DC-DC Converters

Power Delivery Network of a Smartphone

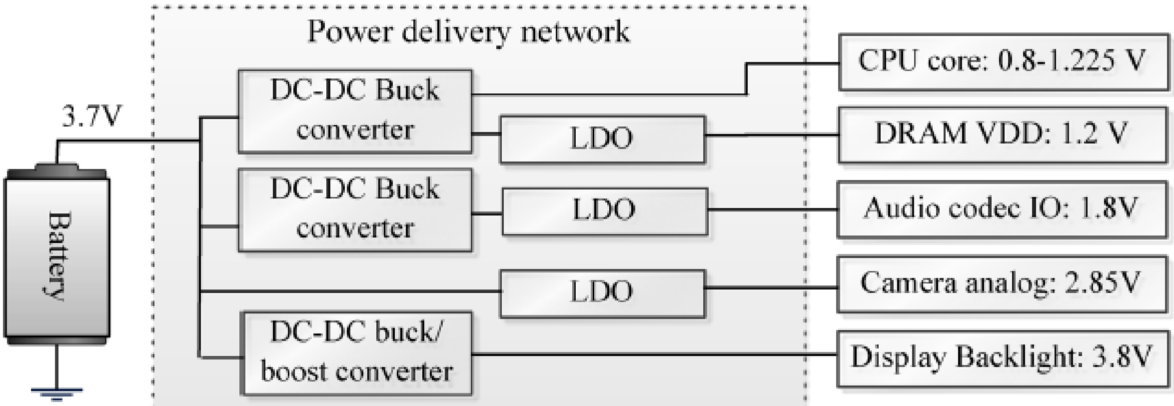

Fig. 1. Conceptual diagram of the PDN in a smartphone platform.

Lee et al., IEEE-TCAD Vol.33, pp, 136, 2015.

DC-DC Converters

Typical uses:

- DC Power supplies
- DC Motor drives
- Portable electronics

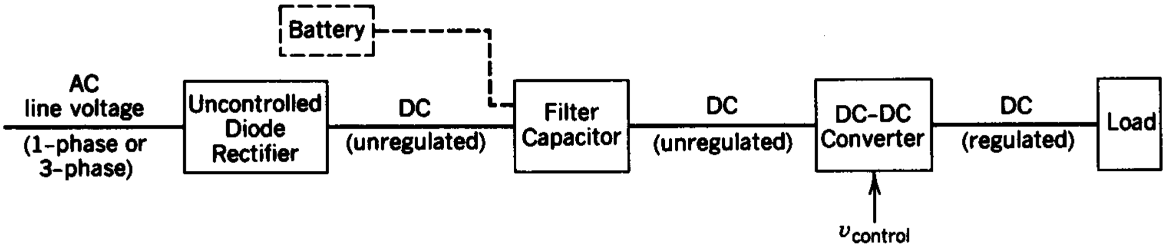

Figure 7-1 A dc–dc converter system.

DC-DC Converters

Typical uses:

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- DC Motor drives
- Portable Electronics

Types of converters

- **Step-down (buck)**
- **Step-up (boost)**
- Buck-boost
- Cuk
- Full-Bridge

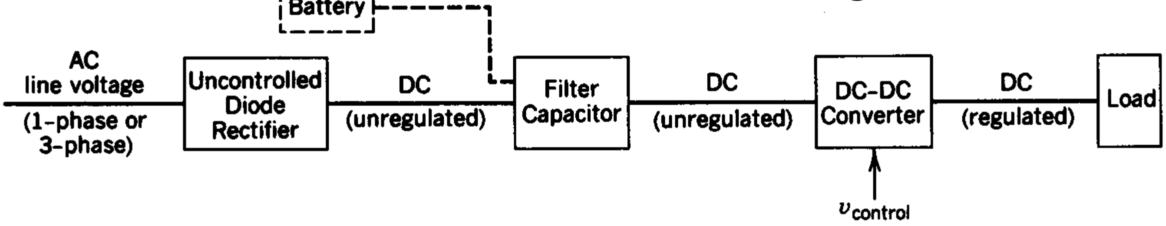

Figure 7-1 A dc–dc converter system.

