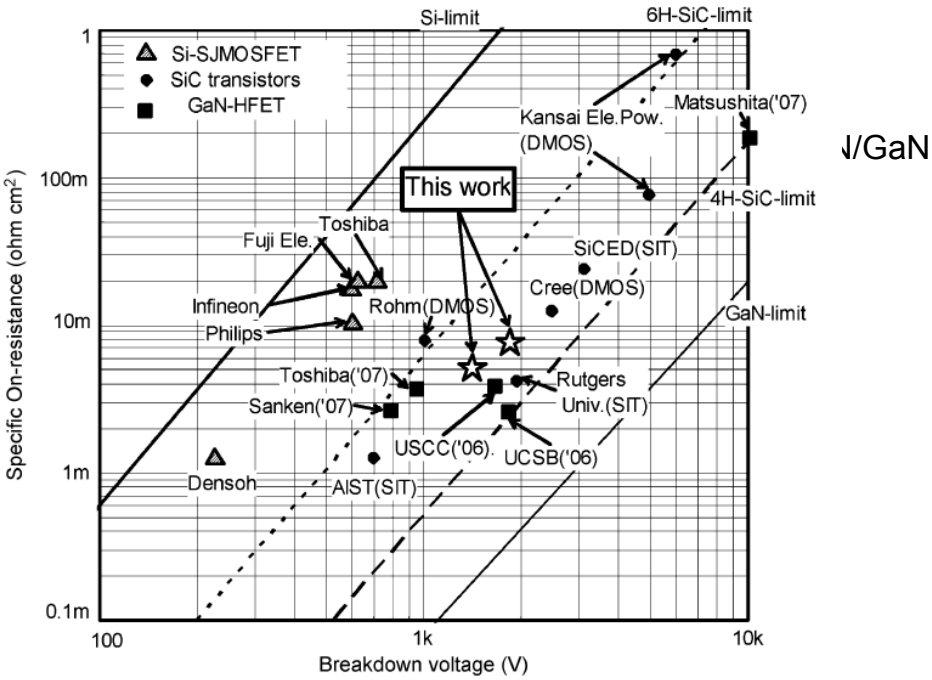


Fig. 12 of Ikeda et al. Proc. IEEE Vol. 98, pp. 1151-1161, 2010.

DC-DC Converters

Typical uses:

- DC Power supplies
- DC Motor drives
- Portable Electronics

Types of converters

- **Step-down (buck)**
- **Step-up (boost)**
- Buck-boost
- Cuk
- Full-Bridge

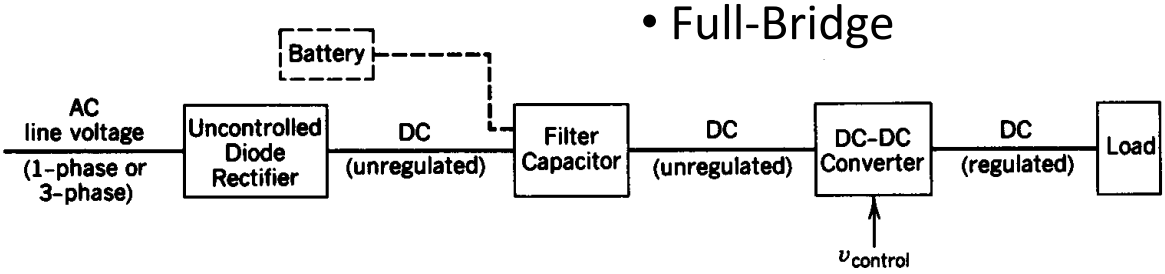
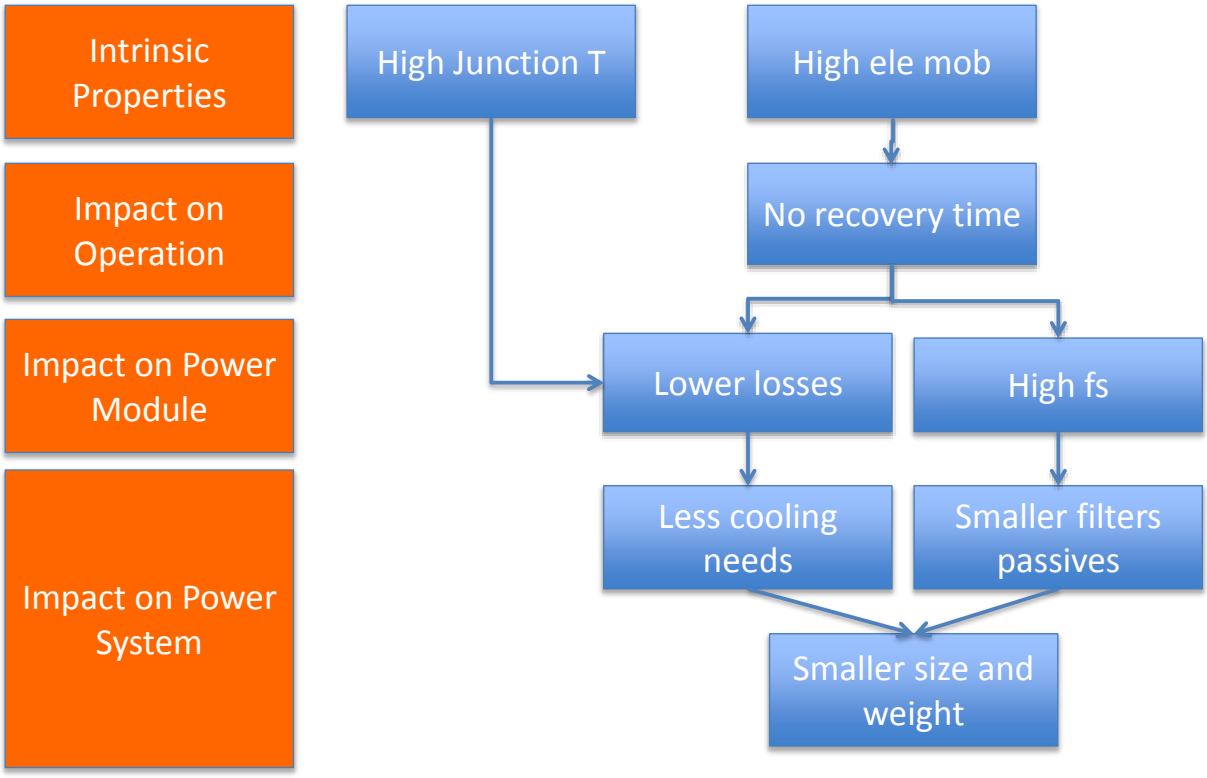


Figure 7-1 A dc–dc converter system.

Added value of SiC and GaN



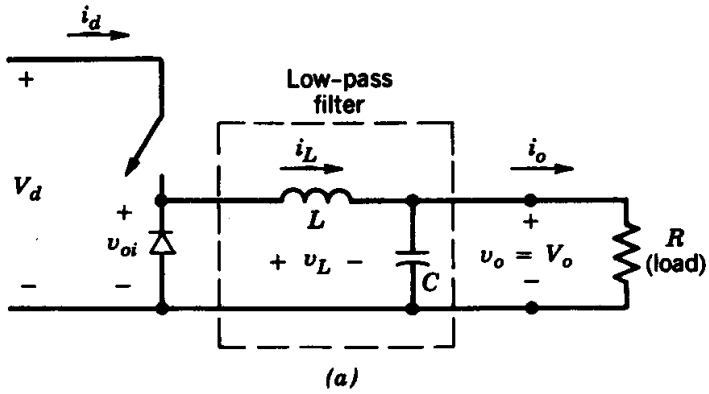
Step-down (buck) converter

DC power supplies, DC motor drives -- $V_o < V_d$

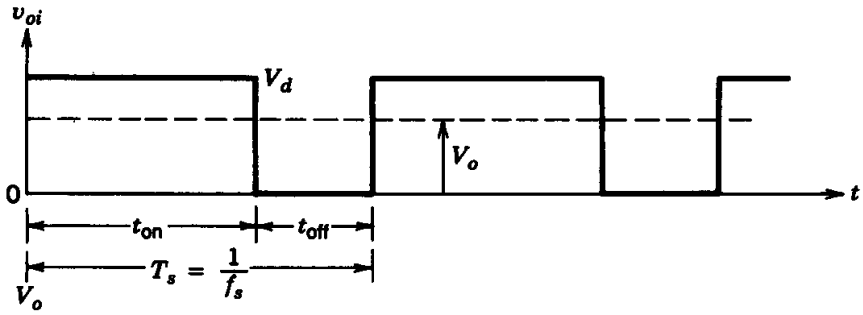
Low-pass filter keeps output voltage constant

Note: 2nd order non dissipative filter

$$f_c = \frac{1}{2\pi}\frac{1}{\sqrt{LC}} \ll f_s$$

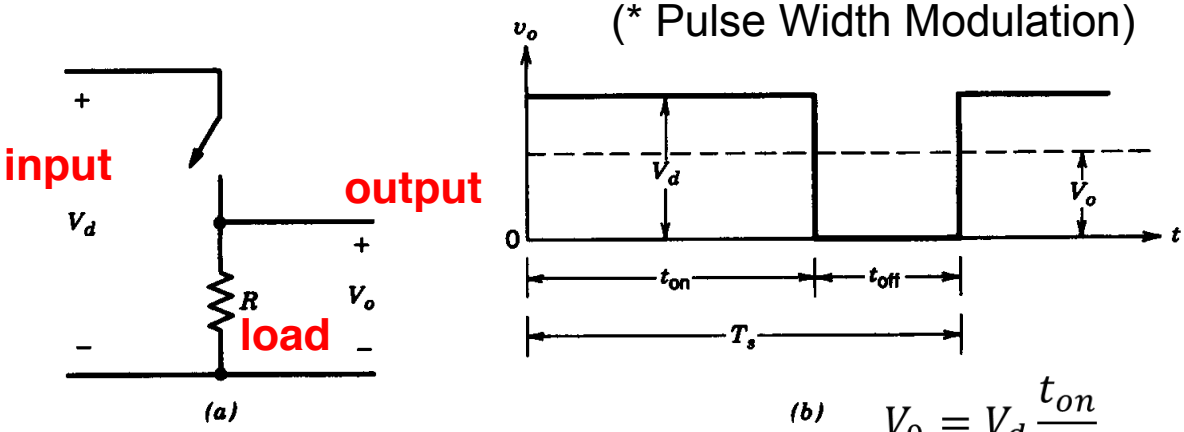


Diode avoids voltage spike on switch (when switch is off, diode provides current to L)



Ideal concept of step-down converter with PWM* switching

(* Pulse Width Modulation)



$$V_0 = V_d \frac{t_{on}}{T_s}$$

Figure 7-2 Switch-mode dc–dc conversion.

Assumptions: Switches, L, C are lossless, DC input has zero internal impedance, load is an equivalent R

This cannot work: 1. Load is inductive and can destroy switch by dissipating all stored energy, 2. output voltage must be continuous

Limit of continuous conduction

If the ripple amplitude $I_{LB} \equiv \frac{I_{peak}}{2} = I_o$, the converter is at the limit of continuous conduction (i.e. $min\{I_L\} = 0$)

$$I_{LB} \equiv \frac{I_{peak}}{2} = \frac{t_{on}(V_d - V_o)}{2L} = \frac{DT_sV_d(1-D)}{2L} = I_{LBmax}4D(1-D)$$

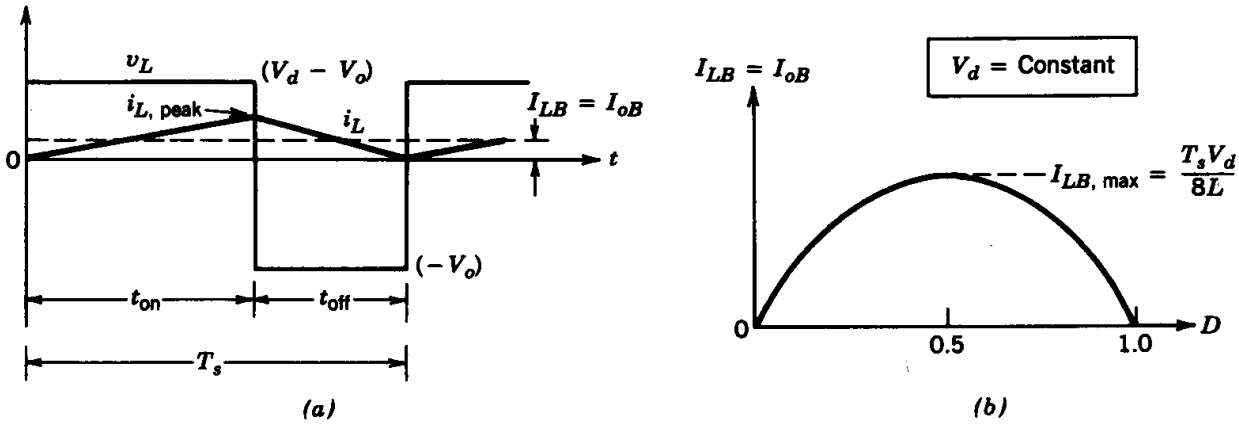


Figure 7-6 Current at the boundary of continuous–discontinuous conduction: (*a*) current waveform; (*b*) I_{LB} versus D keeping V_d constant.

Continuous-conduction mode

Current in *L* is always > 0

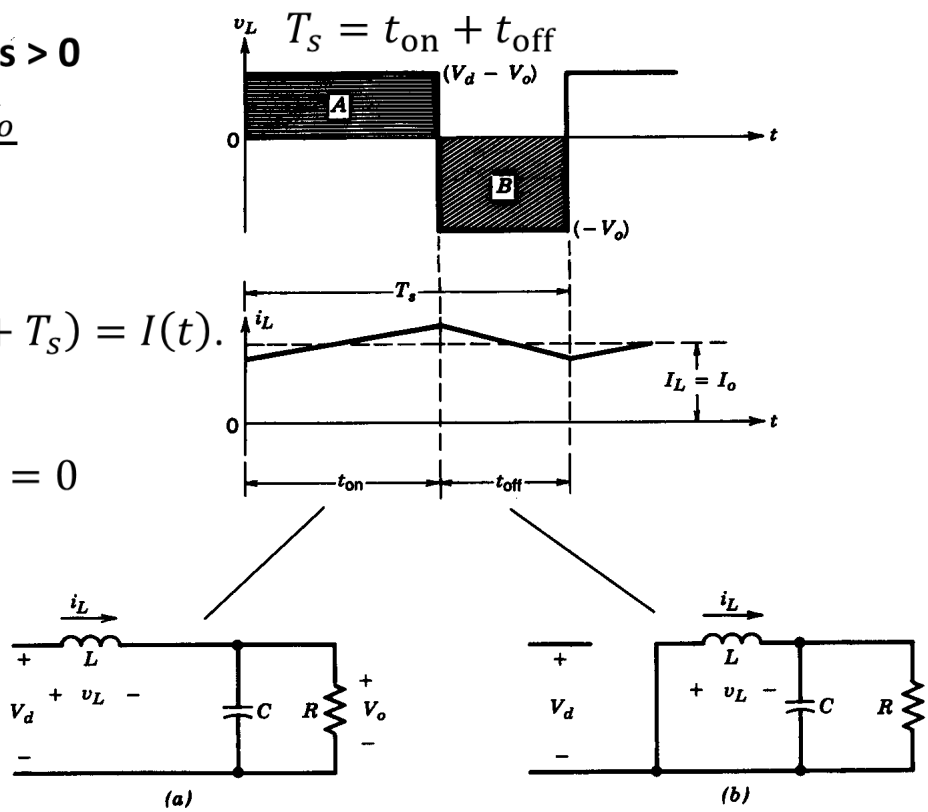
- t_{on}: $\frac{dI}{dt} = \frac{V_d - V_o}{L}$
- t_{off}: $\frac{dI}{dt} = -\frac{V_o}{L}$

At steady state: $I(t + T_s) = I(t)$.

Therefore

$$\frac{V_d - V_o}{L} t_{\text{on}} - \frac{V_0}{L} t_{\text{off}} = 0$$

$$\frac{V_o}{V_d} = \frac{t_{on}}{T_s} = D$$



Discontinuous-conduction mode with constant V_d

Motor drives

$$I_{\text{peak}} = \frac{(V_d - V_o)DT_s}{L} = \frac{V_o \Delta_1 T_s}{L}$$

$$\frac{V_o}{V_d} = \frac{D}{(D + \Delta_1)}$$

$$I_{\text{peak}} = \frac{V_d T_s}{L} \frac{D\Delta_1}{D + \Delta_1}$$

$$I_{\text{peak}} = 8 I_{\text{LBmax}} \frac{D\Delta_1}{D + \Delta_1}$$

$$I_o T_s = \frac{I_{\text{peak}}(D + \Delta_1)T_s}{2}$$

$$I_o = 4 I_{\text{LBmax}} D \Delta_1$$

$$\frac{V_o}{V_d} = \frac{D^2}{D^2 + I_o/(4 I_{LBmax})}$$

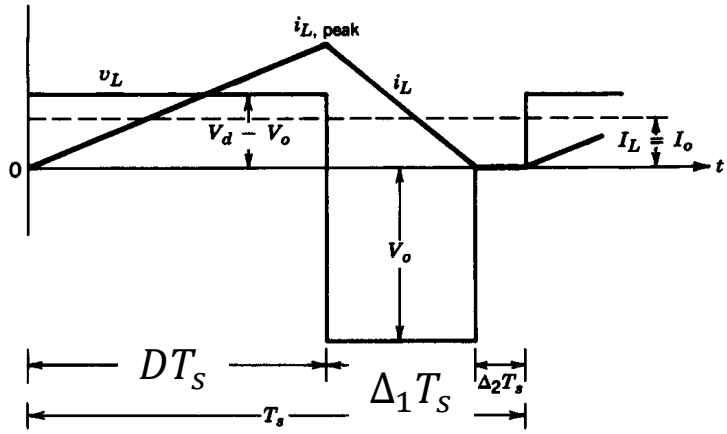


Figure 7-7 Discontinuous conduction in step-down converter.

Limits of continuous-discontinuous conduction (constant V_d)

Continuous

$$\frac{V_o}{V_d} = D$$

$$\frac{I_o}{I_{\text{LBmax}}} > 4D(1 - D)$$

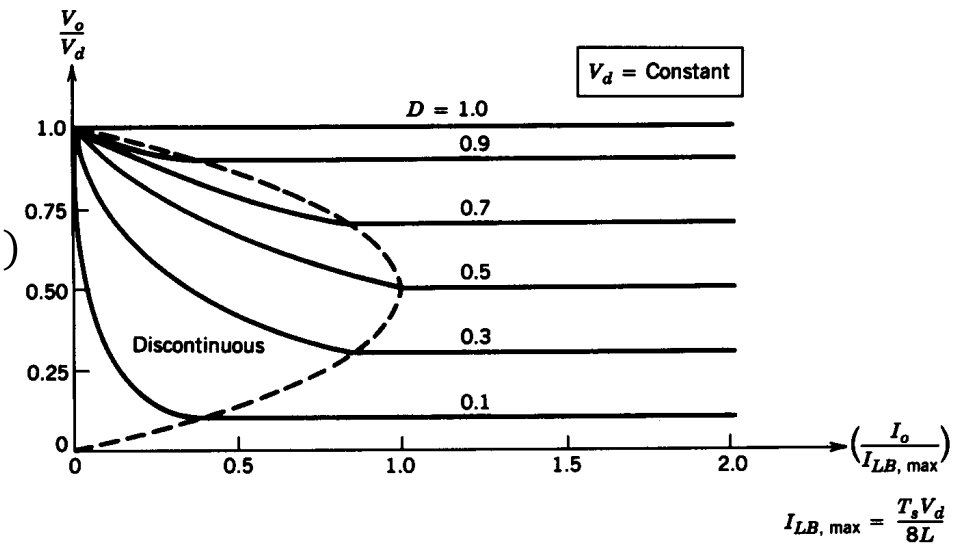


Figure 7-8 Step-down converter characteristics keeping V_d constant.

Discontinuous-conduction with constant Vo

DC voltage supply

At the limit of continuous conduction

$$I_{LB} = \frac{V_o T_s (1-D)}{2L} = I_{\mathrm{LBmax}}(1-D)$$

We can write D explicitly from:

$$I_{\mathrm{peak}} = \frac{V_o \Delta_1 T_s}{L} = 2 I_{\mathrm{LBmax}} \Delta_1$$

$$I_o = \frac{I_{\mathrm{peak}}(D+\Delta_1)}{2} = I_{\mathrm{LBmax}} \Delta_1 (D+\Delta_1) \qquad \frac{V_d}{V_o} = \frac{D+\Delta_1}{D}$$

$$\frac{I_o}{I_{\mathrm{LBmax}}} = D^2 \frac{V_d}{V_o}\left(1-\frac{V_d}{V_o}\right) \Rightarrow D = \left[\frac{V_o}{V_d}\frac{I_o}{I_{LBmax}}\left(1-\frac{V_d}{V_o}\right)^{-1}\right]^{\frac{1}{2}}$$

Limits of continuous-discontinuous conduction (constant Vd)

Continuous

$$\frac{I_o}{I_{\mathrm{LBmax}}} > 4D(1-D)$$

$$\frac{V_o}{V_d} = D$$

Discontinuous

$$\frac{I_o}{I_{\mathrm{LBmax}}} < 4D(1-D)$$

$$\frac{V_o}{V_d} = \frac{D^2}{D^2 + \frac{I_o}{4 I_{\mathrm{LBmax}}}}$$

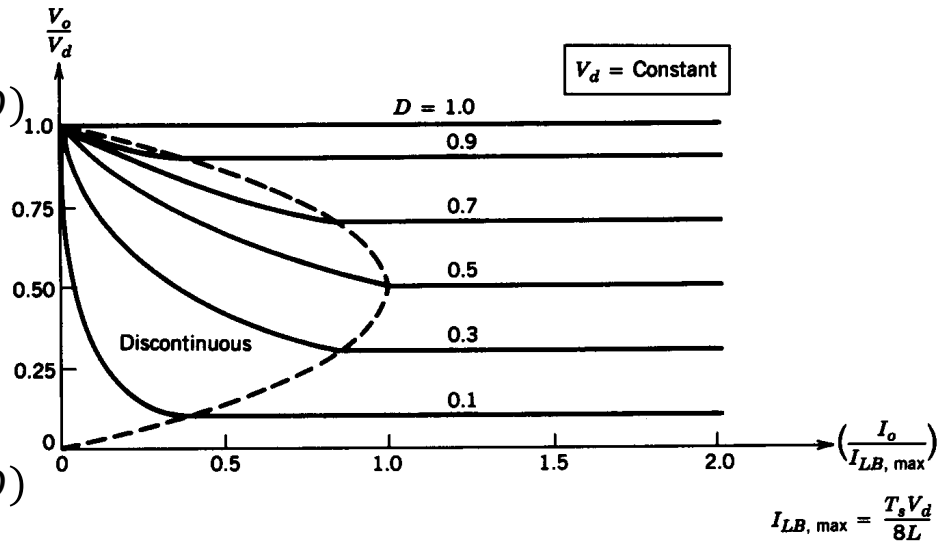
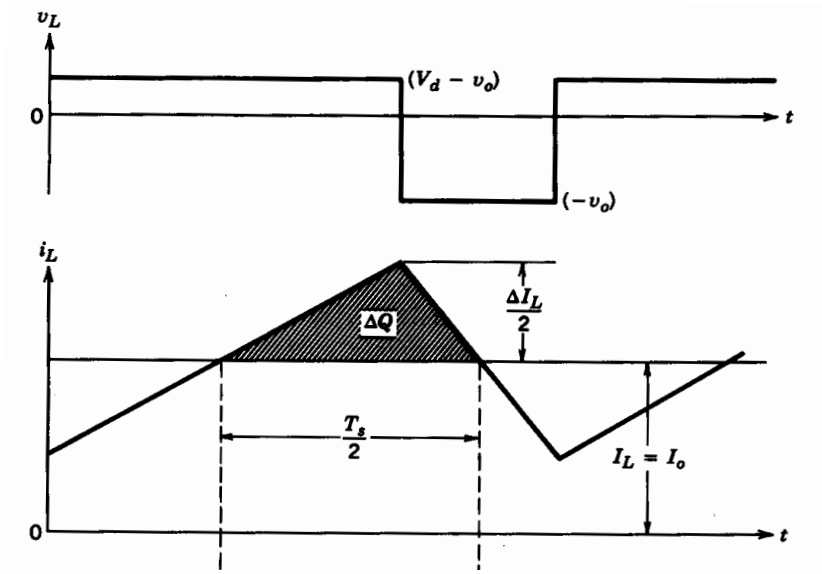


Figure 7-8 Step-down converter characteristics keeping V_d constant.

Output voltage ripple

First order calculation:

The average iL flows in the load, and the ripple component in C.



Additional charge:

$$\Delta Q = \frac{1}{2}\frac{\Delta I_L}{2}\frac{T_s}{2}$$

Current ripple:

$$\Delta I_L = (V_o/L)(1-D)T_s$$

Voltage ripple:

$$\Delta V_o = \frac{\Delta Q}{C} = \frac{V_o}{8LC}T_s^2(1-D)$$

$$\frac{\Delta V_o}{V_o} = \frac{\pi^2}{2}(1-D)\frac{f_c^2}{f_s^2}$$

$$f_c = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$$

Discontinuous-conduction with constant Vo

DC voltage supply

Continuous: $I_o > I_{LB}$

$$D > 1 - \frac{I_o}{I_{LBmax}}$$

$$D = \frac{V_o}{V_d}$$

Discontinuous: $I_o < I_{LB}$

$$D < 1 - \frac{I_o}{I_{LBmax}}$$

$$D = \left[\frac{V_o}{V_d}\frac{I_o}{I_{LBmax}}\left(1 - \frac{V_d}{V_o}\right)^{-1}\right]^{\frac{1}{2}}$$

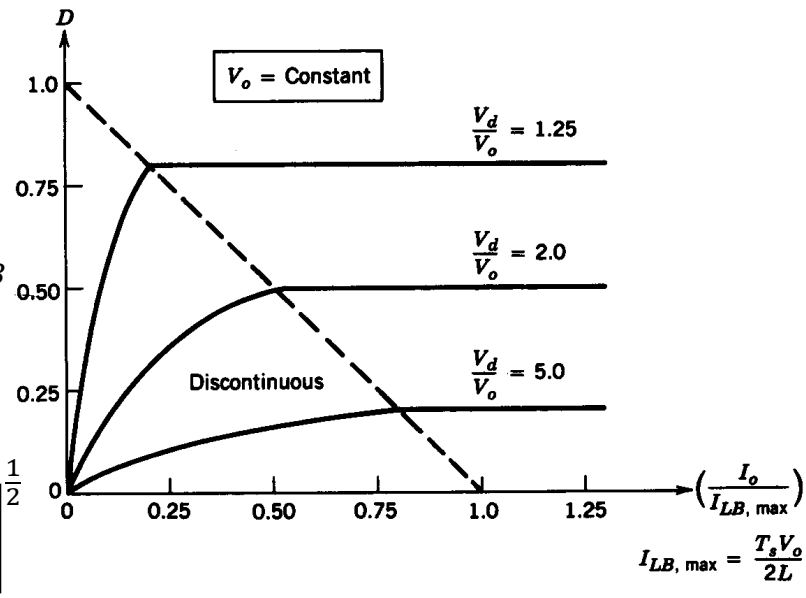


Figure 7-9 Step-down converter characteristics keeping V_o constant.

Continuous-conduction mode

Periodic conditions:

$$\frac{t_{\text{on}}V_d}{L} + \frac{t_{\text{off}}(V_d - V_o)}{L} = 0$$

if $t_{\text{on}} = DT_s$ and

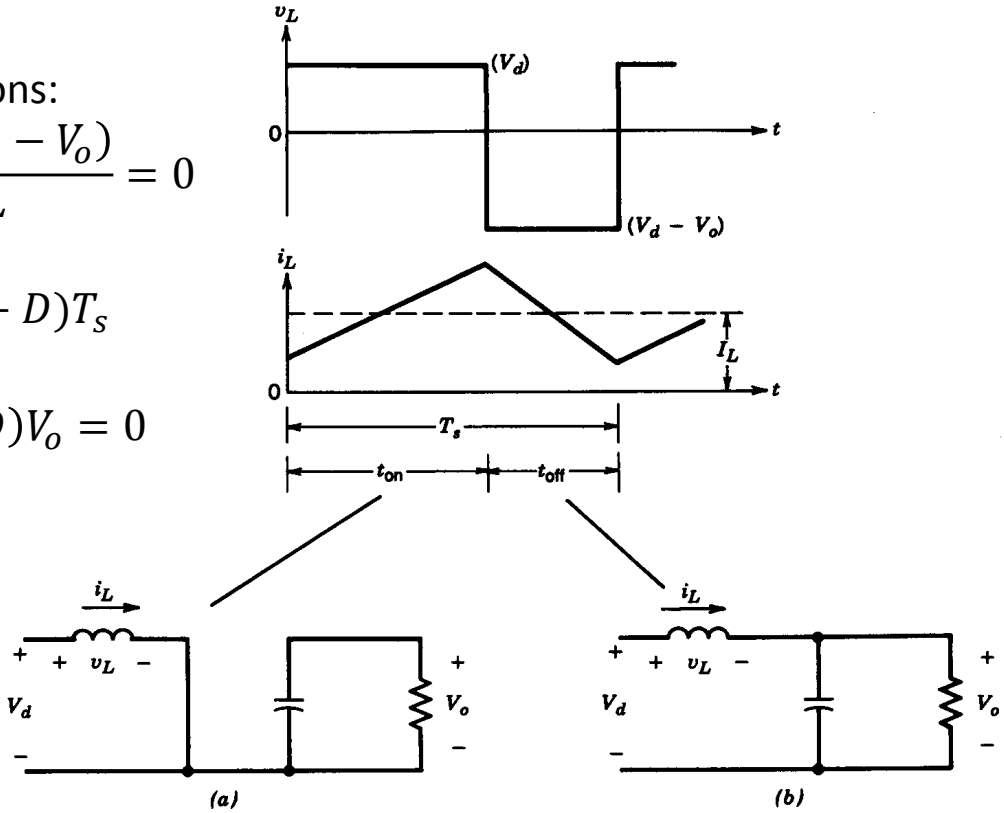
$t_{\text{off}} = (1 - D)T_s$

$$T_s V_d + T_s(1 - D)V_o = 0$$

$$\frac{V_o}{V_d} = \frac{1}{1 - D}$$

No losses:

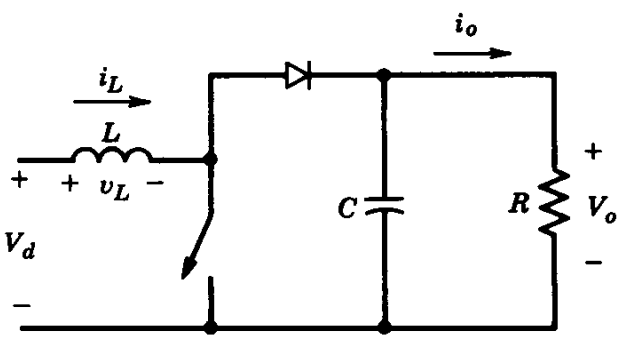
$$V_o I_o = V_d I_d$$



Step-up (boost) converter

- DC power supplies
- Regenerative breaking of DC motors

Output voltage always larger than the input



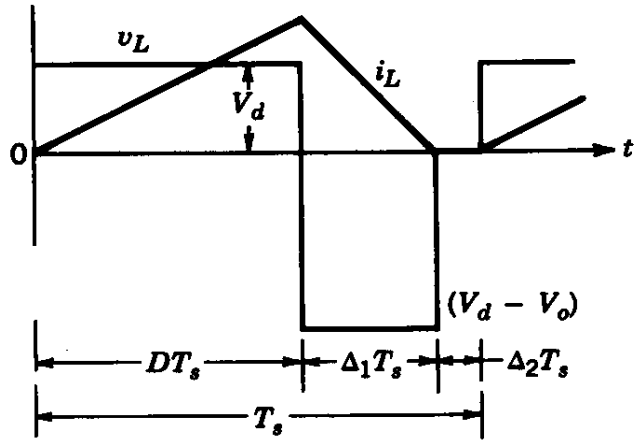
Switch on → diode off, output isolated, L accumulates energy from input

Switch off → diode on, load receives energy from input and from L

Discontinuous conduction mode (constant V_o)

Periodic conditions:

$$\frac{DT_sV_d}{L} + \frac{\Delta_1 T_s(V_d - V_o)}{L} = 0$$



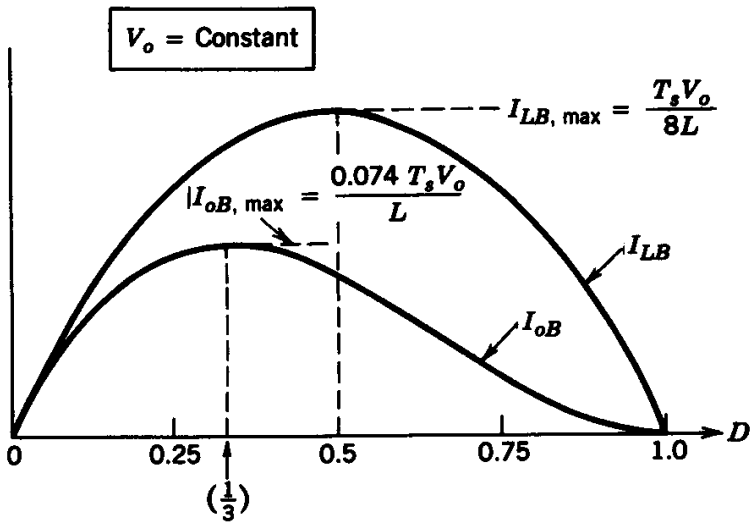
Continuous-discontinuous boundary

Average current in L = ripple :

$$I_{LB} = \frac{V_d t_{on}}{2L} = \frac{V_o(1-D)T_sD}{2L}$$

Average output current at the limit:

$$I_{oB} = I_{LB}(1-D) = \frac{V_oT_s(1-D)^2D}{2L}$$



I_{LB} is max if D=0.5 → $I_{LBmax} = \frac{V_oT_s}{8L}$,

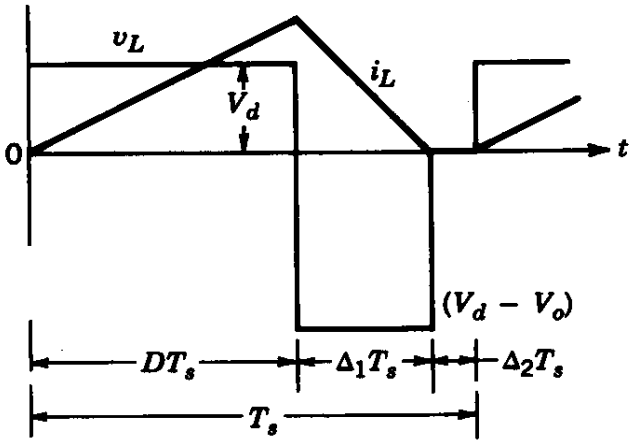
I_{oB} is max if D=1/3 → $I_{oBmax} = \frac{2V_oT_s}{27L}$ → $I_{oB} = \frac{27}{4}(1-D)^2DI_{oBmax}$

Discontinuous conduction mode (constant V_o)

Periodic conditions:

$$\frac{DT_sV_d}{L} + \frac{\Delta_1 T_s(V_d - V_o)}{L} = 0$$

$$\frac{V_o}{V_d} = 1 + \frac{D}{\Delta_1} = \frac{I_d}{I_o}$$



Average current in L

$$I_d T_s = \frac{DT_sV_d}{L}\frac{(D+\Delta_1)T_s}{2}$$

Average output current

$$I_o = I_d\frac{\Delta_1}{D+\Delta_1} = \frac{T_sV_d}{2L}D\Delta_1$$

$$= \frac{27}{4}I_{oBmax}\frac{V_d}{V_o}D^2\frac{V_d}{V_o - V_d}$$

$$D = \left[\frac{4}{27}\frac{V_o}{V_d}\left(\frac{V_o}{V_d} - 1\right)\frac{I_o}{I_{oBmax}}\right]^{\frac{1}{2}}$$

Discontinuous conduction mode (constant V_o)

Periodic conditions:

$$\frac{DT_sV_d}{L} + \frac{\Delta_1 T_s(V_d - V_o)}{L} = 0$$

$$\frac{V_o}{V_d} = 1 + \frac{D}{\Delta_1} = \frac{I_d}{I_o}$$

tions:

$$\frac{(V_d - V_o)}{L} = 0$$

$$- \frac{D}{\Delta_1} = \frac{I_d}{I_o}$$

Losses and ripple

Losses: inductor, capacitor, switch, diode

Ripple: first order assumption: when the switch is on the C is discharged through the load

$$\Delta V_o = \frac{\Delta Q}{C} = \frac{I_o D T_s}{C} = \frac{V_o D T_s}{RC}$$

$$\frac{\Delta V_o}{V_o} = D\frac{T_s}{\tau}$$

Continuous-discontinuous mode (constant V_o)

Continuous mode:

$$I_o > I_{oB}$$

$$= I_{oBmax}\frac{27(1-D)^2 D}{4}$$

$$D = 1 - \frac{V_d}{V_o}$$

Discontinuous mode:

$$I_o < I_{oB}$$

$$D = \left[\frac{4}{27}\frac{V_o}{V_d}\left(\frac{V_o}{V_d} - 1\right)\frac{I_o}{I_{oBmax}}\right]^{\frac{1}{2}}$$

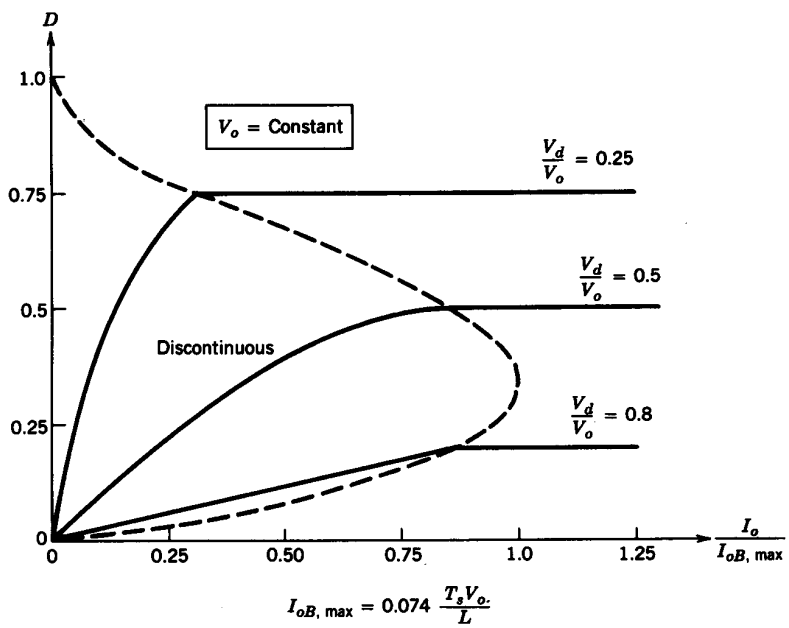


Figure 7-15 Step-up converter characteristics keeping V_o constant.

Continuous-discontinuous boundary

Current in L at the boundary

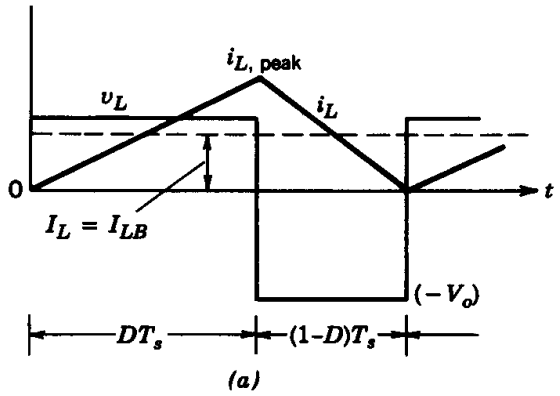
$$I_{LB} = \frac{DT_sV_d}{2L}$$

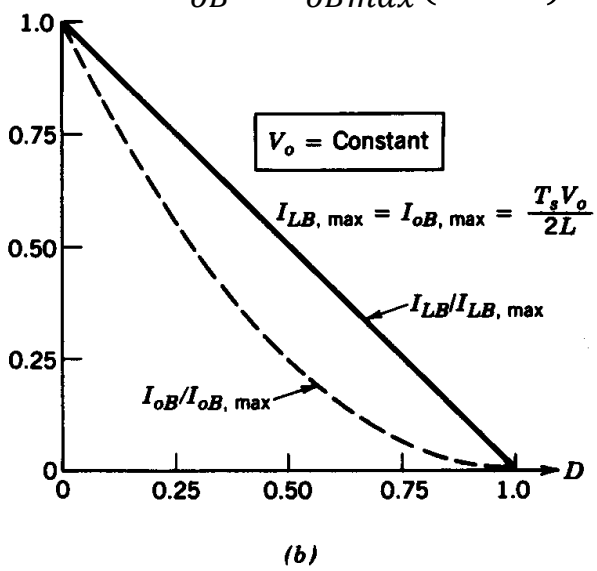
$$I_{LB} = I_{LBmax}(1-D)$$

Output current at the boundary:

$$I_{oB} = I_{LB}(1-D) = \frac{T_sV_o}{2L}(1-D)^2$$

$$I_{oB} = I_{oBmax}(1-D)^2$$



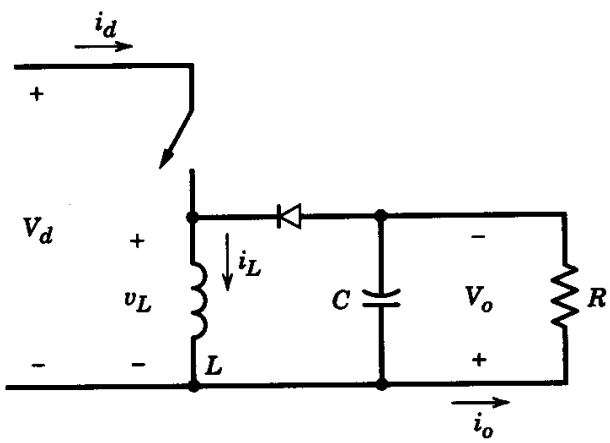


Buck-boost converter

Negative DC power supply

Switch on: inductance accumulates energy, diode off, C supplies the load

Switch off: diode on, inductance transfers energy to the capacitance and to the load



Periodic conditions in continuous conduction mode:

$$\frac{DT_sV_d}{L} - \frac{V_o(1-D)T_s}{L} = 0$$

$$\frac{V_o}{V_d} = \frac{D}{1-D} = \frac{I_d}{I_o}$$

$$I_L = I_o + I_d = \frac{I_o}{1-D}$$

Continuous-discontinuous mode

Continuous operation

$$I_o > I_{oB} = I_{oBmax}(1-D)^2$$

$$D = \frac{V_o}{V_d - V_o}$$

Discontinuous operation

$$I_o > I_{oB}$$

$$D = \frac{V_o}{V_d}\sqrt{\frac{I_o}{I_{oBmax}}}$$

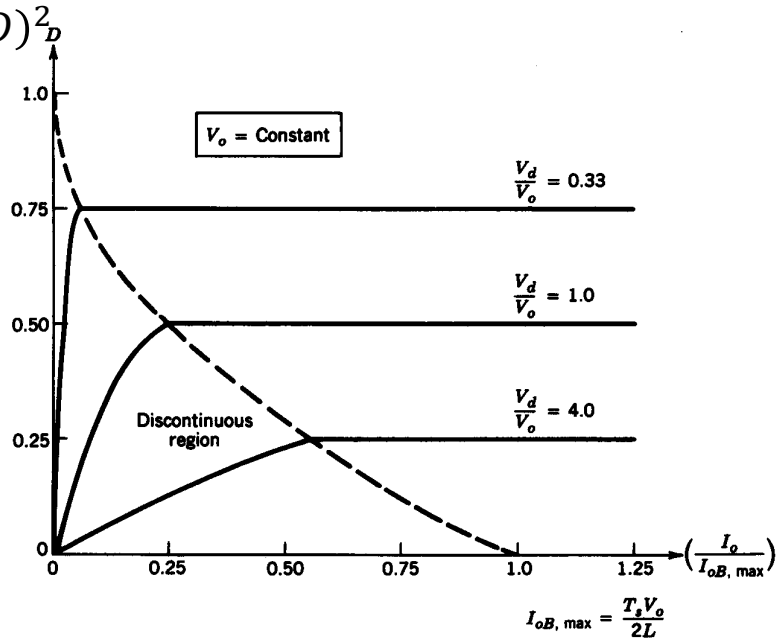


Figure 7-22 Buck–boost converter characteristics keeping V_o constant.