Ideal concept of step-down converter with PWM* switching

(* Pulse Width Modulation)

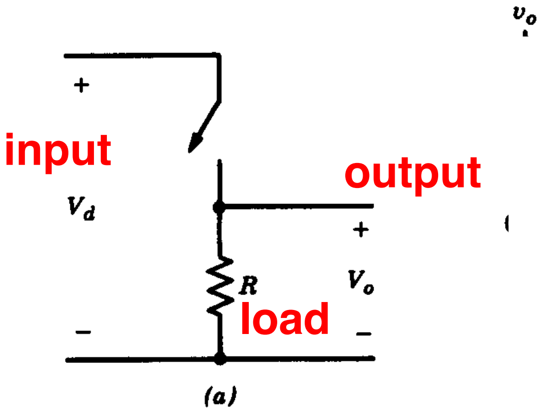

Figure 7-2 Switch-mode dc–dc

Assumptions: Switches, L, C are lossless, DC input has zero internal impedance, load is an equivalent R

Ideal concept of step-down converter with PWM* switching

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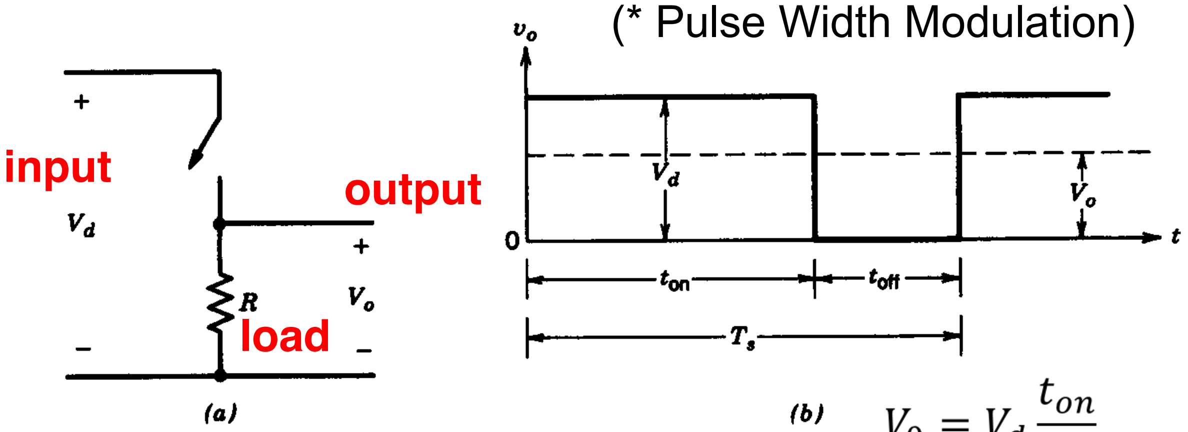

$$V_0 = V_d \frac{t_{on}}{T_s}$$

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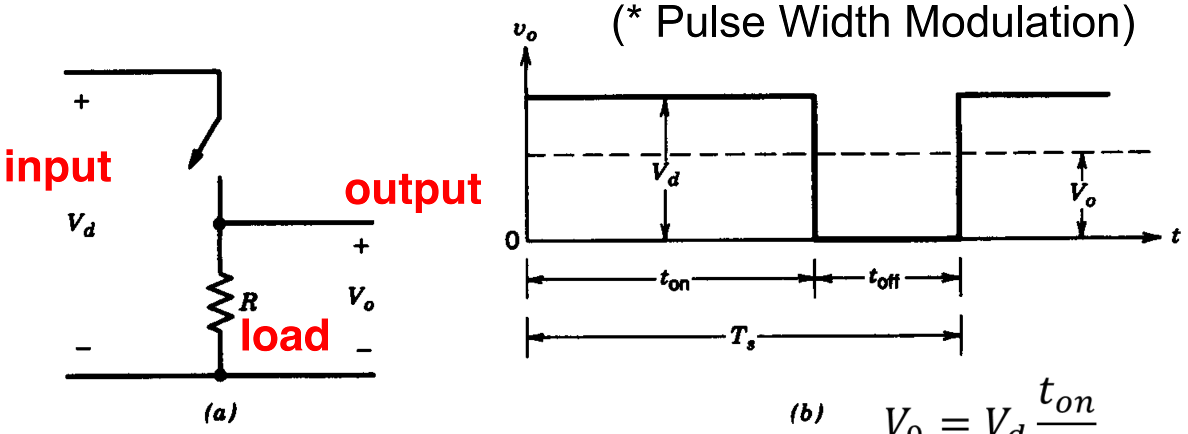

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Figure 7-2 Switch-mode dc−dc conversion.