Discontinuous conduction

Periodic conditions:

$$\frac{DV_dT_s}{L} - \frac{V_o\Delta_1 T_s}{L} = 0$$

$$\frac{V_o}{V_d} = \frac{D}{\Delta_1} = \frac{I_d}{I_o}$$

Average current in L:

$$I_L T_s = \frac{V_d D T_s}{L}\frac{(D+\Delta_1)T_s}{2}$$

Therefore:

$$I_L = I_o\left(1+\frac{D}{\Delta_1}\right) = \frac{V_dT_s}{2L}D(D+\Delta_1)$$

$$\frac{I_o}{I_{oBmax}} = D\Delta_1\frac{V_d}{V_o} = D^2\left(\frac{V_d}{V_o}\right)^2 \rightarrow D = \frac{V_o}{V_d}\sqrt{\frac{I_o}{I_{oBmax}}}$$

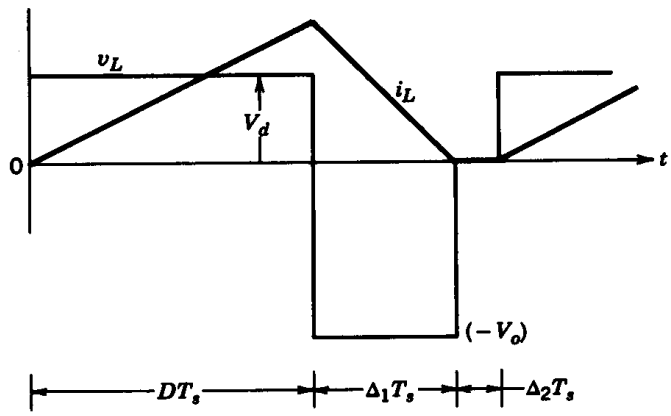


Figure 7-21 Buck–boost converter waveforms in a discontinuous-conduction mode.

Cuk DC-DC converter

Negative DC power supply

DC analysis: $V_{C1} = V_d + V_o$ note: $(V_{C1} > V_d)$

Assumption: Large C1 (Voltage almost constant)

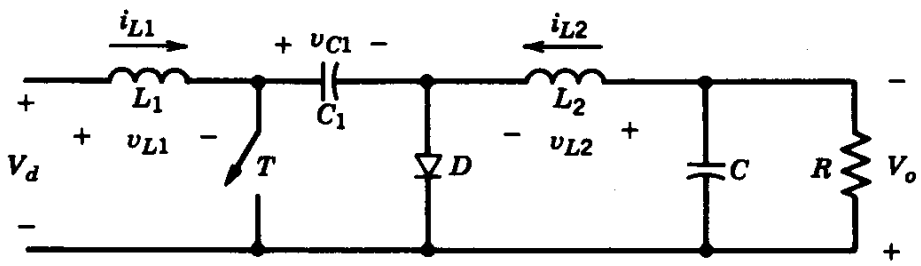


Figure 7-25 Cúk converter.

Output voltage ripple

When the switch is ON, C is discharged through the load

$$\Delta V_o = \frac{\Delta Q}{C} = \frac{D T_s V_o}{RC} \rightarrow \frac{\Delta V_o}{V_o} = D \frac{T_s}{\tau}$$

Cuk DC-DC converter

Negative DC power supply

DC analysis: $V_{C1} = V_d + V_o$ note: $(V_{C1} > V_d)$

Assumption: Large C1 (Voltage almost constant)

Switch OFF: C1 is charged through L1 and the input, Diode ON, L2 supplies energy to R (currents in L1 and L2 decrease)

Switch ON: L1 receives energy, Diode OFF, C supplies current to R, C1 gives energy to L2 (currents in L1 and L2 increase)

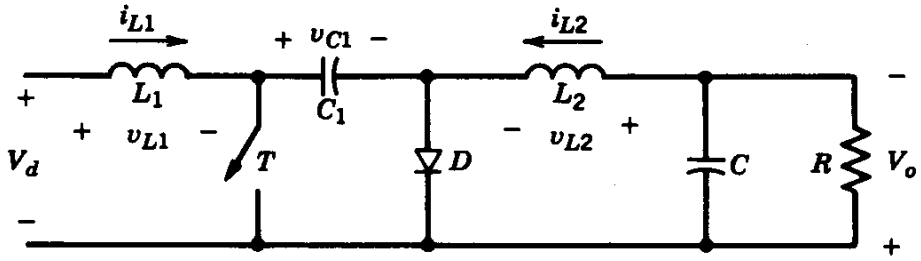


Figure 7-25 Cúk converter.

Cuk DC-DC converter

Negative DC power supply

DC analysis: $V_{C1} = V_d + V_o$ note: $(V_{C1} > V_d)$

Assumption: Large C1 (Voltage almost constant)

Switch OFF: C1 is charged through L1 and the input, Diode ON, L2 supplies energy to R (currents in L1 and L2 decrease)

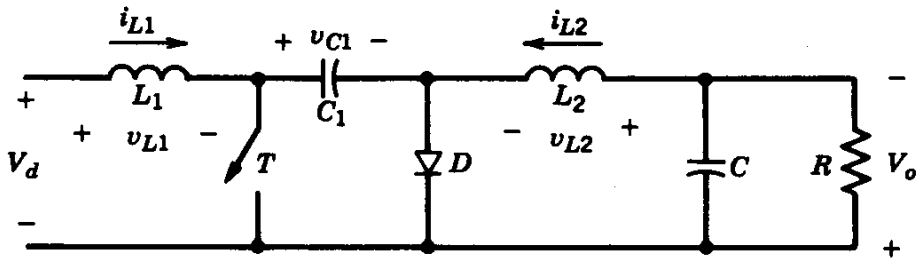


Figure 7-25 Cúk converter.

Full bridge DC-DC converter

Applications:

- DC motor drives
- DC to AC conversion in UPS
- DC to AC conversion in transformer isolated power supply

Fixed V_d.

Control polarity and amplitude of Vo

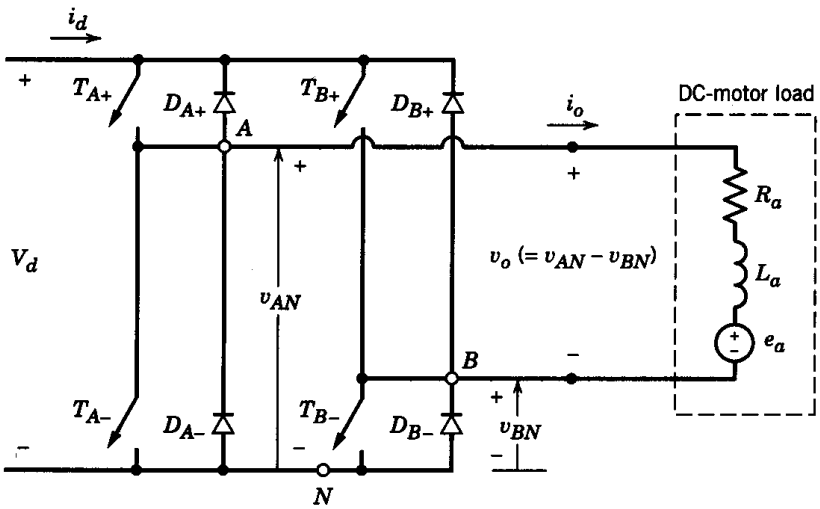


Figure 7-27 Full-bridge dc–dc converter.

Two legs: A and B. **Only one switch** in each leg is ON at any time

Cuk

Periodic conditions in L1

$$V_d D T_s + (1-D)T_s(V_d - V_{C1}) = 0$$

$$V_{C1} = \frac{V_d}{1-D}$$

Periodic conditions in L2

$$(V_{C1} - V_o)DT_s - \mathrm{V_o}(1-\mathrm{D})T_s = 0$$

$$V_{C1} = \frac{V_o}{D}$$

Therefore

$$\frac{V_o}{V_d} = \frac{D}{1-D}$$

Pro: currents in L1 and L2 ripple free

Con: C1 must be large

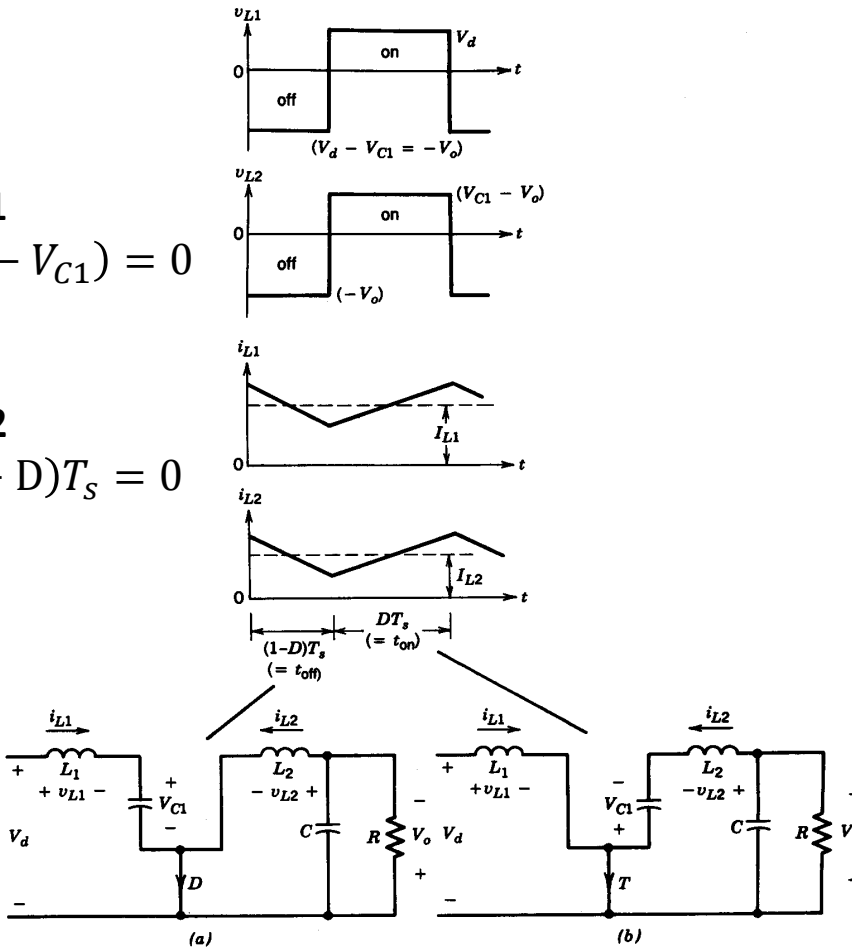


Figure 7-26 Cúk converter waveforms: (*a*) switch off; (*b*) switch on.

PWM with bipolar voltage switching

When $v_{control} > v_{tri}$,

TA+ and TB- are **ON**

Duty cycle

$$D_1 = \frac{1}{2} + \frac{v_{control}}{\widehat{V_{tri}}}\frac{1}{2}$$

When $v_{control} < v_{tri}$,

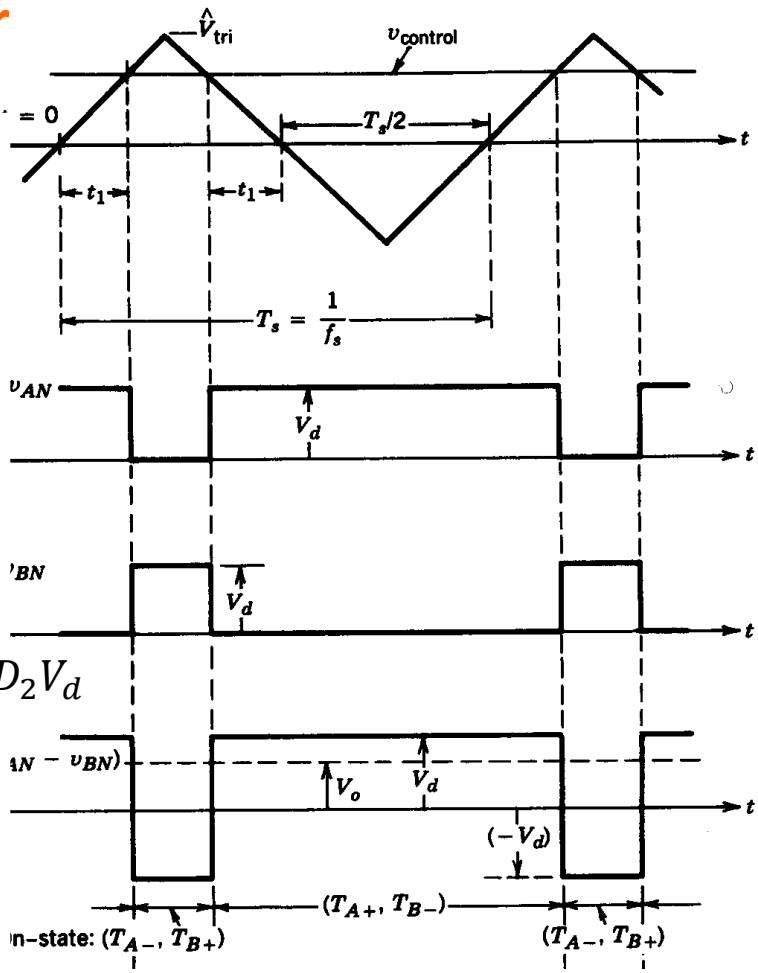
TA- and TB+ are **ON**

$$D_2 = 1 - D_1$$

$$V_o = V_{AN} - V_{BN} = D_1 V_d - D_2 V_d$$

$$= (2D_1 - 1)V_d$$

$$= \frac{V_d}{\widehat{V_{tri}}} v_{control}$$



Full bridge DC-DC converter

When switch TA+ is on:

$i_o > 0$: i_o through TA+

$i_o < 0$: i_o through DA+

$V_{AN} = V_d \text{dutycycle}(TA^+)$

When switch TB+ is on:

$i_o < 0$: i_o through TB+

$i_o > 0$: i_o through DB+

$V_{BN} = V_d \text{dutycycle}(TB^+)$

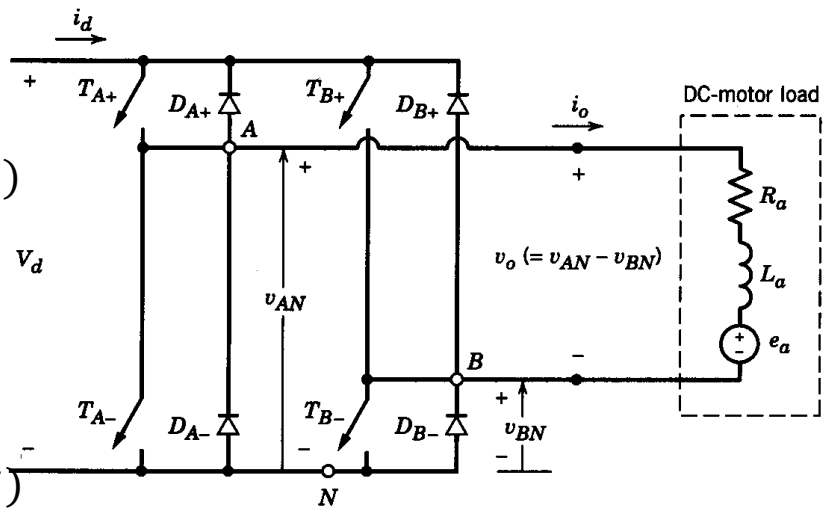
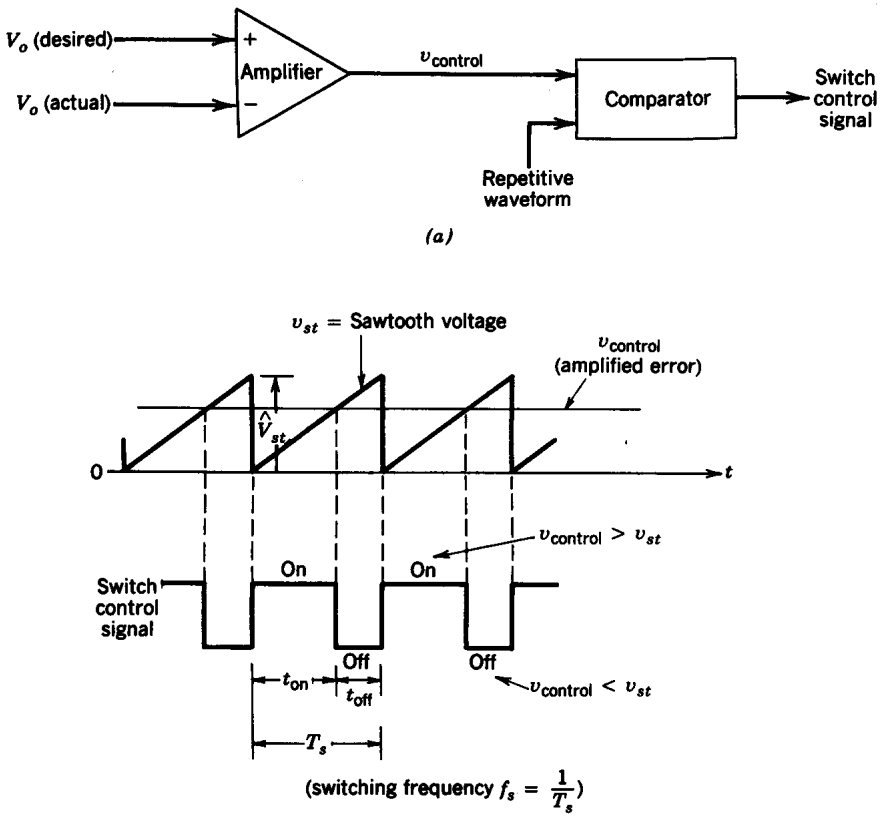


Figure 7-27 Full-bridge dc–dc converter.

$$V_o = V_{AN} - V_{BN}$$

Four quadrant operation on V_o, I_o

PWM signal generation



PWM with unipolar voltage switching

When $v_{control} > v_{tri}$,

TA+ and TB- are **ON**

Duty cycle

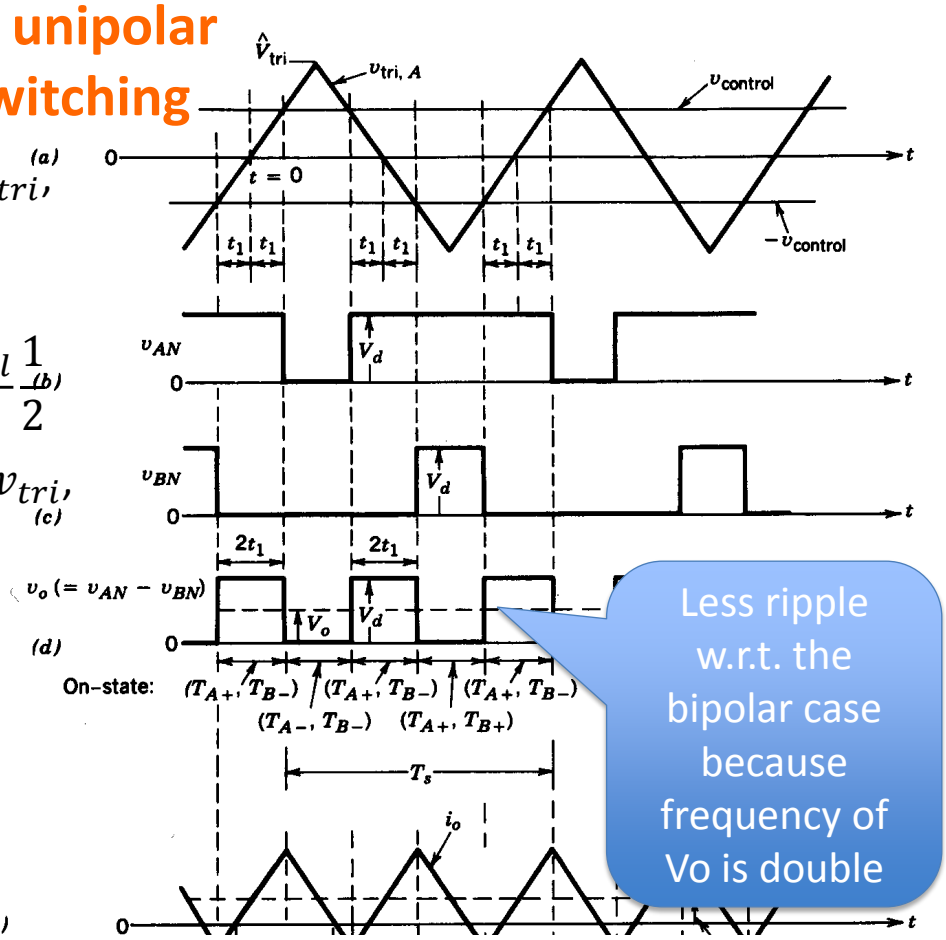
$$D_1 = \frac{1}{2} + \frac{v_{control}}{\widehat{V_{tri}}}\frac{1}{2}$$

When $-v_{control} < v_{tri}$,

TA- and TB+ are **ON**

$$D_2 = 1 - D_1$$

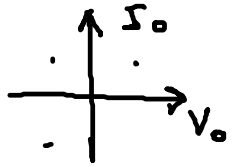
$$V_o = V_{AN} - V_{BN}$$
$$= D_1 V_d - D_2 V_d$$
$$= (2D_1 - 1)V_d$$
$$= \frac{V_d}{\widehat{V_{tri}}} v_{control}$$

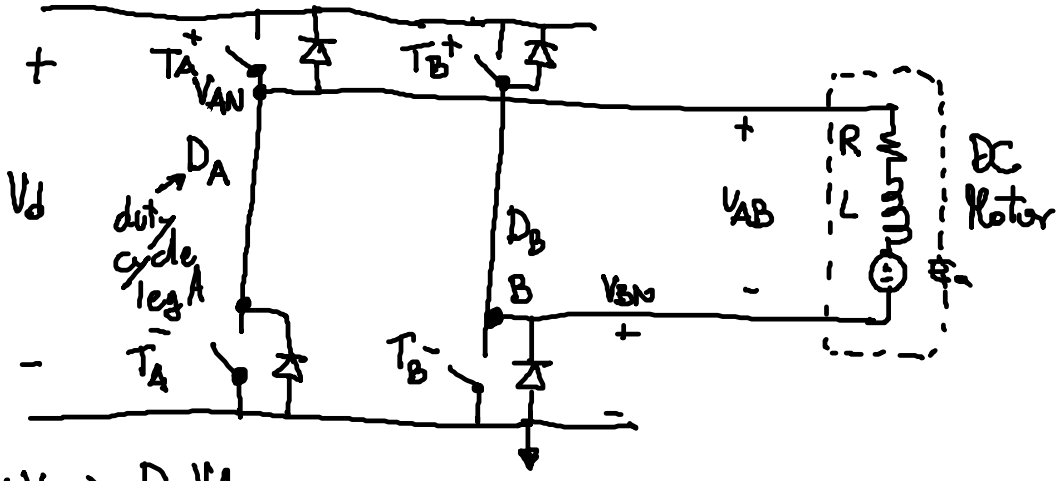


Less ripple w.r.t. the bipolar case because frequency of Vo is double

Full Bridge DC DC Converter (4Q)