Assumptions: Switches, L, C are lossless, DC input has zero internal impedance, load is an equivalent R

This cannot work: 1. Load is inductive and can destroy switch by dissipating all stored energy, 2. output voltage must be continuous

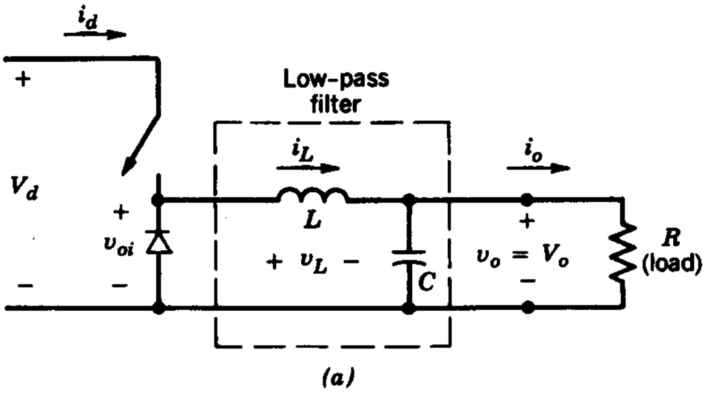
Step-down (buck) converter

DC power supplies, DC motor drives -- $V_o < V_d$

Low-pass filter keeps output voltage constant

Note: 2nd order non dissipative filter

$$f_c = \frac{1}{2\pi}\frac{1}{\sqrt{LC}} \ll f_s$$



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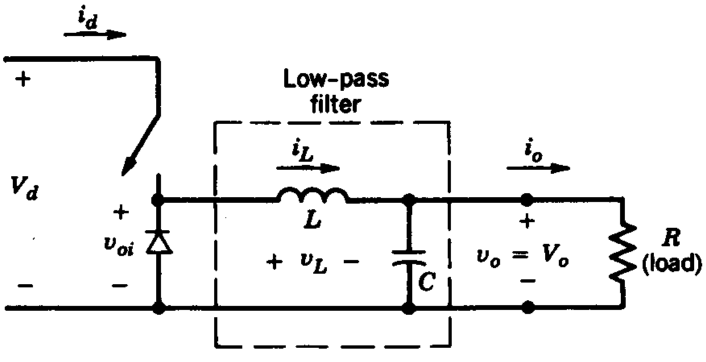
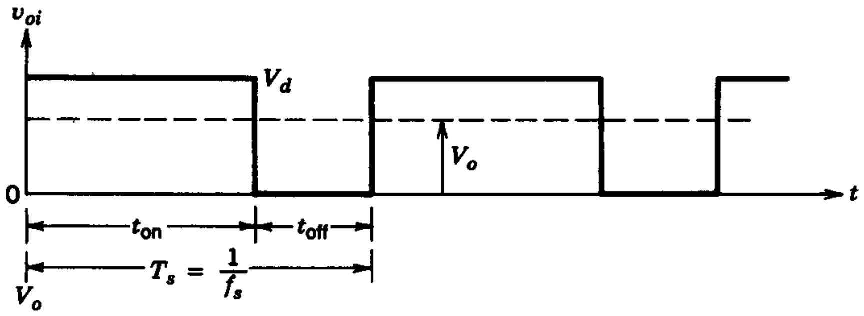


(a)

Diode avoids voltage spike on switch (when switch is off, diode provides current to L)

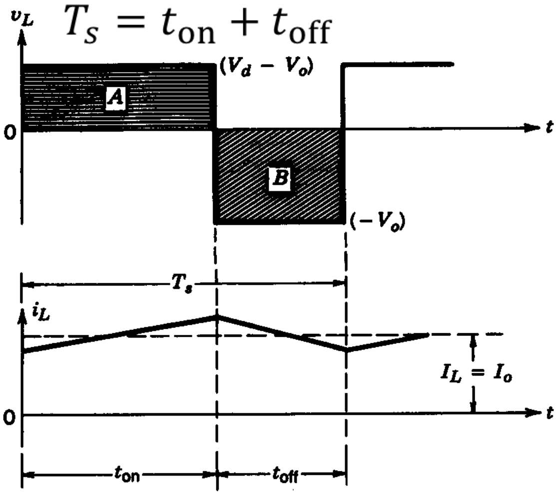


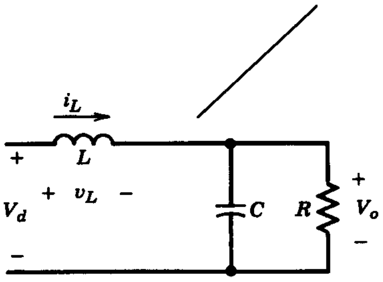
Continuous-conduction mode

Current in *L* is always > 0

- t_{on}: $\frac{dI}{dt} = \frac{V_d - V_o}{L}$

$$T_s = t_{on} + t_{off}$$





(a)