11 May 2015 14:08





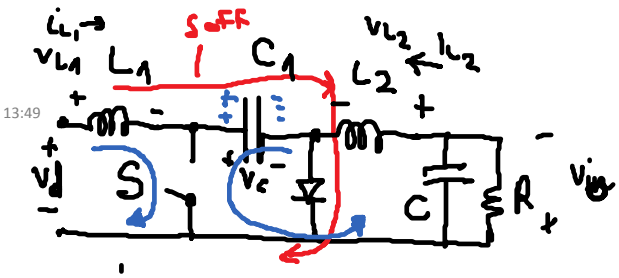
$\langle V_{AN} \rangle = D_A V_d$

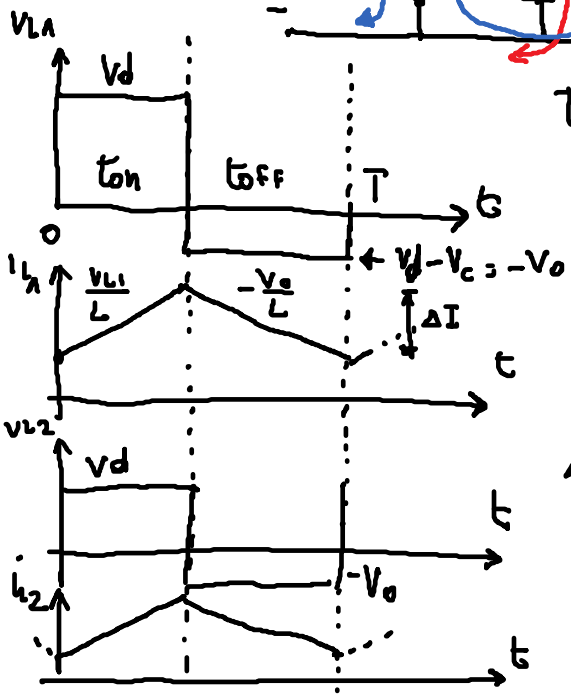
$\langle V_{BN} \rangle = D_B V_d \rightarrow \langle V_{AB} \rangle = \langle V_{AN} \rangle - \langle V_{BN} \rangle = (D_A - D_B) V_d$

Controllo → PWM Pulse Width Modulation → BIPOLAR V / UNIPOLAR V

ĆUK

11 May 2015 13:49





$t_{on} = DT$

$V_C = V_d + V_o$

$V_c > V_d \rightarrow V_o < 0$

$V_C = V_o + V_{L2}$

$V_{L2} = V_c - V_o = V_d$

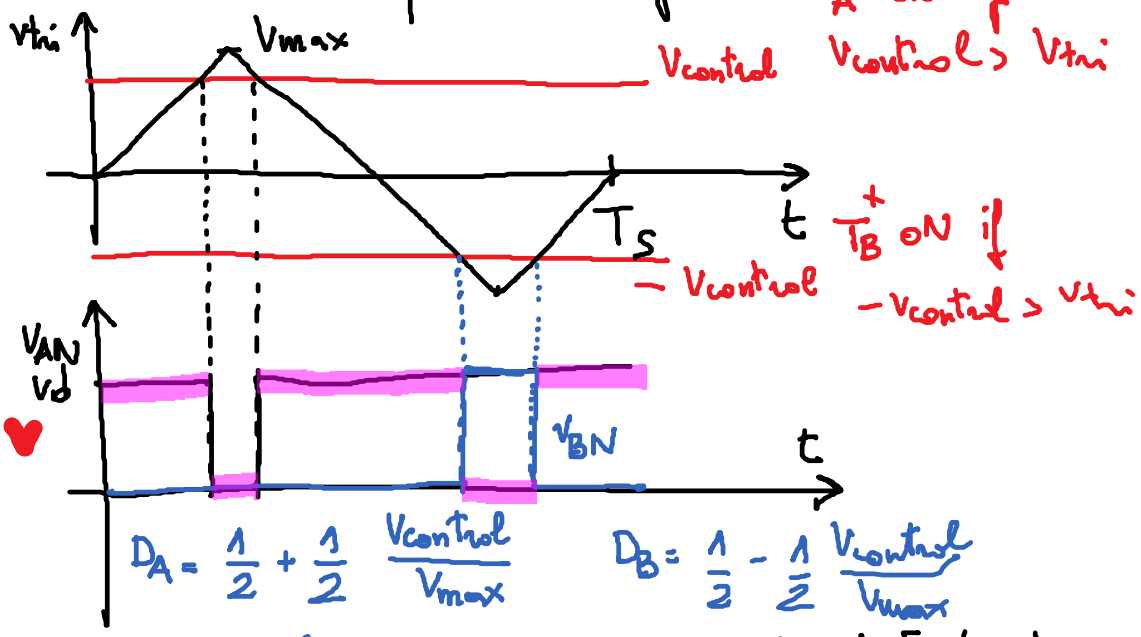
$\Delta I = \frac{V_d}{L} \cdot t_{on} = \frac{V_o}{L} t_{off}$

$V_d D T = V_o (1 - D) T$

$$\boxed{V_o = V_d \frac{D}{1-D}}$$

11 May 2015 14:25

PWM with unipolar voltage



T_A^+ ON if $V_{control} > V_{tri}$

T_B^+ ON if $-V_{control} > V_{tri}$

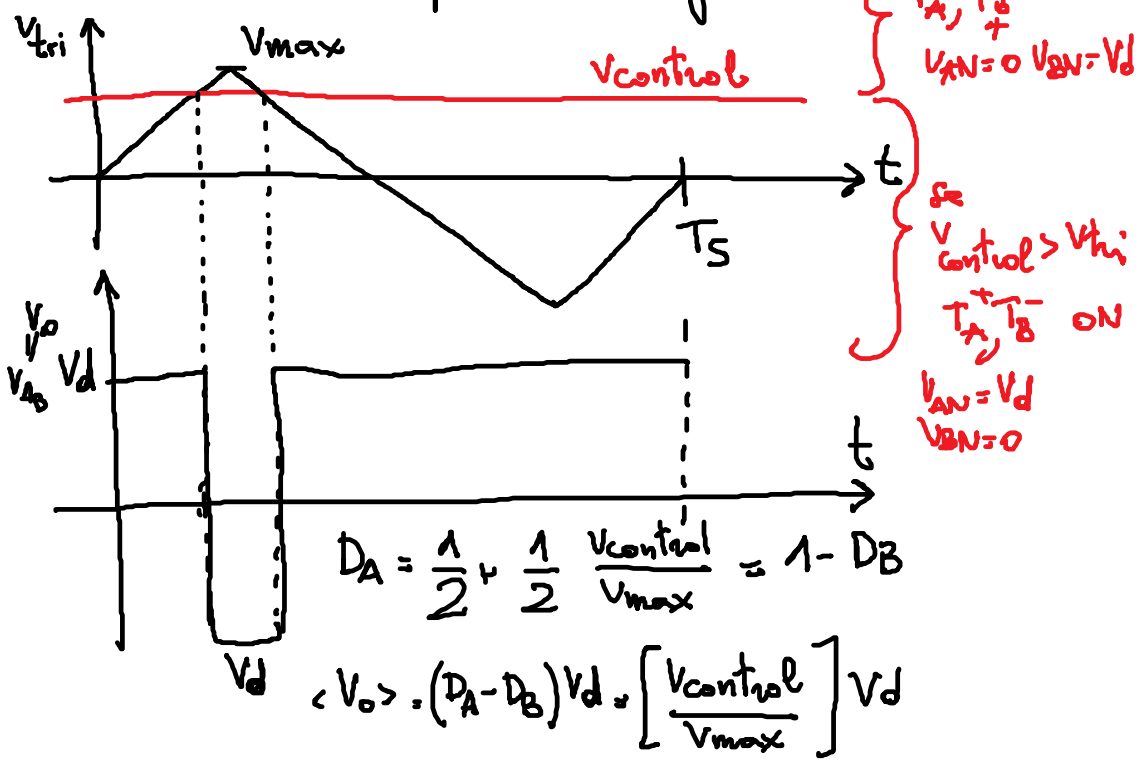
$$D_A = \frac{1}{2} + \frac{1}{2}\frac{V_{control}}{V_{max}} \qquad D_B = \frac{1}{2} - \frac{1}{2}\frac{V_{control}}{V_{max}}$$

$$V_o = \langle V_{AN} - V_{BN} \rangle = (D_A - D_B) V_d = \frac{V_{control}}{V_{max}} V_d$$

RIPPLE A F doppia, frequenza la metà → più facile da filtrare

11 May 2015 14:16

PWM with Bipolar Voltage



T_A^-, T_B^+
$V_{AN} = 0$ $V_{BN} = V_d$

se $V_{control} > V_{tri}$
T_A^+, T_B^- ON
$V_{AN} = V_d$
$V_{BN} = 0$

$$D_A = \frac{1}{2} + \frac{1}{2}\frac{V_{control}}{V_{max}} = 1 - D_B$$

$$\langle V_o \rangle = (D_A - D_B) V_d = \left[\frac{V_{control}}{V_{max}}\right] V_d$$

11 May 2015 15:03

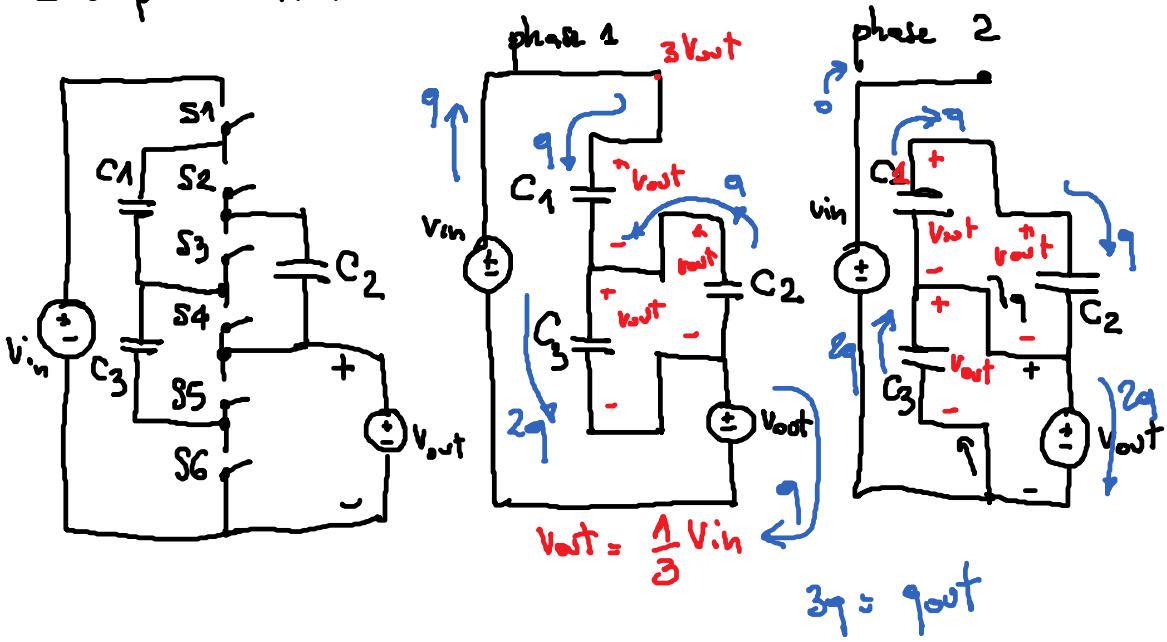
Limits

- SSL Slow switching limit [low f_s]
  - C's reach their final charge state during each phase
  - we can discard power dissipation in R_S
- FSL Fast Switching limit [high f_s]
  - C's charge does not change during each phase.

11 May 2015 14:42

Convertitori DCDC · inductorless

Esempio · LADDER 3:1



SSL

11 May 2015 15:13

1) charge multiplier vectors

phase 1) $\vec{a}^{(1)} = \left[a_{out}^{(1)} \quad a_{c_1}^{(1)} \quad a_{c_2}^{(1)} \quad \dots \quad a_{c_N}^{(1)} \quad a_{in}^{(1)} \right]$

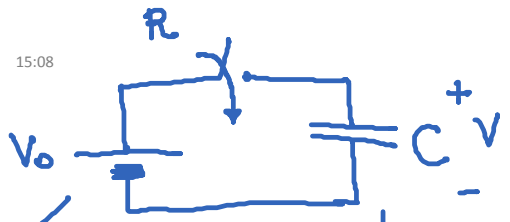
frazione della carica che viene fornita in uscita per ciascun periodo

per ladder 3:1

$$\vec{a}^{(1)} = \left[\frac{1}{3} \quad \frac{1}{3} \quad -\frac{1}{3} \quad \frac{2}{3} \quad \frac{1}{3} \right]$$

$$\vec{a}^{(2)} = \left[\frac{2}{3} \quad -\frac{1}{3} \quad \frac{1}{3} \quad -\frac{2}{3} \quad 0 \right]$$

11 May 2015 15:08



$$i(t) = \frac{V_0}{R} e^{-\frac{t}{RC}}$$

$$E = \frac{1}{2} C V_0^2$$

$$\int_0^\infty V_0 i(t)\, dt = \frac{V_0^2}{R} RC = C V_0^2$$

$$\int_0^\infty R i^2(t)\, dt = \frac{V_0^2}{R} \int_0^\infty e^{-\frac{2t}{RC}} dt = \frac{V_0^2}{R} \frac{RC}{2} = \frac{1}{2} C V_0^2$$

11 May 2015 15:29

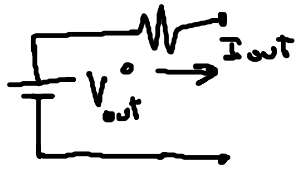
$$\Delta V_{out} = V_{out} - V_{out}^{\circ} = -\sum_{i \in cap} \underbrace{\left[V_{c_i}^{(1)} - V_{c_i}^{(2)} \right]}_{\frac{a_{c,i}\, q_{out}}{C_i}} a_{c_i}$$

$$\Delta V_{out} = -\sum_{i \in cap} \frac{(a_{c,i})^2 q_{out}}{C_i} \qquad q_{out} = \frac{I_{out}}{f_s}$$

$$\Delta V_{out} = -\left[\sum_{i \in cap} \frac{a_{c_i}^2}{C_i f_s} \right] I_{out}$$

$$\rightarrow R_{out}^{SSL}$$

R_{out}^{SSL}



Teorema di Tellegen

11 May 2015 15:19

$$\sum v_i i_i = 0$$

$$V_{out}\, i_{out} + \sum_{i \in cap} v_{c_i} i_{c_i} + V_{in} i_{in} = 0$$

phase 1. $\quad V_{out}\, a_{out}^{(1)} + \sum_{i \in cap} V_{c_i}^{(1)} a_{c_i}^{(1)} + V_{in}\, a_{in}^{(1)} = 0$

$+$

phase 2 $\quad V_{out}\, a_{out}^{(2)} + \sum_{i \in cap} V_{c_i}^{(2)} a_{c_i}^{(2)} + V_{in}\, a_{in}^{(2)} = 0$

sum: phase 1 + phase 2

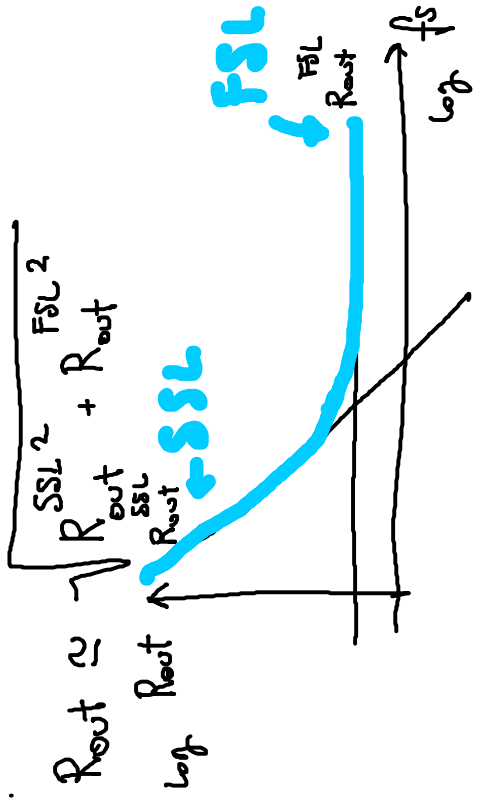
NB. $a_{out}^{(1)} + a_{out}^{(2)} = 1$

$a_{c_i}^{(1)} = -a_{c_i}^{(2)} = a_{c,i}$

$$V_{out} + \sum_{i \in cap} a_{c_i} \left[V_{c_i}^{(1)} - V_{c_i}^{(2)} \right] + V_{in} \underbrace{\left[a_{in}^{(1)} + a_{in}^{(2)} \right]}_{=0} = -V_{out}^{\circ}$$

11 May 2015 15:48

$$R_{out}^{FSL} = 4\sum_{i \in switch} R_i\, a_{s,i}^2 = 4\sum_{i \in switch} R_i \left[a_{s,i}^{(1)2} + a_{s,i}^{(2)2} \right]$$



FSL

11 May 2015 15:39

R_i resistance of Switch S_i

- charge multiplier vector

$$\vec{a}_s^{(1)} = \begin{array}{c} \begin{array}{cccccc} S_1 & S_2 & S_3 & S_4 & S_5 & S_6 \end{array} \\ \begin{bmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & -\frac{2}{3} & 0 \end{bmatrix} \end{array}$$

$$\vec{a}_s^{(1)} = \begin{bmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & -\frac{2}{3} \end{bmatrix}$$

switch S_i $\quad i_{s,i}^{(1)} = 2\, a_{s,i}^{(1)}\, \overbrace{q_{out}}^{I_{out}} f_s$ $\leftarrow$ average current in S_i during phase 1

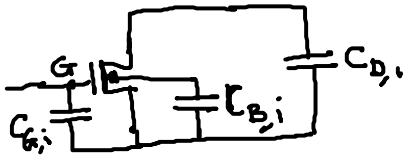
$$P_{FSL} = \sum_{i \in Switch} R_i\, i_{s,i}^2 = \underbrace{\sum_{i \in Switch} 4 R_i\, a_{s,i}^2}_{R_{out}^{FSL}}\, I_{out}^2$$

Other Power losses.

Switching Loss

$$P_{SW} = f_S \sum_{i \in switch} C_{G_i} V_{G_i}^2 + C_{D_i} V_{D_i}^2 + C_{B_i} V_{B_i}^2$$

$P_{SW} \propto f_S, A_{SW}$



Bottom plate parasitic capacitance

$$P_{CAP} = f_S \sum_{i \in cap} C_{BP_i} V_{BP_i}^2$$

$P_{CAP} \propto f_S, A_{cap}$

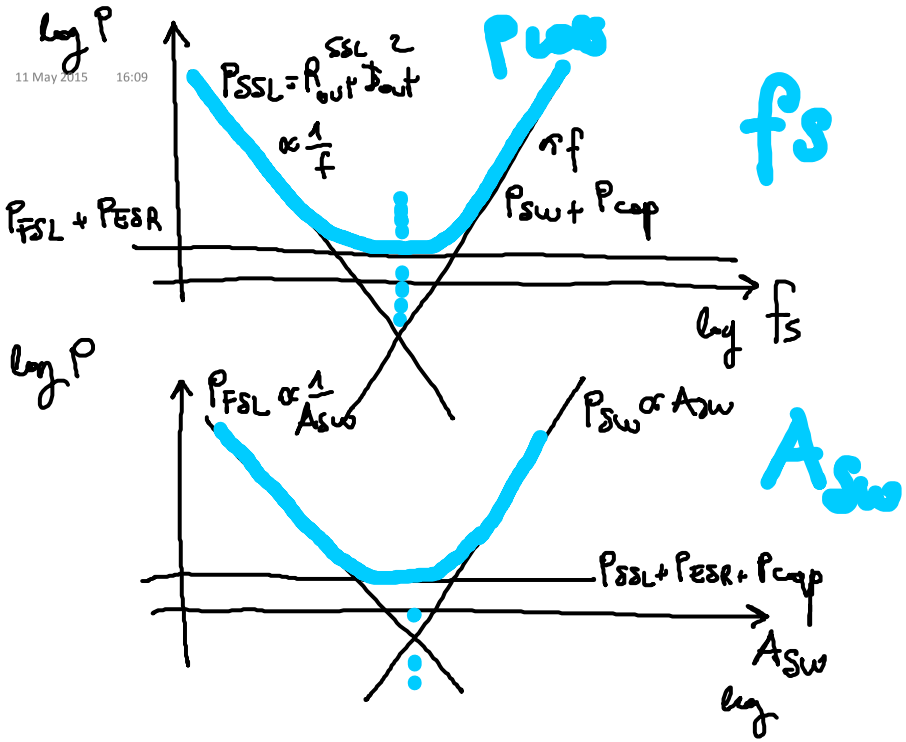
Equivalent Series Resistance

$$P_{ESR} = R_{ESR} I_{out}^2$$

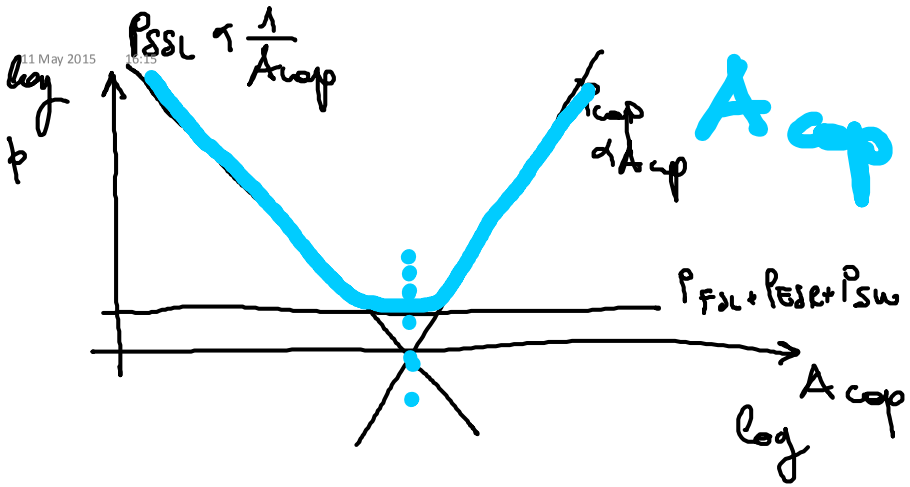
$$P_{LOSS} = R_{out} I_{out}^2 + P_{SW} + P_{ESR} + P_{cap}$$