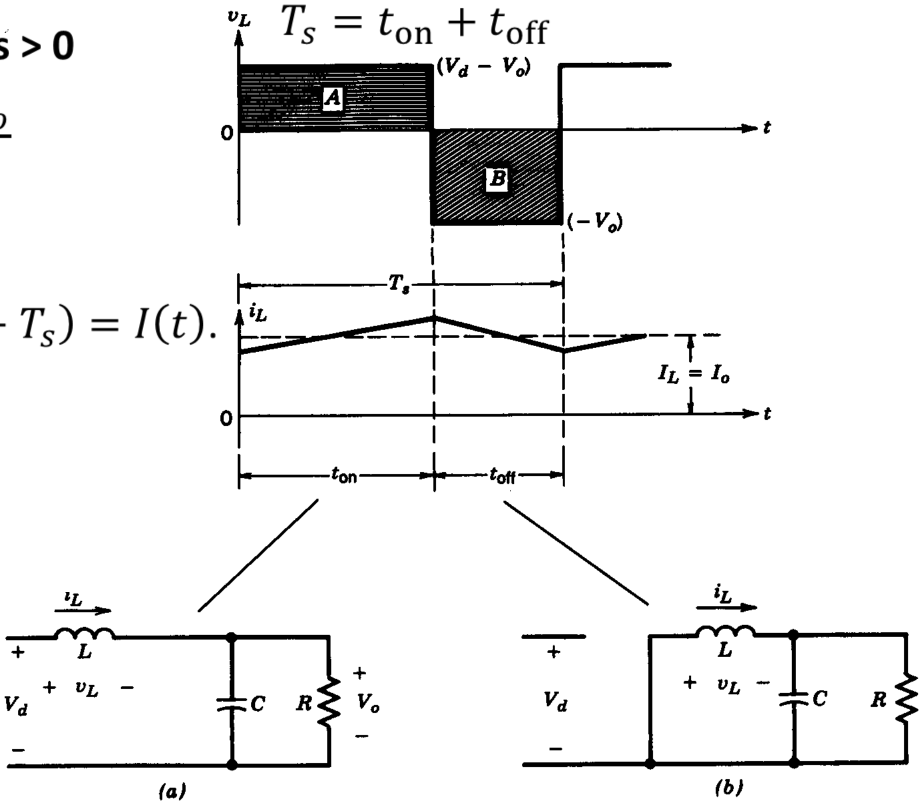
Continuous-conduction mode

Current in *L* is always > 0

- t_{on}: $\frac{dI}{dt} = \frac{V_d - V_o}{L}$
- t_{off}: $\frac{dI}{dt} = -\frac{V_o}{L}$

At steady state: $I(t + T_s) = I(t)$.

$T_s = t_{\text{on}} + t_{\text{off}}$



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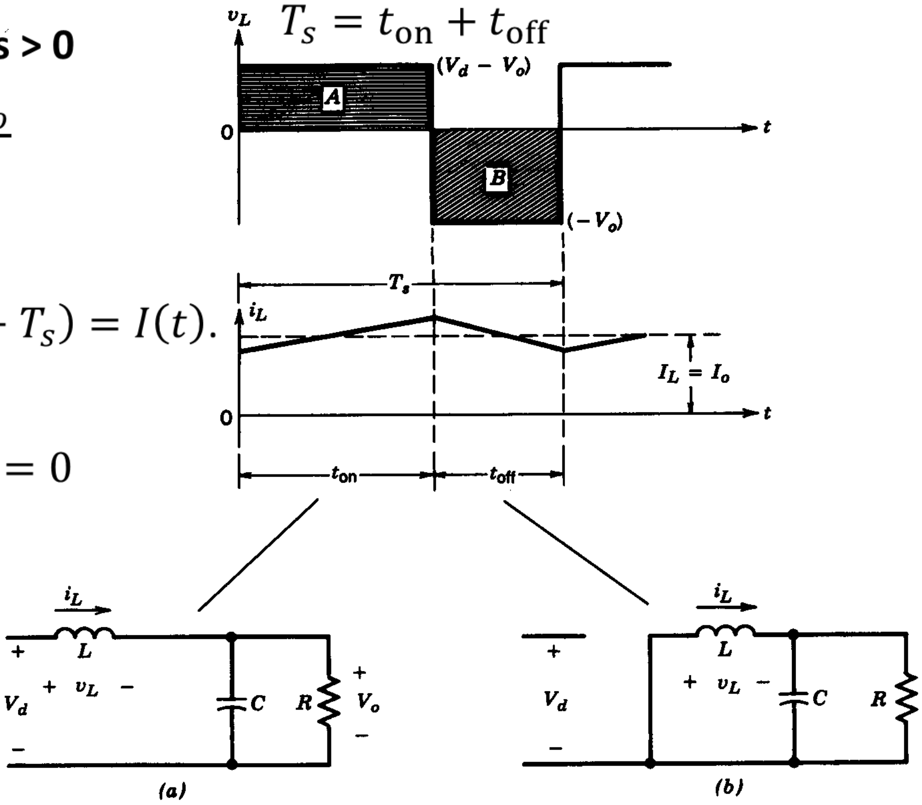
Therefore