$$\eta = \frac{V_{out} I_{out}}{V_{out} I_{out} + P_{LOSS}} \simeq 50\% - 90\%$$

11 May 2015 16:09



11 May 2015 16:15



13 May 2015 08:56

Raggiungibilità · Controllabilità · Osservabilità

→ di un sistema lineare e stazionario

TC:
$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases}$$

$\vec{u}$: m ingressi

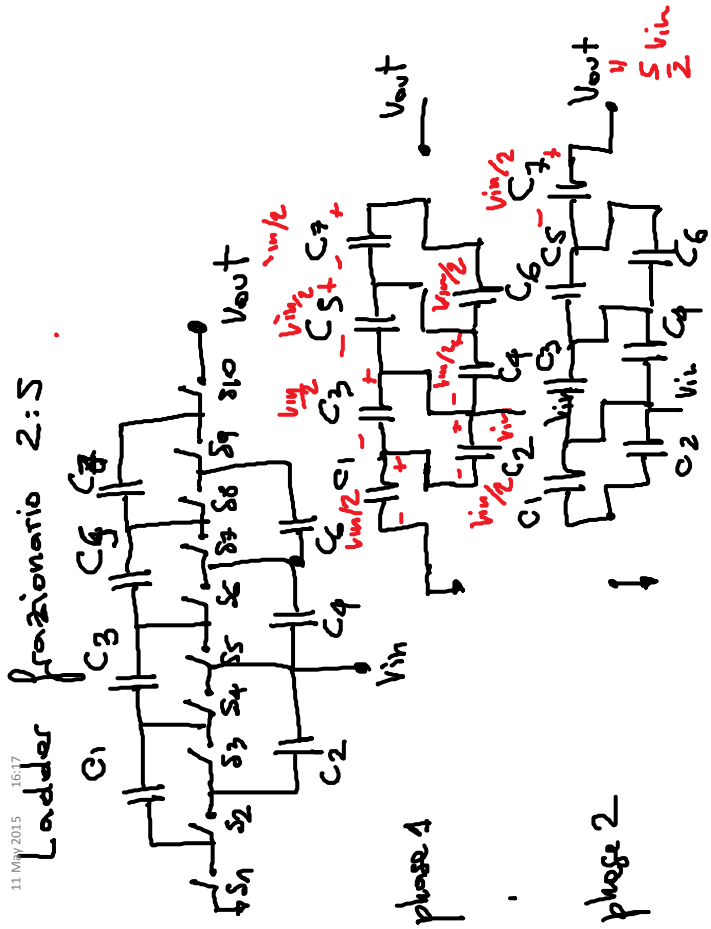
$\vec{x}$: n stati

$\vec{y}$: l uscite

TD:
$$\begin{cases} x(k+1) = Ax(k) + Bu(k) \\ y(k) = Cx(k) + Du(k) \end{cases}$$

11 May 2015 16:17

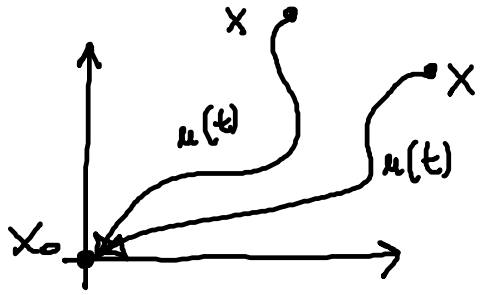
Ladder frazionario 2:5



Controllabilità

13 May 2015 09:06

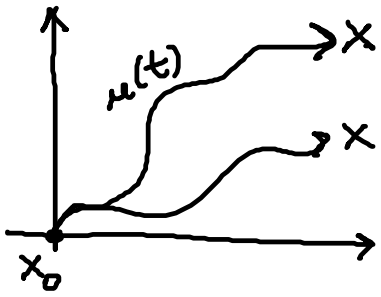
DEF un sistema è CONTROLLABILE se a partire da un qualunque stato x esiste una opportuna azione di controllo in grado di portare il sistema nello stato x_0



Raggiungibilità

13 May 2015 09:03

DEF Un sistema è RAGGIUNGIBILE se partendo da un qualunque stato iniziale x_0 si può raggiungere un qualunque stato finale x con una opportuna azione di controllo



13 May 2015 09:19

il sistema è RAGGIUNGIBILE se $\underline{\underline{\text{Rank}(R) = n}}$

↳ ogni stato può essere raggiunto in $\underline{n \text{ passi}}$

se $\text{Rank}[R] < n$ sono raggiungibili solo gli stati $\in \text{Image}\{R\}$

[PARZIALMENTE RAGGIUNGIBILE]

Condizioni di raggiungibilità

13 May 2015 09:08

$$x(0) = x_0$$
$$x(1) = Ax(0) + Bu(0) = Ax_0 + Bu(0)$$
$$x(2) = Ax(1) + Bu(1) = A^2x_0 + ABu(0) + Bu(1)$$
$$x(3) = Ax(2) + Bu(2) = A^3x_0 + A^2Bu(0) + ABu(1) + Bu(2)$$
$$\vdots$$
$$x(k) = A^k x_0 + \sum_{i=1}^{k} A^{k-i} B u(i-1)$$

se $k = n$

$$\underbrace{x(n) - A^n x_0}_{n \times 1} = \underbrace{\begin{bmatrix} B & AB & \cdots & A^{n-1}B \end{bmatrix}}_{n \times mn} \underbrace{\begin{bmatrix} u(n-1) \\ u(n-2) \\ \vdots \\ u(0) \end{bmatrix}}_{mn \times 1}$$

MATRICE DI RAGGIUNGIBILITÀ (R)

il sistema è raggiungibile se $\forall x(n)$ esiste una soluzione

Osservabilità

13 May 2015 09:32

DEF un sistema è OSSERVABILE se

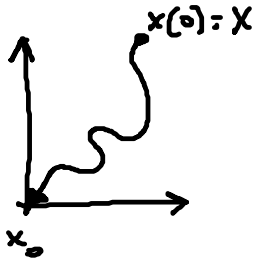
- conoscendo $u(t)$ da $t=t_0$ a $t=t_f$
  conoscendo $y(t)$ da $t=t_0$ a $t=t_f$

siamo in grado di ricavare lo stato iniziale $x(t_0)$

Controllabilità

13 May 2015 09:23

$$\left[x(n) - A^n x(0)\right] = R \begin{bmatrix} u(n-1) \\ \vdots \\ u(0) \end{bmatrix}$$



$x(0) = X \leftarrow$ generico

$x(n) = x_0 \leftarrow$ origine

$$x_0 - A^n X = R \begin{bmatrix} u(n-1) \\ \vdots \\ u(0) \end{bmatrix}$$

la soluzione esiste se $\text{Image}(A) \subset \text{Image}(R)$

[il sistema è controllabile se ↗

- se $\det(N) \neq 0$ Raggiungibilità e Controllabilità coincidono
- se un sistema è Raggiungibile allora è anche controllabile

se $\text{Rank}(\mathcal{O}) = n$ il sistema è

[completamente] osservabile

(la soluzione è unica)

se $\text{Rank}(\mathcal{O}) < n$ il sistema è

PARZIALMENTE OSSERVABILE

[gli stati $\in \text{Ker}(\mathcal{O})$ non sono osservabili]

$$\mathcal{O}(\tilde{x}) = \mathcal{O}(\tilde{x} + x_{n0})$$

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) \\ y(k) = Cx(k) + Du(k) \end{cases}$$

$$y(0) = Cx(0) + Du(0)$$

$$y(1) = Cx(1) + Du(1) = CAx(0) + CBu(0) + Du(1)$$

$$y(2) = Cx(2) + Du(2) = CAx(1) + CBu(1) + Du(2)$$

$$= CA^2x(0) + CABu(0) + CBu(1) + Du(2)$$

$$y(k) = CA^k x(0) + \underbrace{\left[\sum_{i=1}^{k} CA^{k-i} Bu(i-1)\right]}_{\ell n \times 1} + Du(k)$$

Sistema

$$\begin{bmatrix} y(0) - Du(0) \\ y(1) - CBu(0) - Du(1) \\ \vdots \\ y(n-1) - \sum_{i=1}^{n-1} CA^{n-1-i} Bu(i-1) - Du(n-1) \end{bmatrix} = \underbrace{\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}}_{\ell n \times n} \underbrace{x(0)}_{n \times 1}$$

MATRICE DI OSSERVABILITÀ $\mathcal{O}$

Forma standard di raggiungibilità

13 May 2015 09:58

$$T = \left[T_1 \,,\; T_2 \right] \qquad x = T x'$$

T_1: base dello spazio raggiungibile (Image (R))

T_2: base dello spazio n.r.

$$x' \begin{cases} x_1 \leftarrow r \\ x_2 \leftarrow n.r \end{cases}$$

$$\begin{cases} \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \left[\begin{array}{c:c} A_{11} & A_{12} \\ \hdashline \underset{0}{A_{21}} & A_{22} \end{array}\right] \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} = \left[\begin{array}{c} B_1 \\ \hdashline \underset{0}{B_2} \end{array}\right] u(k) \\ y(k) = \left[C_1 \vdots C_2 \right] \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} = D u(k) \end{cases}$$

gli stati x_2 (n.r) evolvono liberamente e non dipendono da u

Cambio di base degli stati

13 May 2015 09:52

$$\underline{x = T x'} \qquad x' = T^{-1} x$$

$$\cdot T^{-1} \begin{cases} x(k+1) = A x(k) + B u(k) \\ y(k) = C x(k) + D u(k) \end{cases}$$

$$\begin{cases} x'(k+1) = T^{-1} A T x'(k) + T^{-1} B u(k) \\ y(k) = C T x'(k) + D u(k) \end{cases}$$

$$A' = T^{-1} A T$$
$$B' = T^{-1} B$$
$$C' = C T$$
$$D' = D$$

$$\begin{cases} x'(k+1) = A' x'(k) + B' u(k) \\ y(k) = C' x'(k) + D u(k) \end{cases}$$

lo stesso sistema dopo il cambiamento di base degli stati

Forma canonica di Kalman

13 May 2015 10:15

$$T = \left[\begin{array}{c:c:c:c} T_1 & T_2 & T_3 & T_4 \end{array}\right] \rightarrow x' = \begin{bmatrix} x_1 \\ \hdashline x_2 \\ \hdashline x_3 \\ \hdashline x_4 \end{bmatrix} \begin{array}{cc} n.o & r \\ o & r \\ no & nr \\ o & nr \end{array}$$

T_1: no r — T_2: o r — T_3: no nr — T_4: o nr

$[\ker(O) \cap \mathrm{Im}(R)]$

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \\ x_4(k+1) \end{bmatrix} = \left[\begin{array}{c:c:c:c} A_{11} & A_{12} & A_{13} & A_{14} \\ \hdashline 0 & A_{22} & 0 & A_{24} \\ \hdashline 0 & 0 & A_{33} & A_{34} \\ \hdashline 0 & 0 & 0 & A_{44} \end{array}\right] \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \end{bmatrix} + \begin{bmatrix} B_1 \\ \hdashline B_2 \\ \hdashline 0 \\ \hdashline 0 \end{bmatrix} u(k)$$

$$y(k) = \left[\begin{array}{c:c:c:c} 0 & C_2 & 0 & C_4 \end{array}\right] \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \end{bmatrix} + D u(k)$$

Forma standard di osservabilità

13 May 2015 10:10

$$x' = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad T = \left[\begin{array}{c:c} T_1 & T_2 \end{array}\right]$$

T_1: base spazio Oss. — T_2: base spazio n.o. ($\ker(O)$)

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \left[\begin{array}{c:c} A_{11} & \overset{0}{A_{12}} \\ \hdashline A_{21} & A_{22} \end{array}\right] \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} = \begin{bmatrix} B_1 \\ \hdashline B_2 \end{bmatrix} u(k)$$

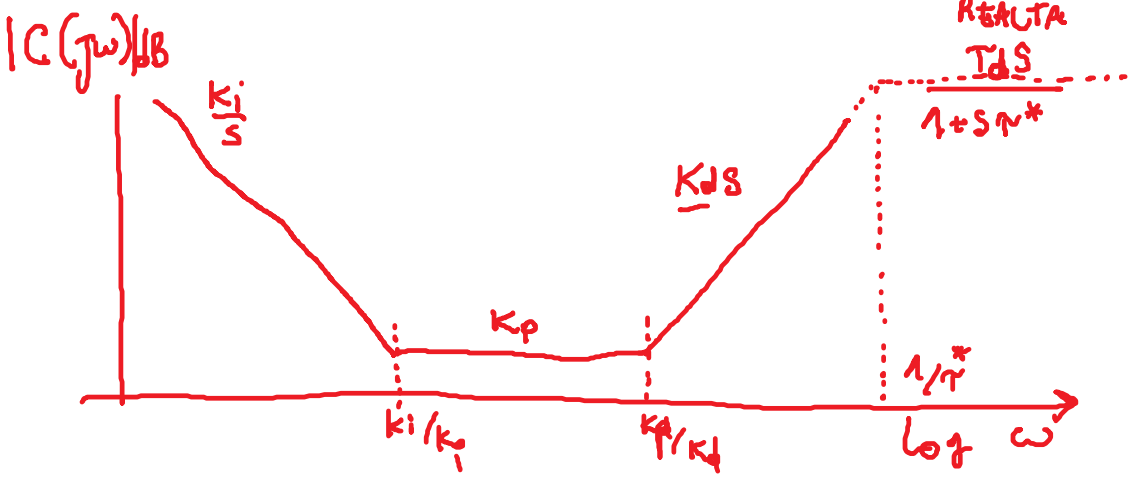
$$y(k) = \left[\begin{array}{c:c} C_1 & \underset{0}{C_2} \end{array}\right] \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + D u(k)$$

Controllore PID

18 May 2015 14:14

$$C(s) = \underset{\uparrow \atop P}{K_p} + \underset{\uparrow \atop I}{\frac{K_i}{s}} + \underset{\uparrow \atop D}{K_d s} = K_p\left[1 + \frac{1}{s\tau_i} + \tau_d s\right]$$

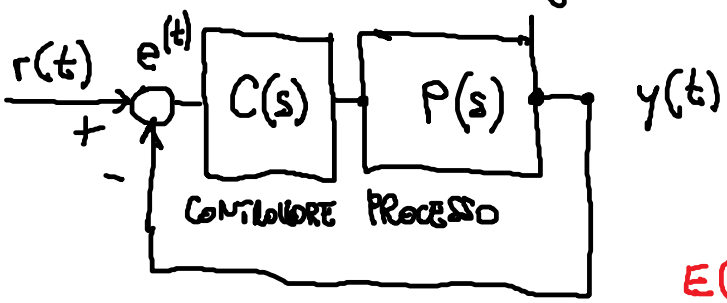
NELLA REALTÀ $\frac{T_d s}{1+s\tau^*}$



Controllori PID

18 May 2015 14:06

proporzionale, integrativo, derivativo



$e(t) = r(t) - y(t)$ errore

$E(s) = R(s) - Y(s)$

$Y(s) = PC\,E(s)$

$Y = PC(R - Y)$

$(1 + PC)Y = PCR$

$Y = \left[\frac{PC}{1+PC}\right]R$ $E = \frac{R}{1+PC}$

OBIETTIVI

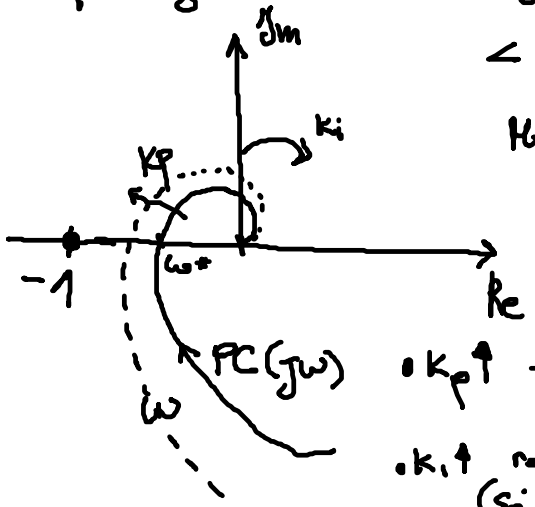
1) minimizzare $|e|$, $|e^2|$, ...
2) sistema stabile
3) minimizzare il tempo di risposta

Effetto sulla stabilità

18 May 2015 14:25

→ P passabasso

$$C = k_p + \frac{k_i}{s} + k_d s = k_p\left[1 + \frac{1}{T_i s} + T_d s\right]$$



ω^* tale che

$\angle PC(j\omega^*) = \pi$

Margine di guadagno è

$$\frac{1}{|PC(j\omega^*)|}$$

- $k_p \uparrow$ → si riduce il margine di guadagno
- $k_i \uparrow$ rotazione oraria (si riduce il margine di g.)
- $k_d \uparrow$ rotazione antioraria (aumenta margine g.)

Risposta al gradino

18 May 2015 14:19

$$R = \frac{1}{s} \qquad E = \frac{R}{1+CP}$$

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE = \lim_{s\to 0} \frac{1}{1+C(s)P(s)}$$

- se C ha la componente <u>integrativa</u> allora

$$\lim_{s\to 0} C \to \infty \quad \text{e quindi} \quad \lim_{t\to\infty} e(t) = 0$$

- se C non ha componente integrativa

$$\lim_{t\to\infty} e(t) = \frac{1}{1+k_p P(0)}$$ ← OFFSET del sistema

CONTROLLORE INDUSTRIALE

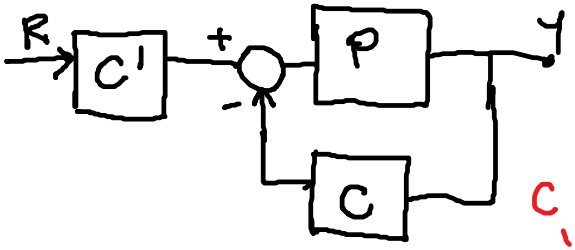
18 May 2015 14:40

$$u(s) = K_p\left[R(s) - Y(s)\right] + \frac{K_i}{s}\left[\alpha R(s) - Y(s)\right] + K_d s\left[\beta R(s) - Y(s)\right]$$

2 gradi di libertà in più

$$u(s) = \underbrace{\left[K_p + \frac{K_i \alpha}{s} + K_d \beta s\right]}_{C'} R(s) - \underbrace{\left[K_p + \frac{K_i}{s} + K_d s\right]}_{C} Y(s)$$

C' invece di C



$$Y = \frac{PC'}{1+PC} R$$

C agisce sui poli della fdt

C' agisce sugli zeri della fdt

PSEUDOCODICE

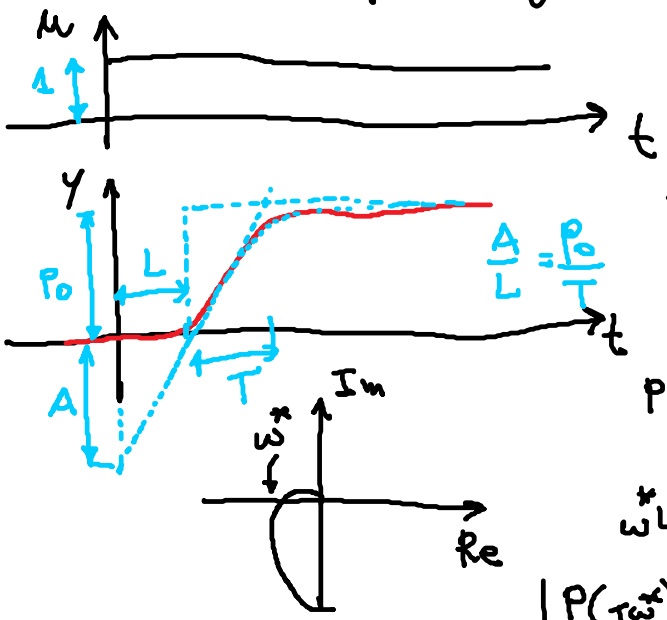
18 May 2015 14:36

```
i = 0
e_old = 0
forever do
    e = setpoint - actual_position
    i = i + e * dt
    d = (e - e_old)/dt
    u = Kp*e + Ki*i + Kd*d
    e_old = e
    wait (dt)
end do
```

Zigler - Nichols a CICLO APERTO

18 May 2015 14:58

:) misurare la risposta al gradino di $P(s)$



$$\frac{A}{L} = \frac{P_0}{T}$$

$$P(j\omega) = \frac{e^{-j\omega L} P_0}{1 + j\omega T}$$

per $\omega \sim \omega^*$

$$P(j\omega) \simeq \frac{e^{-j\omega L} P_0}{j\omega T}$$

$$\omega^* L = \frac{\pi}{2} \rightarrow \omega^* = \frac{\pi}{2L}$$

$$|P(j\omega^*)| \sim \frac{P_0}{\pi T} 2L = \frac{A2}{\pi}$$

Metodo di Ziegler-Nichols (41)

18 May 2015 14:49

:) CICLO CHIUSO

Hp $P(s)$ stabile e $P(0) > 0$

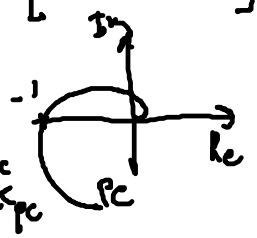
1) si chiude il sistema in reazione con C proporzionale e si aumenta k_p finché il sistema comincia a oscillare

2) prendo nota di $k_p = k_{pc}$, T_c [periodo di oscillazione]

3) P : $k_p = 0.5\, k_{pc}$ [margine di guadagno: 2]

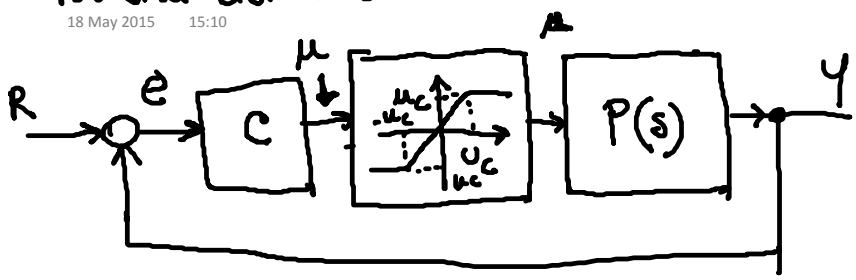
PI : $k_p = 0.45\, k_{pc}$ $\tau_i = 0.8\, T_c$

PID : $k_p = 0.6\, k_{pc}$, $\tau_i = 0.5\, T_c$, $\tau_d = 0.125\, T_c$



Problema del "WIND UP"

18 May 2015 15:10



⇒ c'è un problema se C ha una componente integrativa

⇒ bisogna inibire l'integratore se $u > u_c$.