$$\frac{V_d - V_o}{L} t_{\text{on}} - \frac{V_0}{L} t_{\text{off}} = 0$$

$T_s = t_{\text{on}} + t_{\text{off}}$



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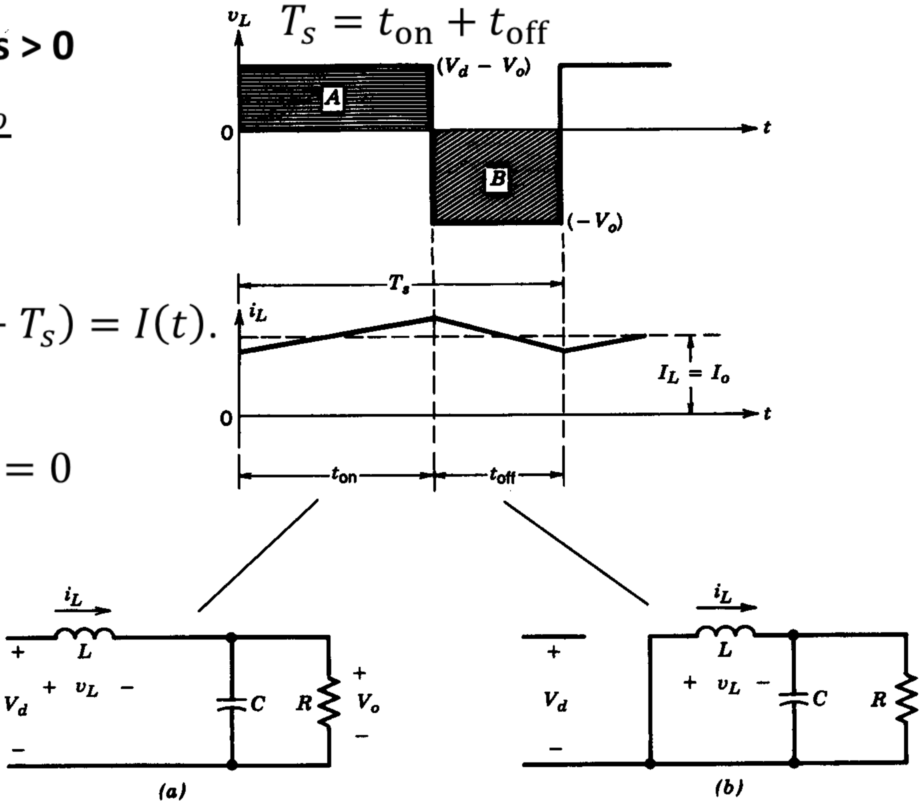
Therefore

$$\frac{V_d - V_o}{L} t_{\text{on}} - \frac{V_0}{L} t_{\text{off}} = 0$$

$$\frac{V_o}{V_d} = \frac{t_{on}}{T_s} = D$$

$T_s = t_{\text{on}} + t_{\text{off}}$



Limit of continuous conduction

If the ripple amplitude $I_{LB} \equiv \frac{I_{peak}}{2} = I_o$, the converter is at the limit of continuous conduction (i.e. $min\{I_L\} = 0$)

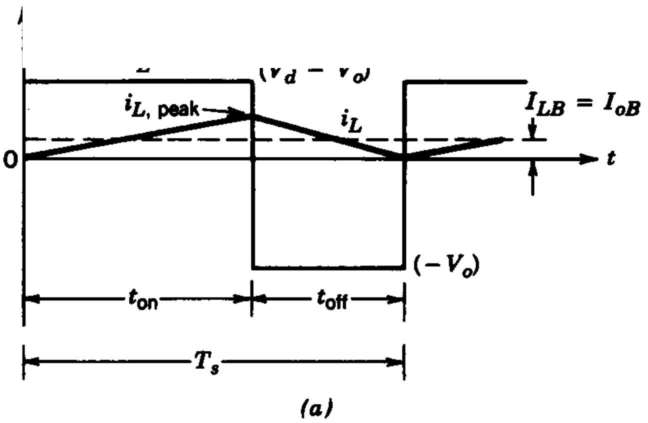


Figure 7-6 Current at the boundary of continuous–discontinuous conduction: (*a*) current waveform; (*b*) I_{LB} versus D keeping V_d constant.