18 May 2015 15:07

se $\boxed{P}$ $k_p = \frac{1}{A}$ $PC(j\omega^*) = \frac{2}{\pi}$

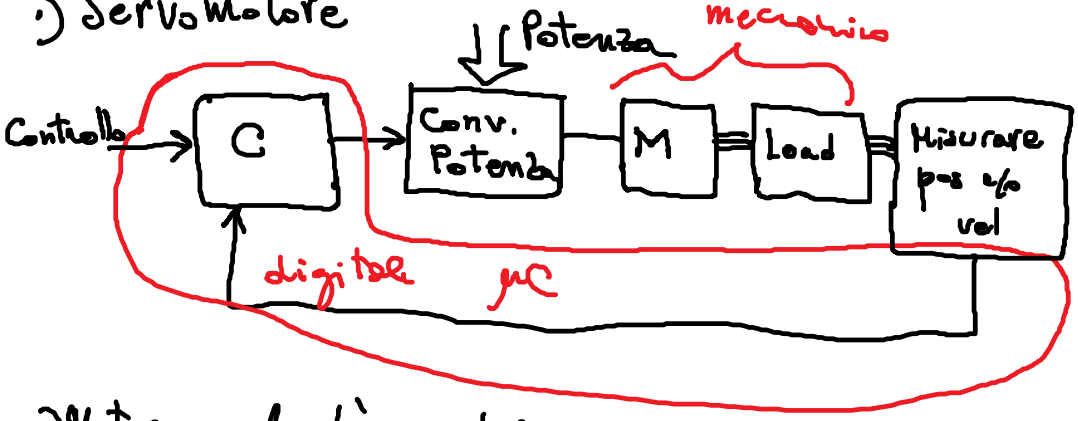
Margine di guadagno $= \frac{\pi}{2}$

	k_p	τ_i	τ_d
P	$1/A$		
PI	$0.9/A$	$3L$	
PID	$1.2/A$	$2L$	$L/2$

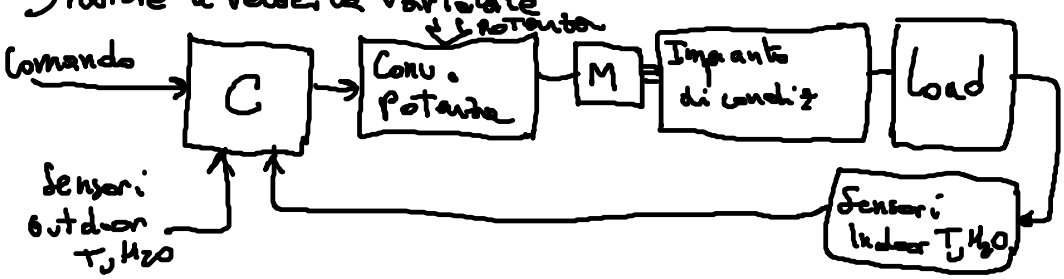
Sistema di controllo

18 May 2015 15:38

·) Servomotore



·) Motore a velocità variabile



Controllo di MOTORI

18 May 2015 15:32

W ÷ KW

·) Servomotori

[Azionamenti meccanici, robotica industriale]

→ risposta veloce

→ posizione e/o velocità precise

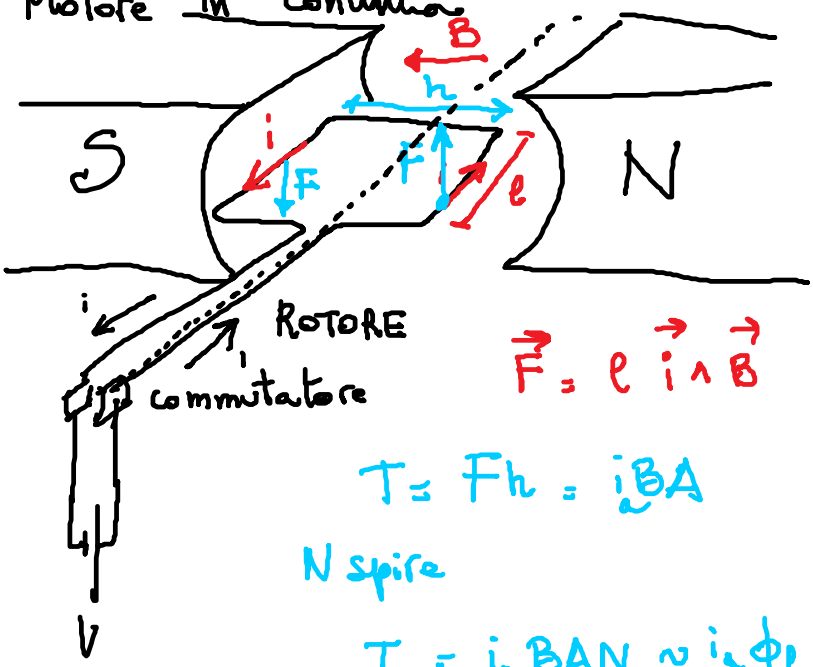
·) Motori a velocità variabile (es. impianto di condizionamento)

→ risposta lenta (il sistema è lento)

→ controllo di velocità

18 May 2015 15:47

Motore in continua



$$\vec{F} = l\,\vec{i} \wedge \vec{B}$$

$$T = Fh = i_a BA$$

N spire

$$T = i_a \underbrace{BAN}_{\text{flusso concatenato con l'avvolgimento del rotore}} \sim i_a \phi_f$$

18 May 2015 15:46

- Motori in continua
- Motore a induzione (asincrono)
- Motore sincrono

18 May 2015 16:04

Potenza meccanica

$$T_{em}\,\omega = K_T\,\phi_f\,i_a\,\omega$$

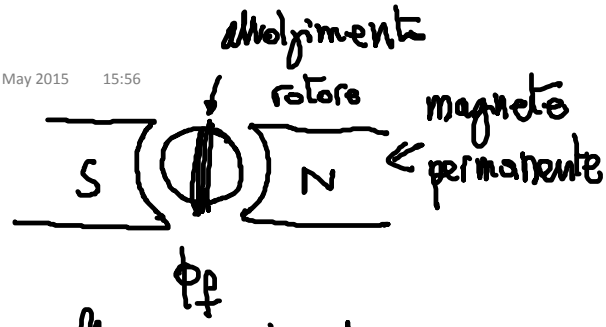
Potenza elettrica assorbita dal rotore

$$e_a i_a = \underbrace{K_e\,\phi_f\,\omega}_{e_a}\,i_a$$

in assenza di perdite. $K_T = K_e$ [Potenza elettrica = Potenza meccanica]

in presenza di perdite $K_T < K_e$

18 May 2015 15:56



flusso concatenato con avvolgimento rotore

$$T_{em} = K_t\,\phi_f\,i_a$$

K_t: costante di coppia

(si può modulare ϕ_f)

FORZA ELETTROMOTRICE INDOTTA SULL'AVVOLGIMENTO DEL ROTORE

$$e_a \propto \phi_f\,\omega$$

$$e_a = K_e\,\phi_f\,\omega$$

K_e: costante di tensione

Condizioni stazionarie

18 May 2015 16:14

$$\begin{cases} T_{em} = K_T \phi_f I_a \\ E_a = K_e \phi_f \omega \end{cases}$$

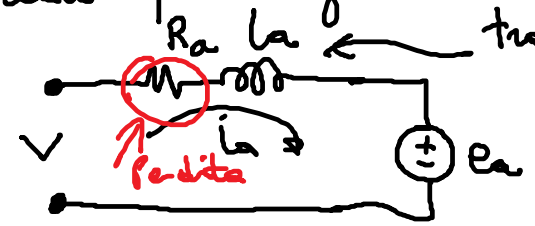
$$V = R_a I_a + E_a$$

$$V = \frac{R_a T_{em}}{K_T \phi_f} + K_e \phi_f \omega$$

$$\omega = \frac{1}{K_e \phi_f}\left[V - \frac{R_a T_{em}}{K_T \phi_f}\right]$$

18 May 2015 16:07

circuito eq. avvolgimento del rotore



a valle del commutatore

a monte del commutatore

$$V = R I_a + E_a$$

$$T_{em} = T_w + \underbrace{B\omega}_{\text{attrito}} + J\dot{\omega}$$

Perdite

coppia trasferita al carico

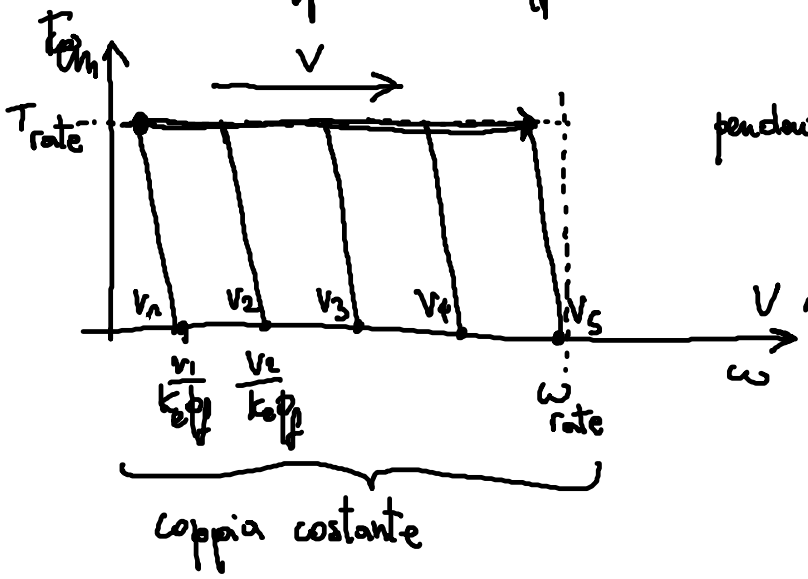
momento d'inerzia

20 May 2015 12:56

Motore in continua con magnete permanente

$$\omega = \frac{1}{K_e \phi_f}\left[V - \frac{R_a}{K_T \phi_f} T_{em}\right]$$

ϕ_f fisso



pendenza: $-\frac{K_T K_e \phi_f^2}{R_a}$

$V \propto E_a \propto \omega$

$i_a \propto T_{em}$

FRENATA

18 May 2015 16:20

$V = R_a I_a + E_a$

poniamo $\omega > 0$ $E_a > 0$

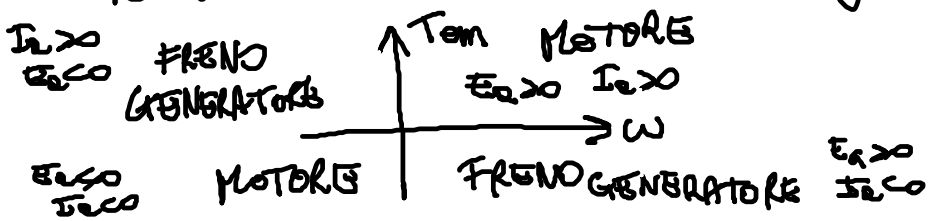
se $V > E_a \rightarrow I_a > 0$ MOTORE ($T_{em} > 0$)

se $V < E_a \leftarrow I_a < 0$ FRENO ($T_{em} < 0$)

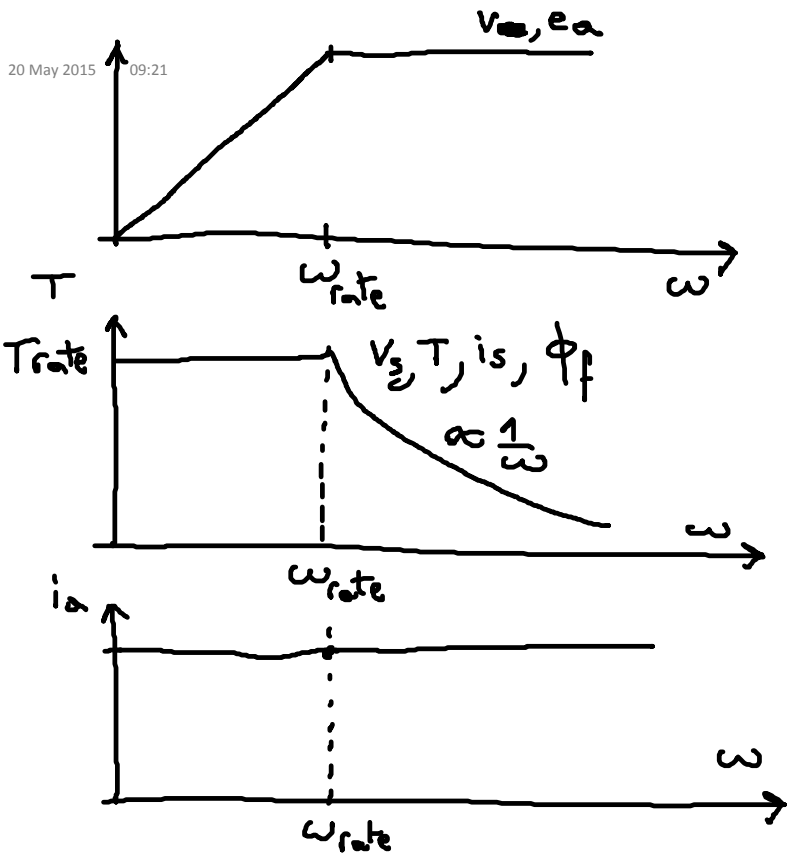
se $\omega < 0$ $E_a < 0$

se $V < E_a \rightarrow I_a < 0$ MOTORE ($T_{em} < 0$)

se $V > E_a \rightarrow I_a > 0$ FRENO ($T_{em} > 0$)



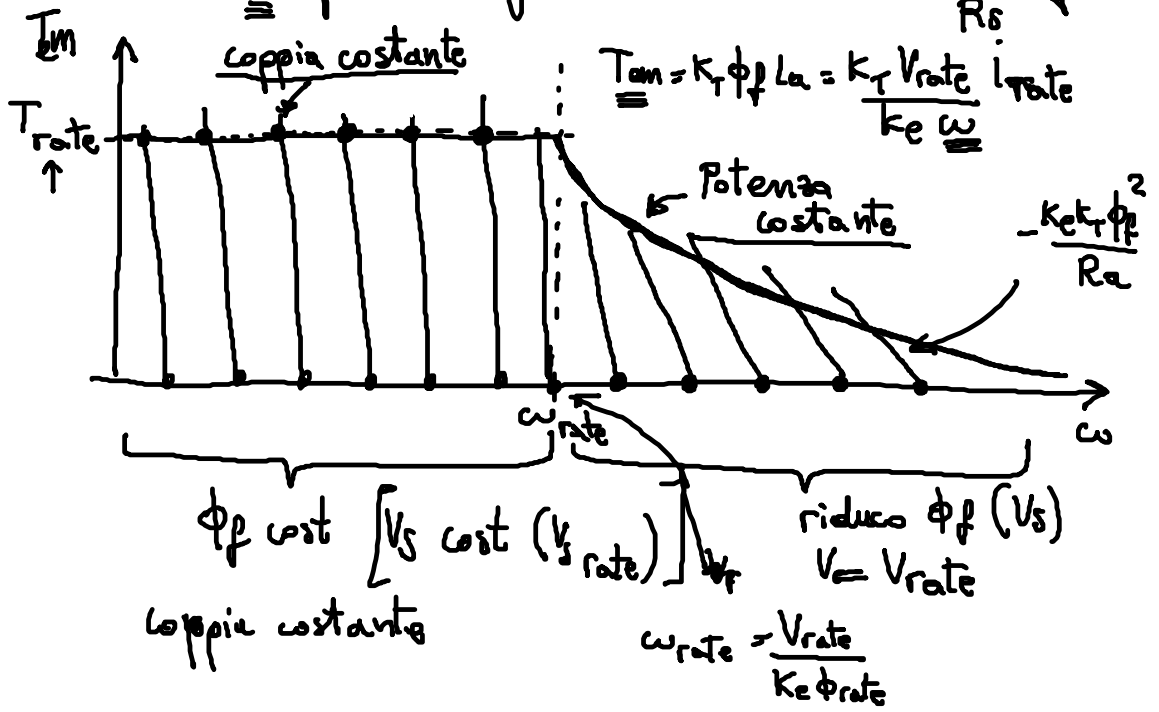
20 May 2015 09:21



20 May 2015 09:10

Motore con eccitazione indipendente

- v per il rotore: i_a ← (avvolgimento di armatura)
- V_s per l'avvolgimento di statore: $i_s = \frac{V_s}{R_s}$ $\phi_f \propto i_s$



20 May 2015 09:31

$$\omega_m = V \frac{k_T \phi_f}{(R_a + L_a s)(Js + B) + k_T k_e \phi_f^2} \cdots$$

2 poli
nessuno zero

approx: $B \sim 0$

$$\frac{L_a}{R_a} = \tau_e \qquad \frac{\omega_m}{V} = \frac{k_T \phi_f}{R_a\left[\left[1+\tau_e s\right] Js + \frac{k_e k_T \phi_f^2}{R_a}\right]}$$

$$= \frac{k_T \phi_f}{k_e k_T \phi_f^2 \left[(1+\tau_e s)\underbrace{\frac{R_a J}{k_e k_T \phi_f^2}}_{\tau_m} + 1\right]}$$

$$= \frac{\cancel{k_T \phi_f}\ 1}{k_e \cancel{k_T} \phi_f^{\cancel{2}} \left[(1+\tau_e s)\tau_m s + 1\right]}$$

Modello di piccolo segnale

20 May 2015 09:24

$$\Rightarrow V = (R_a + L_a s) i_a + e_a$$

$$e_a = k_e \phi_f \omega_m$$

$$T_{em} = k_T \phi_f i_a = T_w + J\omega_m s + B\omega_m$$

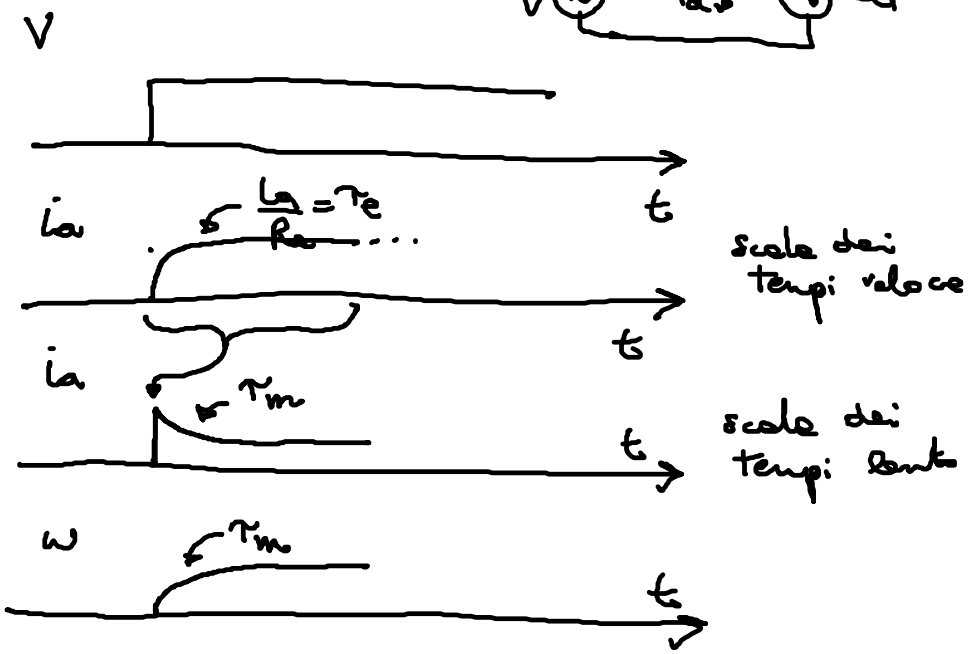
$$i_a = \frac{T_w + J\omega_m s + B\omega}{k_T \phi_f}$$

$$V = \frac{(R_a + L_a s)(T_w + J\omega_m s + B\omega_m)}{k_T \phi_f} + k_e \phi_f \omega_m$$

$$\omega_m \left\{\left[\frac{(B + Js)(R_a + L_a s)}{k_T \phi_f}\right] + k_e \phi_f\right\} = V - \frac{T_w (R_a + L_a s)}{k_T \phi_f}$$

20 May 2015 09:44

Risposta al gradino



20 May 2015 09:40

$$\tau_e = \frac{L_a}{R_a} \qquad \tau_m = \frac{R_a J}{k_e k_T \phi_f^2}$$

costante di tempo elettrica — costante di tempo meccanica

nella pratica $\tau_m \gg \tau_e$

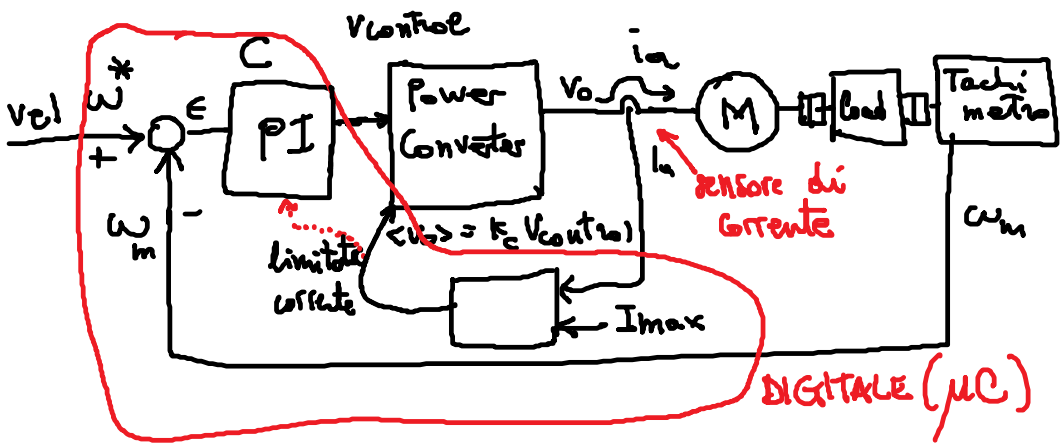
$$\frac{\omega_m}{V} = \frac{1}{k_e \phi_f \left[(1+\tau_e s)\tau_m s + 1\right]} \sim \frac{1}{k_e \phi_f (1+\tau_e s)(1+\tau_m s)}$$

$+\tau_e s$

2 poli REALI molto separati

Sistema di controllo

20 May 2019 09:49



Requisiti del convertitore DC DC

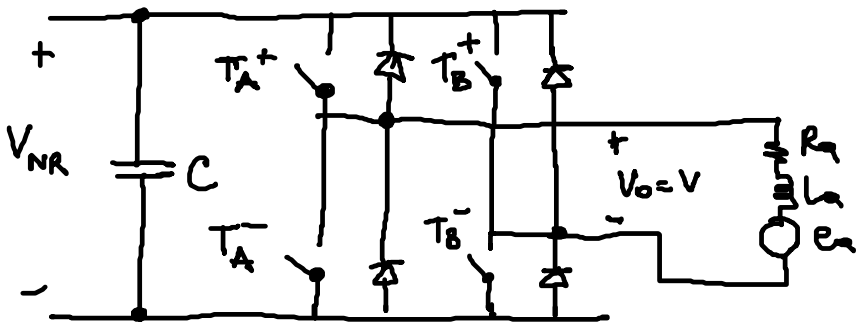
20 May 2019 09:49

- 4 quadranti $v \gtrless 0$, $i_a \gtrless 0$

→ - controllo della corrente

- tensione V lineare con la tensione di controllo

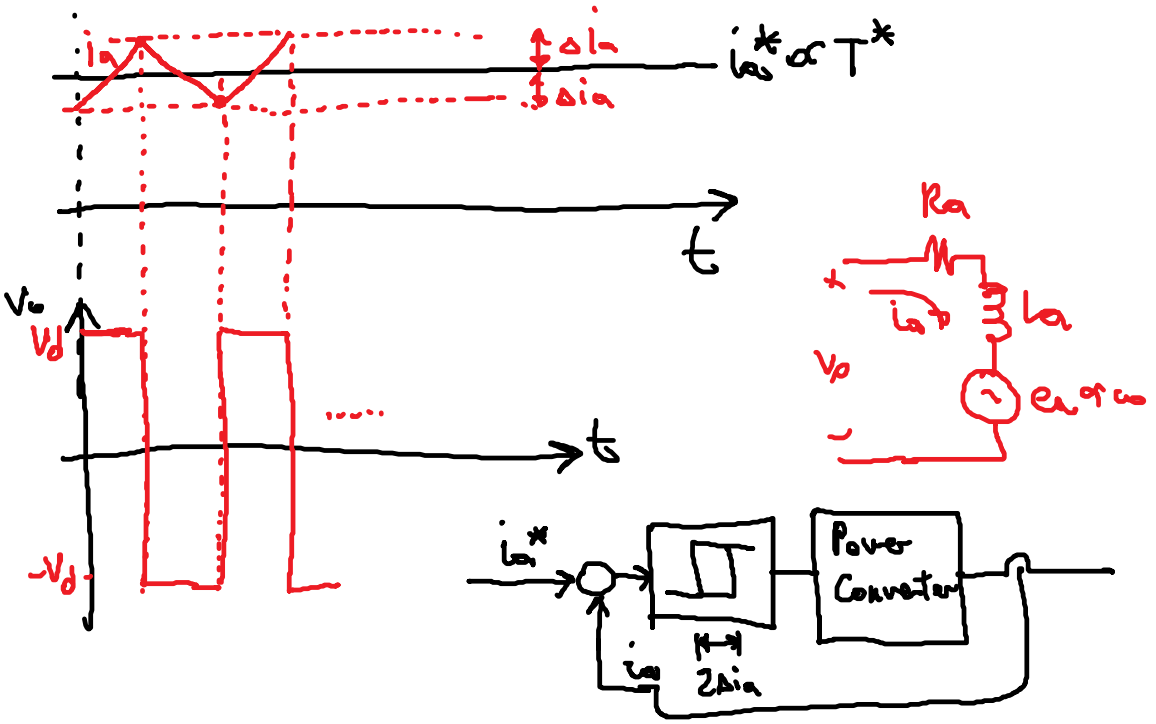
FULL BRIDGE



$$\langle V \rangle = V_{control} \left(\frac{V_{NR}}{V_{max}} \right) = k_c V_{control}$$

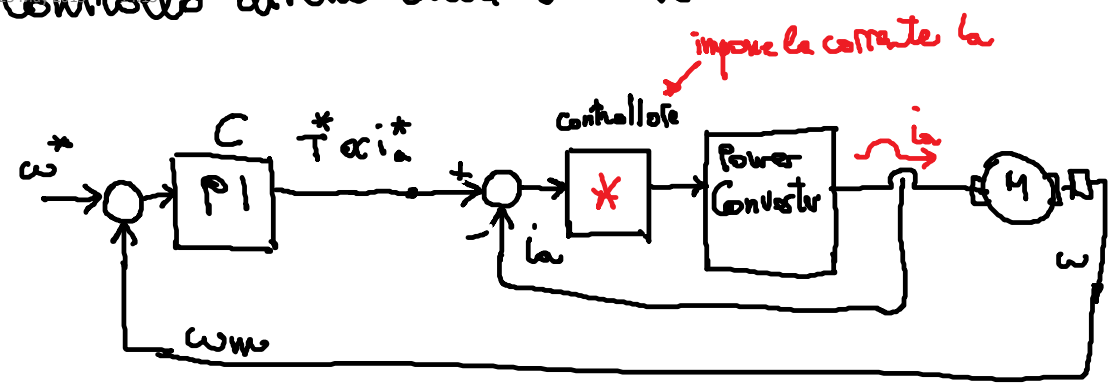
Tolerance band

25 Maggio 2015 14:24



Controllo diretto della corrente

25 Maggio 2015 14:42



Tema Kuo pag 104

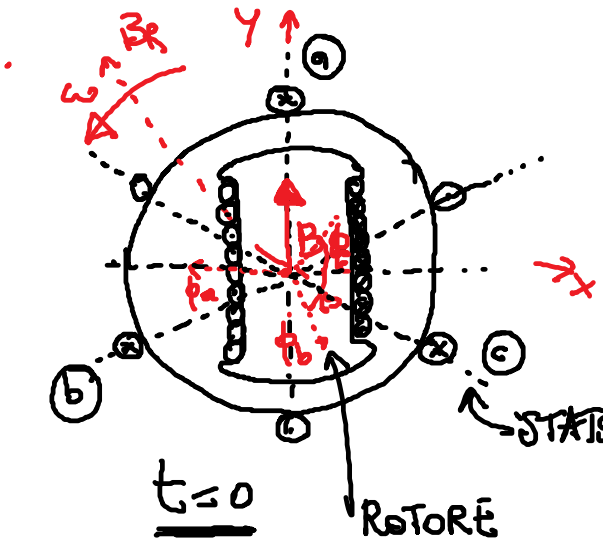
* → 1) Tolerance band (Banda di Tolleranza)

↳ 2) Controllore a frequenza fissa

Motore SINCRONO

25 May 2015 14:36

- servomotori
- motori a velocità variabile



Rotore
- magnete permanente (< kW)
- avvolgimento di campo BRUSHLESS (> kW)

STATORE a 1 polo

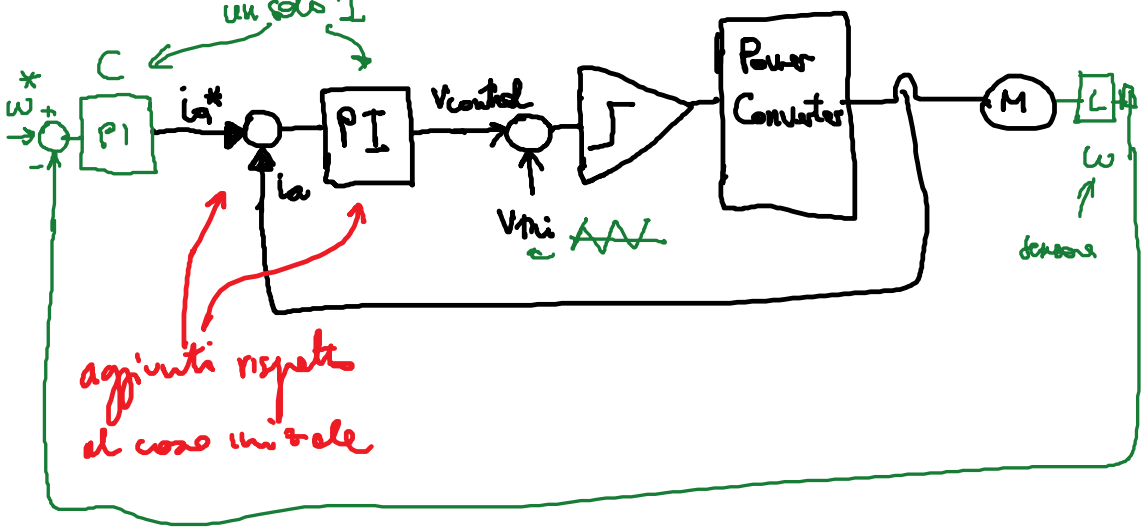
$t = 0$

Motore Sincrono Pagina 114

25 May 2015 14:27

Controllo a Frequenza fissa

un solo "I"



Motori DC Pagina 113

$$+\frac{B}{2}\sin(\omega t+\delta)\underbrace{\cos\left(\frac{4}{3}\pi\right)}_{-1/2}+\frac{B}{2}\cos(\omega t+\delta)\underbrace{\sin\left(\frac{4}{3}\pi\right)}_{-\sqrt{3}/2}$$

Avvolgimenti di statore (di armatura)

25 May 2015 14:45

i_a, i_b, i_c terna di correnti trifase

$$i_a = \sqrt{2}\, I_a \sin(\omega t+\delta)$$
$$i_b = \sqrt{2}\, I_a \sin\left(\omega t + \frac{2}{3}\pi+\delta\right)$$
$$i_c = \sqrt{2}\, I_a \sin\left(\omega t + \frac{4}{3}\pi+\delta\right)$$

CAMPO MAGNETICO RISULTANTE

direzione x (componente x) [B amp. max. campo magnetico di 1 avv]

$$B_{Rx} = \underbrace{-B\sin(\omega t+\delta)}_{a} + \underbrace{\frac{1}{2}B\sin\left(\omega t+\frac{2}{3}\pi+\delta\right)}_{b} + \underbrace{\frac{1}{2}B\sin\left(\omega t+\frac{4}{3}\pi+\delta\right)}_{c}$$

$$B_{Rx} = -B\sin(\omega t+\delta) + \frac{B}{2}\sin(\omega t+\delta)\overset{-1/2}{\cos\left(\frac{2}{3}\pi\right)} + \frac{B}{2}\cos(\omega t+\delta)\overset{\sqrt{3}/2}{\sin\left(\frac{2}{3}\pi\right)} =$$

$$+B\sin(\omega t+\delta)\cos\left(\frac{4}{3}\pi\right) + B\cos(\omega t+\delta)\sin\left(\frac{4}{3}\pi\right)$$

ϕ_{fa} flusso del campo magnetico del ROTORE concatenato con l'avvolgimento di statore (a)

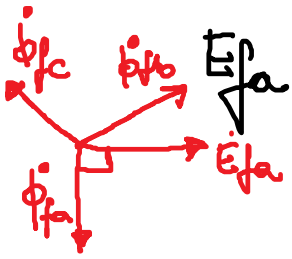
25 May 2015 15:03

$$\phi_{fa} = \phi_f \sin \omega t$$

ampiezza

e_{fa} forza elettromotrice indotta sull'avv. (a)

$$e_{fa} = N_S \frac{d\phi_{fa}}{dt} = N_S \phi_f \omega \cos \omega t$$



E_{fa} ← valore efficace $E_{fa} = \frac{N_S \phi_f \omega}{\sqrt{2}}$

$$B_{Rx} = -\frac{3}{2} B \sin(\omega t + \delta)$$

25 May 2015 14:53

Componente (y)

$$B_{Ry} = \underbrace{-\frac{\sqrt{3}}{2} B \sin\left(\omega t + \frac{2}{3}\pi + \delta\right)}_{b} + \underbrace{\frac{\sqrt{3}}{2} B \sin\left(\omega t + \frac{4}{3}\pi + \delta\right)}_{c}$$

$$B_{Ry} = \cancel{-\frac{\sqrt{3}}{2} B \sin(\omega t + \delta) \cos\frac{2}{3}\pi} - \frac{\sqrt{3}}{2} B \cos(\omega t + \delta) \sin\frac{2}{3}\pi$$

$$\cancel{+\frac{\sqrt{3}}{2} B \sin(\omega t + \delta) \cos\frac{4}{3}\pi} + \frac{\sqrt{3}}{2} B \cos(\omega t + \delta) \sin\frac{4}{3}\pi$$

$-\frac{1}{2}$ $\frac{\sqrt{3}}{2}$ $-\frac{1}{2}$ $-\frac{\sqrt{3}}{2}$

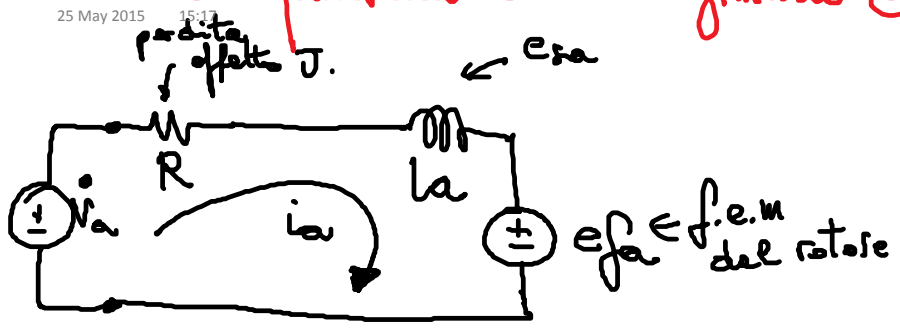
$$B_{Ry} = -\frac{3}{2} B \cos(\omega t + \delta)$$

velocità angolare ω

ampiezza $\frac{3}{2} B$

Circuito equivalente dell'avvolgimento ⓐ

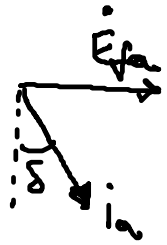
25 May 2015 15:17



$$P_{em} = \dot{i}_a \dot{e}_{fa} = I_a E_{fa} \cos\left(\frac{\pi}{2} - \delta\right) =$$

$$= I_a E_{fa} \sin(\delta) =$$

$$= \frac{I_a N_s \phi_f}{2} \omega \underbrace{\sin(\delta)}_{\text{MAX se } \delta = \pi/2}$$



ϕ_{sa} flusso del campo magnetico rotante concatenato con l'avvolgimento ⓐ

25 May 2015 15:09

$$\rightarrow N_s \phi_{sa} = L_a i_a$$

L_a → $\frac{3}{2} L$ ← dell'avvolgimento ⓐ

↖ comprende l'effetto dell'accoppiamento mutuo con ⓑ e con ⓒ

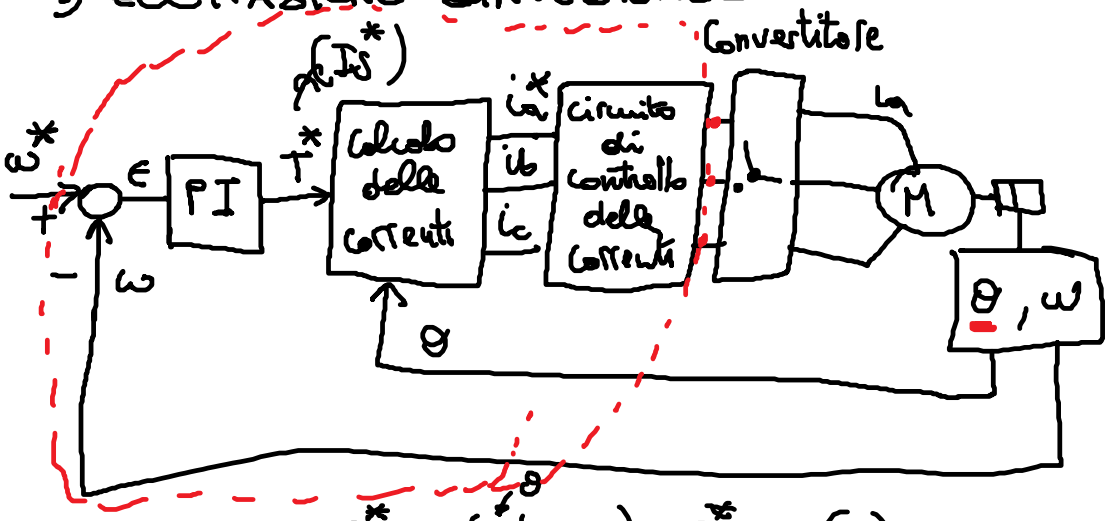
e_{sa} forza e.m indotta sull'avvolgimento ⓐ

$$e_{sa} = N_s \frac{d\phi_{sa}}{dt} = L_a \frac{di_a}{dt}$$

Controllo di motore sincrono

25 May 2015 15:28

3) ECCITAZIONE SINUSOIDALE



$$i_a^*(t) = I_\delta^* \sin\left(\omega t + \frac{\pi}{2}\right) = I_\delta^* \cos(\theta)$$

$$i_b^*(t) = I_\delta^* \sin\left(\omega t + \frac{\pi}{2} + \frac{2}{3}\pi\right) = I_\delta^* \cos\left(\theta + \frac{2}{3}\pi\right)$$

δ

25 May 2015 15:24

$$P_{em_{TOT}} = \frac{3}{2} I_a N_s \phi_f \omega \sin\delta = T_{em}\omega$$

(su 3 avvolgimenti)

$$T_{em} = \frac{3}{2} I_a N_s \phi_f \sin\delta$$

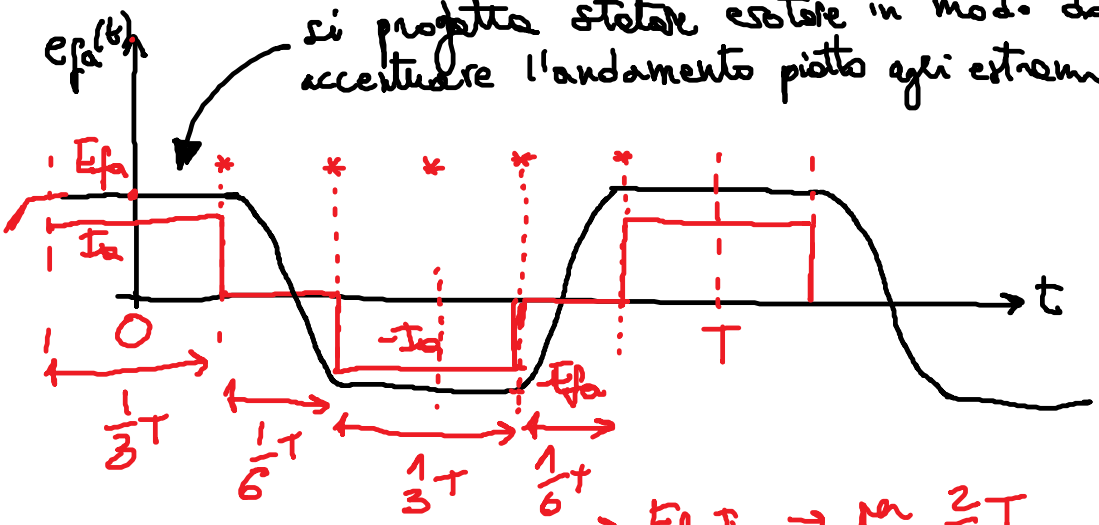
NON DIPENDE DALLA VELOCITÀ

MAX se $\delta = \frac{\pi}{2}$

•) ECCITAZIONE TRAPEZOIDALE

25 May 2015 15:39

si progetta statore esciatore in modo da accentuare l'andamento piatto agli estremi



$P_{em}(t) = e_{fa}(t)\, i_a(t)$ → $= E_{fa} I_a$ → per $\frac{2}{3}T$

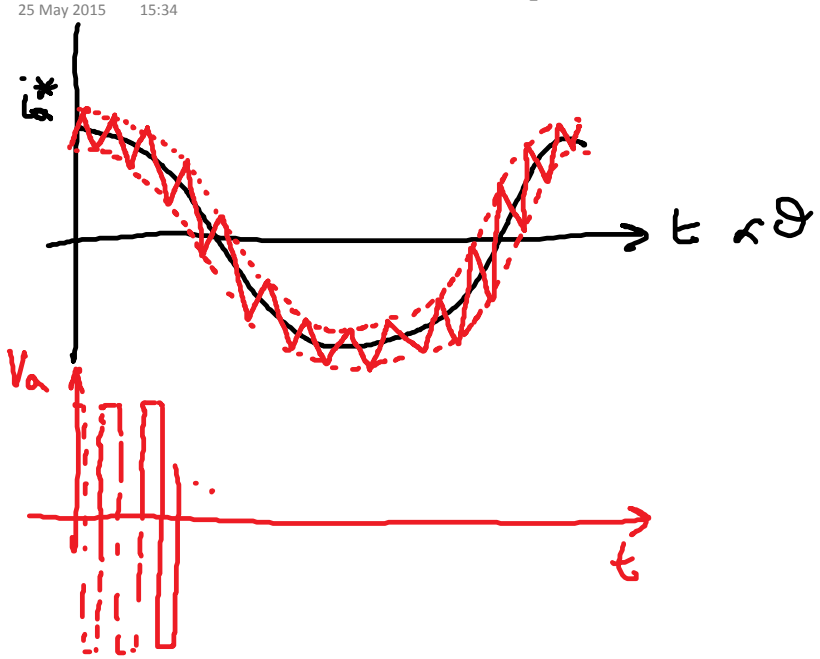
→ $= 0$ per $\frac{1}{3}T$

SOMMANDO LE 3 FASI

$P_{em\,TOT}(t) = 2 E_{fa} I_a \quad \forall t$

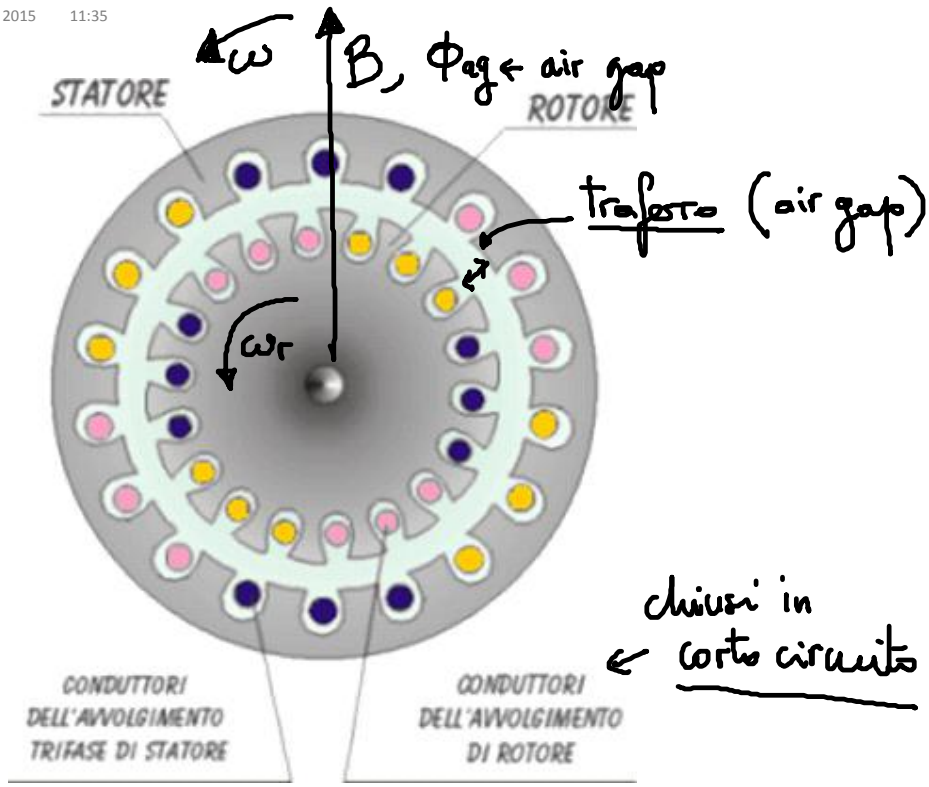
Controllo della corrente con banda di Tolleranza