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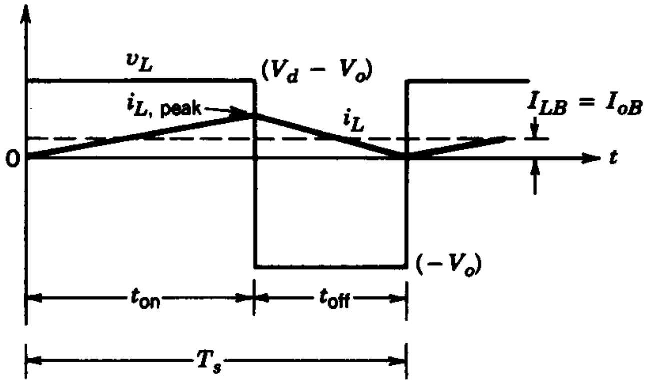
$$I_{LB} \equiv \frac{I_{peak}}{2} = \frac{t_{on}(V_d - V_o)}{2L} =$$



(a) *(b)*

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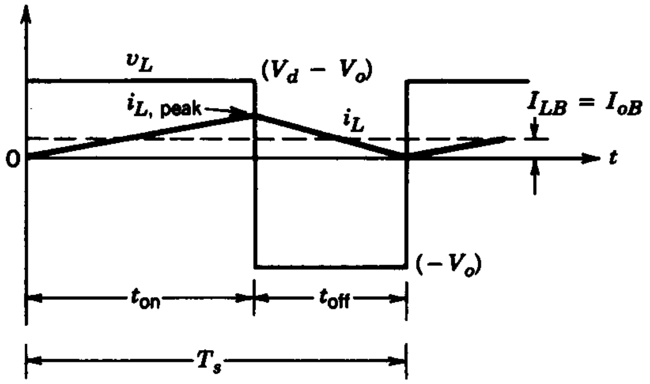
$$I_{LB} \equiv \frac{I_{peak}}{2} = \frac{t_{on}(V_d - V_o)}{2L} = \frac{DT_sV_d(1 - D)}{2L}$$



(a) *(b)*

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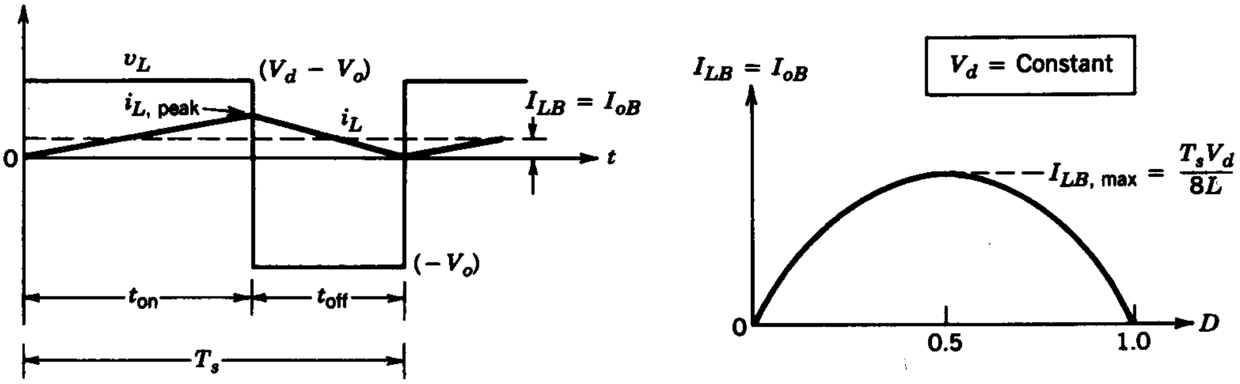


(a) (b)

Figure 7-6 Current at the boundary of continuous–discontinuous conduction: (*a*) current waveform; (*b*) I_{LB} versus D keeping V_d constant.

Limits of continuous-discontinuous conduction (constant V_d)

Continuous

$$\frac{V_o}{V_d} = D$$

$$\frac{I_o}{I_{\text{LBmax}}} > 4D(1-D)$$

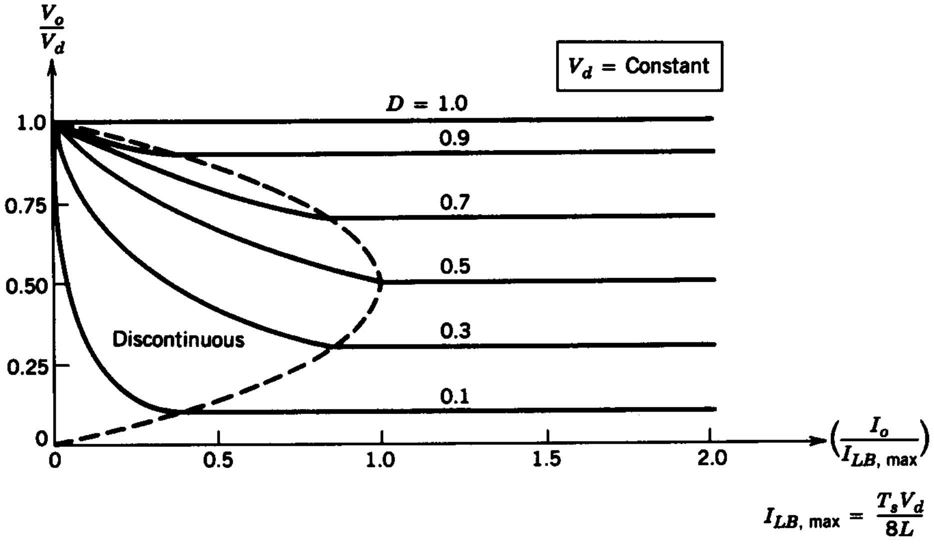


Figure 7-8 Step-down converter characteristics keeping V_d constant.

Discontinuous-conduction mode with constant V_d

Motor drives

$$I_{\text{peak}} = \frac{(V_d - V_o)DT_s}{L} = \frac{V_o\Delta_1 T_s}{L}$$

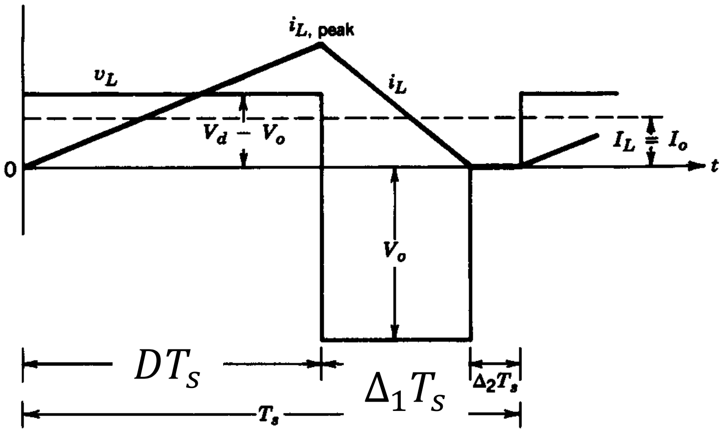


Figure 7-7 Discontinuous conduction in step-down converter.

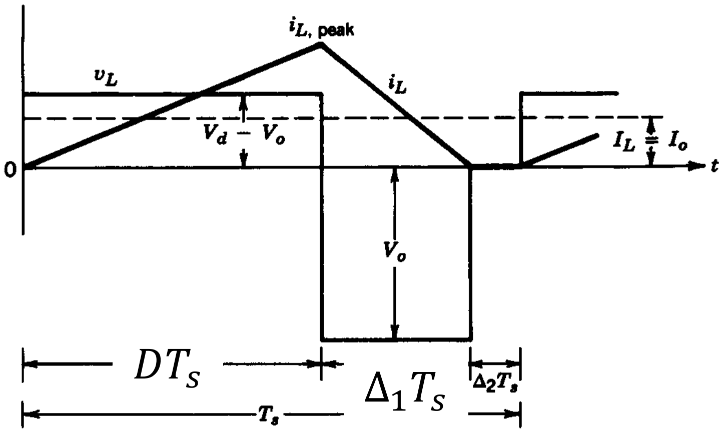
Discontinuous-conduction mode with constant V_d

Motor drives

$$I_{\text{peak}} = \frac{(V_d - V_o)DT_s}{L} = \frac{V_o\Delta_1 T_s}{L}$$

⇒

$$\frac{V_o}{V_d} = \frac{D}{(D + \Delta_1)}$$



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Discontinuous-conduction mode with constant V_d

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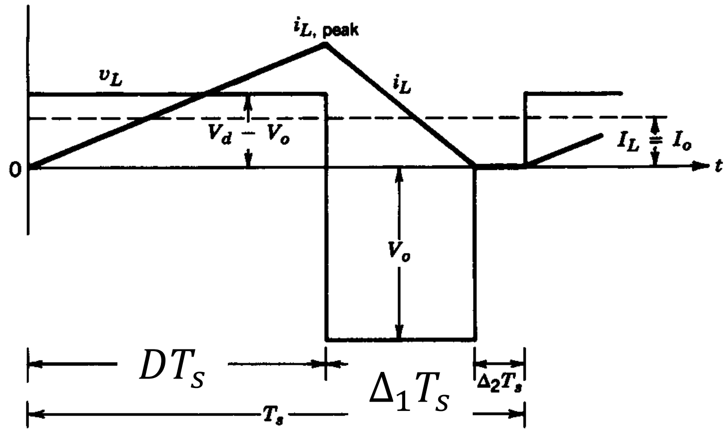
$$I_{\text{peak}} = \frac{(V_d - V_o)DT_s}{L} = \frac{V_o \Delta_1 T_s}{L} \quad \Rightarrow \quad \frac{V_o}{V_d} = \frac{D}{(D + \Delta_1)}$$

$$I_{\text{peak}} = \frac{V_d T_s}{L} \frac{D\Delta_1}{D + \Delta_1}$$



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Discontinuous-conduction mode with constant V_d