25 May 2015 15:34

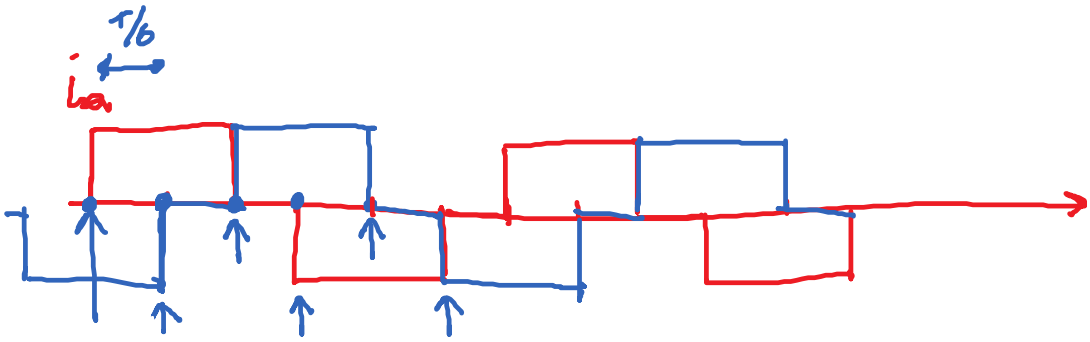


Motore Asincrono (a induzione)

26 May 2015 11:35



25 May 2015 15:55



26 May 2015 11:53

ω velocità angolare di rotazione del campo magnetico di statore

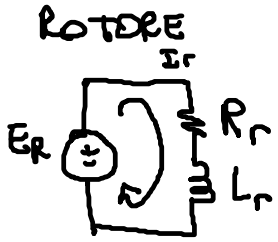
ω_r velocità angolare di rotazione del rotore

$\omega - \omega_r = \omega_{sl}$ velocità (angolare) di slittamento (slipping)

flusso del campo magnetico attraverso l'avvolgimento del rotore

$$\phi_{ag}^{(r)} = \Phi_{ag} \sin(\omega_{sl} t)$$

$$E_r = N_R \frac{d\phi_{ag}}{dt} = N_R \Phi_{ag} \omega_{sl} \cos(\omega_{sl} t)$$



$$\dot{E}_R = \dot{I}_R \left[R_r + j\omega_{sl} L_r \right]$$

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$$N_S \phi_{ag} = L_m I_m$$

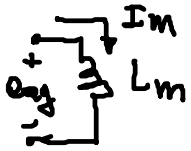
numero di spire di un avvolgimento dello statore

$$\phi_{ag} = \Phi_{ag} \sin \omega t$$

$$e_{ag} = N_S \frac{d\phi_{ag}}{dt} = N_S \Phi_{ag} \omega \cos \omega t$$

f.e.m. indotta

$$= L_m I_m \omega \cos \omega t$$



Circuito equivalente riportato sul primario

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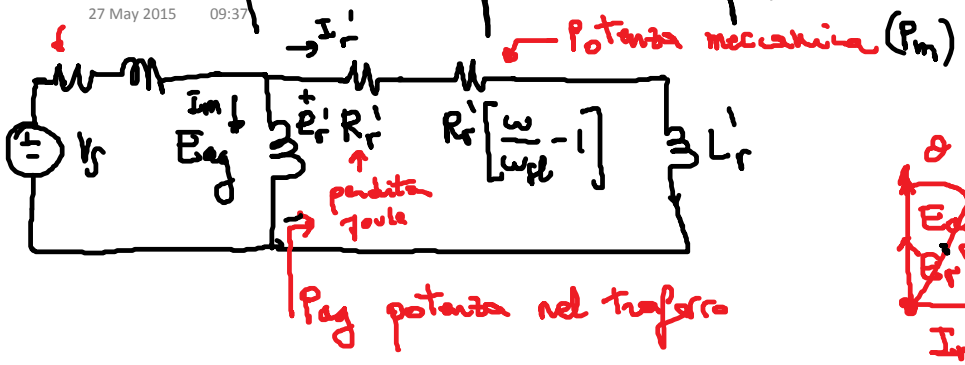
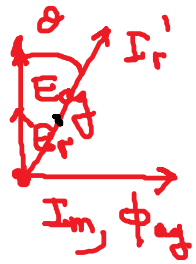




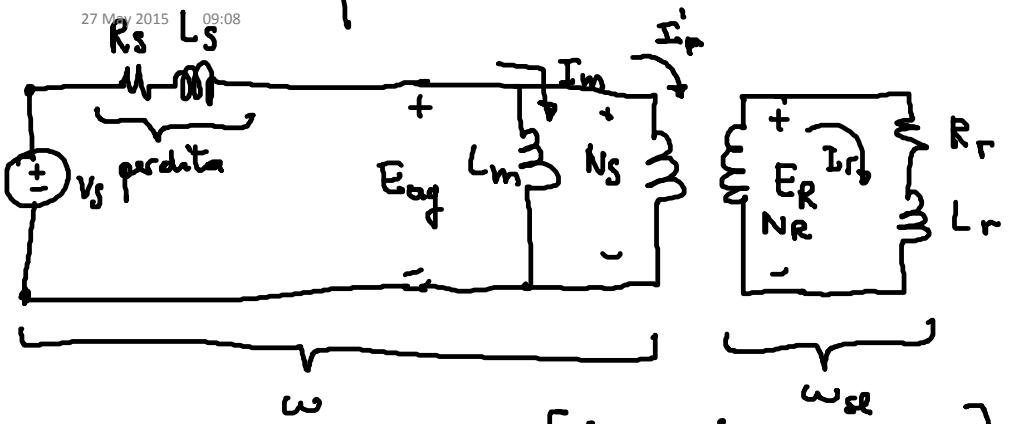
$$\theta = \operatorname{arctg}\left[\frac{\omega_{sl} L_r}{R_r}\right]$$

nell'uno motore $\omega_{sl} \propto \omega$ $\Leftarrow$ $\theta \sim 0$

- $P_{ag} = E_{ag} I_r' \cos\theta$
- $P_m = P_{ag} \dfrac{R_r'\left[\frac{\omega}{\omega_{sl}}-1\right]}{R_r' \omega/\omega_{sl}} = P_{ag}\dfrac{\omega-\omega_{sl}}{\omega} = P_{ag}\dfrac{\omega_r}{\omega}$

Circuito equivalente

27 May 2015 09:08



rapporto spire $a = \dfrac{N_R}{N_S}$ $\quad \left[\dot{E}_R = \dot{E}_{ag}\, a\, \dfrac{\omega_{sl}}{\omega}\right]$

$$\dot{E}_R = \dot{I}_R\left(R_r + j\omega_{sl} L_r\right) \qquad \dot{I}_R' = a\,\dot{I}_R$$

$$\dot{E}_{ag}\, a\, \frac{\omega_{sl}}{\omega} = \frac{\dot{I}_R'}{a}\left(R_r + j\omega_{sl} L_r\right) \Rightarrow \dot{E}_{ag} = \dot{I}_R'\left[\overbrace{\frac{R_r}{a^2}}^{R_r'}\frac{\omega}{\omega_{sl}} + j\omega\overbrace{\frac{L_r}{a^2}}^{L_r'}\right]$$

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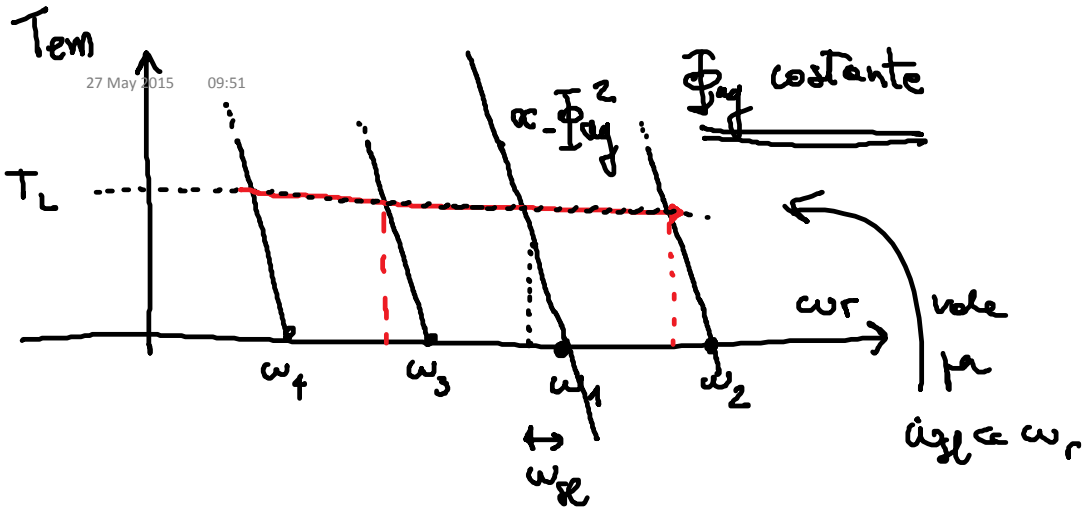


Pilotaggio a coppia costante → ω_{sl} costante

$V_s \simeq E_{ag} = \omega \Phi_{ag}$ $\quad I_s \propto \omega^2 \omega_r$

Dipendenze funzionali:

27 May 2015 09:47

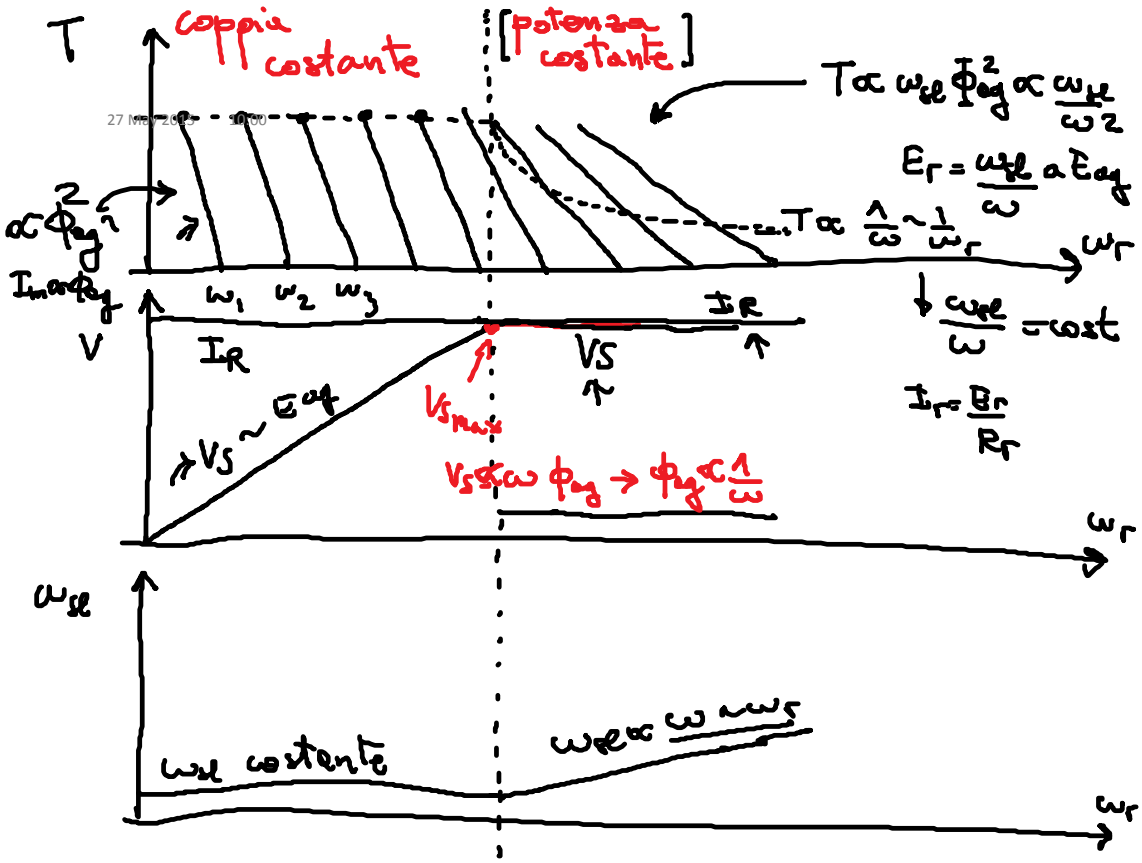
$E_{ag} \propto \omega \Phi_{ag}$ $\qquad E_r = E_{ag} \frac{\omega_{sl}}{\omega} \cdot a \propto \omega_{sl} \Phi_{ag}$

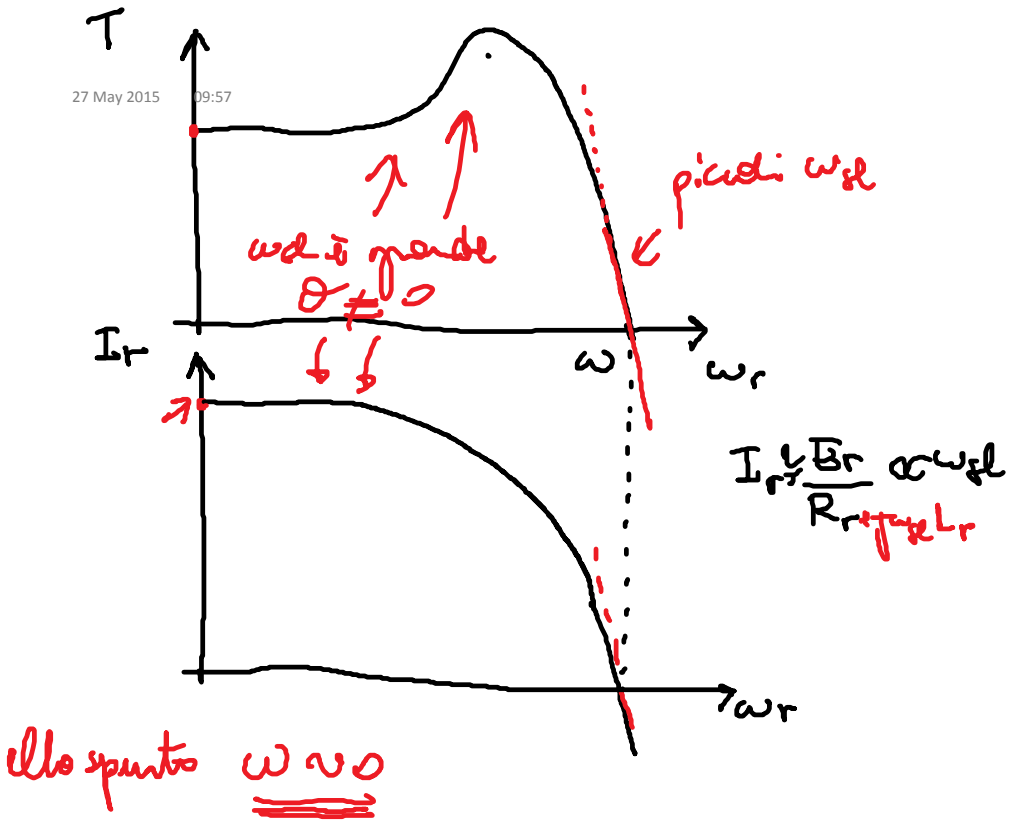
$P_{ag} \propto \omega \omega_{sl} \Phi_{ag}^2$ $\qquad I_r \propto \omega_{sl} \Phi_{ag}$

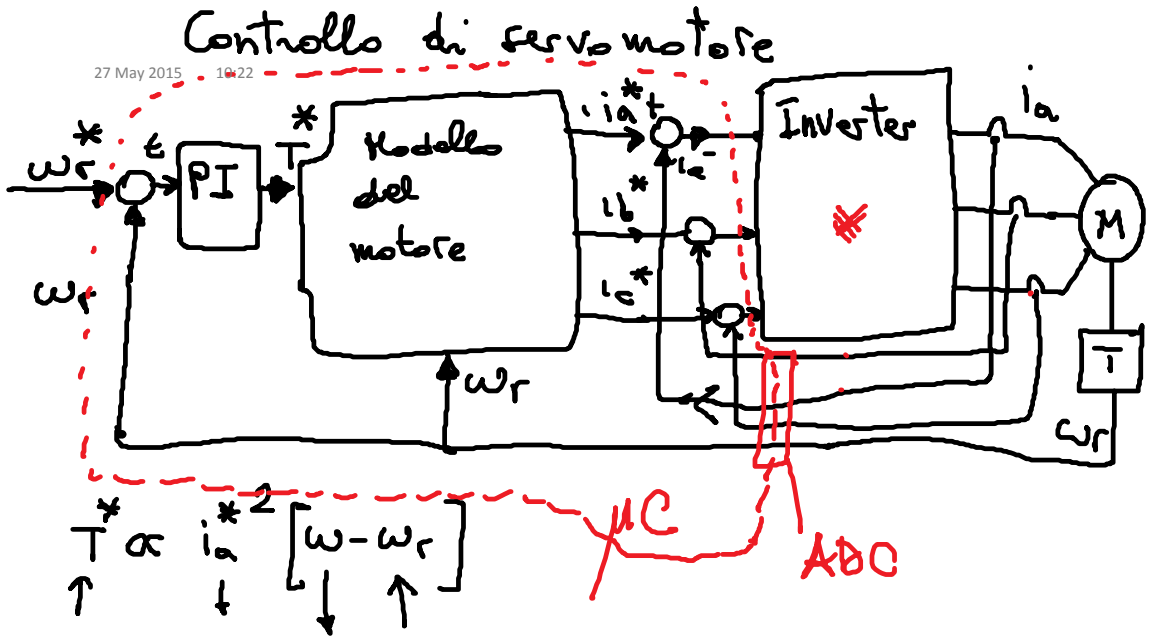
$P_{em} = \frac{P_{ag}\, \omega_r}{\omega} \propto \omega_r \omega_{sl} \Phi_{ag}^2$

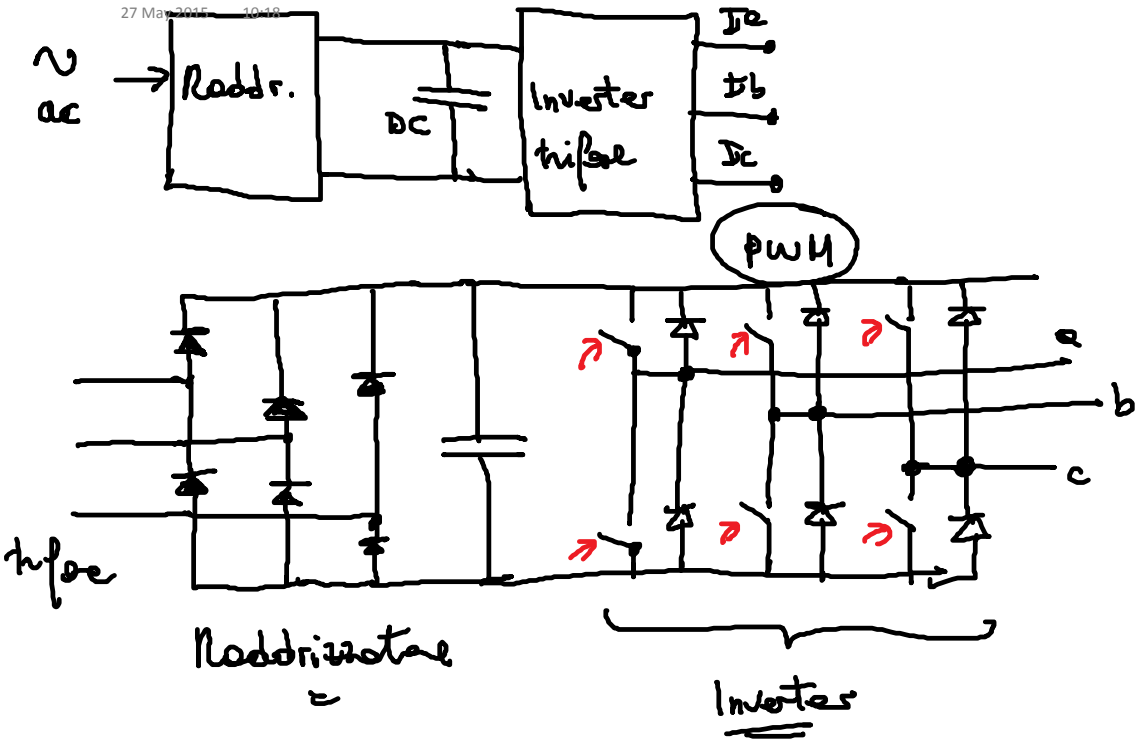
$Tem = \frac{P_{em}}{\omega_r} \propto \omega_{sl} \Phi_{ag}^2$

$Tem = \frac{P_{ag}}{\omega}$


T
coppia costante
[potenza costante]
$T \propto \omega_{sl} \Phi_{ag}^2 \propto \frac{\omega_{sl}}{\omega^2}$
$E_r = \frac{\omega_{sl}}{\omega} a E_{ag}$
$T \propto \frac{1}{\omega} \sim \frac{1}{\omega_r}$
ω_r
$\propto \Phi_{ag}^2$
$I_m \propto \Phi_{ag}$
ω_1 ω_2 ω_3
I_R
$\frac{\omega_{sl}}{\omega} = cost$
V
I_R
V_S
$V_{S_{max}}$
$V_S \sim E_{ag}$
$I_r = \frac{E_r}{R_r}$
$V_S \propto \omega \Phi_{ag} \rightarrow \Phi_{ag} \propto \frac{1}{\omega}$
ω_r
ω_{sl}
ω_{sl} costante
$\omega_{sl} \propto \omega \sim \omega_r$
ω_r


T
27 May 2015 09:57
ω_{sl} è grande
$\theta \neq 0$
piccoli ω_{sl}
I_r
ω
ω_r
$I_r = \frac{E_r}{R_r + j\omega_{sl} L_r} \propto \omega_{sl}$
allo spunto $\omega \sim 0$
ω_r


Controllo di servomotore
27 May 2015 10:22
PI
Modello del motore
Inverter
M
T
μC
ADC


27 May 2015 10:18
Raddr.
DC
Inverter trifase
PWM
a
b
c
trifase
Raddrizzatore
Inverter