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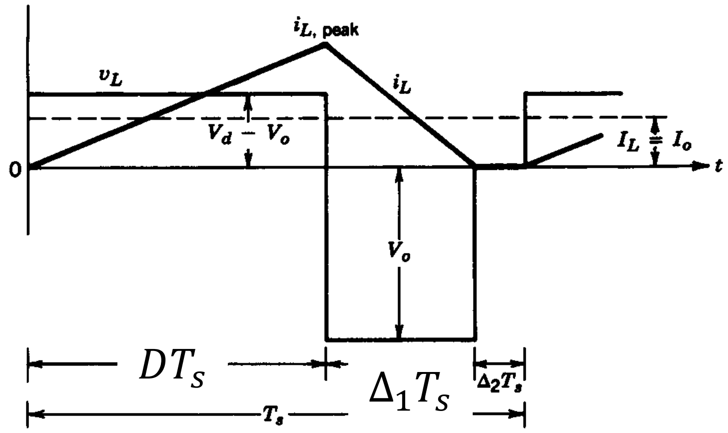
$$I_{\text{peak}} = \frac{V_d T_s}{L} \frac{D\Delta_1}{D + \Delta_1}$$

$$I_{\text{peak}} = 8 I_{\text{LBmax}} \frac{D\Delta_1}{D + \Delta_1}$$



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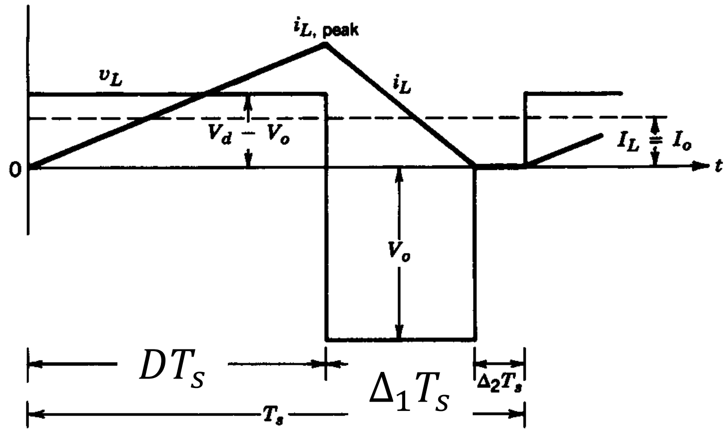
$$I_{\text{peak}} = 8 I_{\text{LBmax}} \frac{D\Delta_1}{D + \Delta_1}$$

$$I_o T_s = \frac{I_{\text{peak}}(D + \Delta_1)T_s}{2}$$



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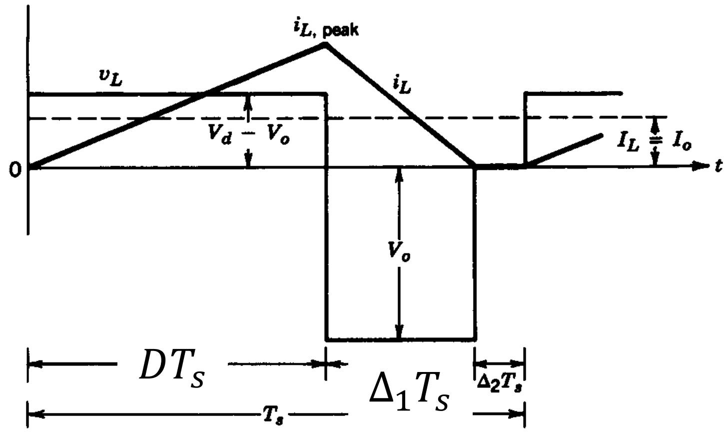

Figure 7-7 Discontinuous conduction in step-down converter.

$$I_o = 4 I_{\text{LBmax}} D \Delta_1$$

Discontinuous-conduction mode with constant V_d

Motor drives

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$$\frac{V_o}{V_d} = \frac{D}{(D + \Delta_1)}$$

$$I_{\text{peak}} = \frac{V_d T_s}{L} \frac{D\Delta_1}{D + \Delta_1}$$

$$I_{\text{peak}} = 8I_{\text{LBmax}} \frac{D\Delta_1}{D + \Delta_1}$$

$$I_o T_s = \frac{I_{\text{peak}}(D + \Delta_1)T_s}{2}$$

$$I_o = 4I_{\text{LBmax}} D\Delta_1$$

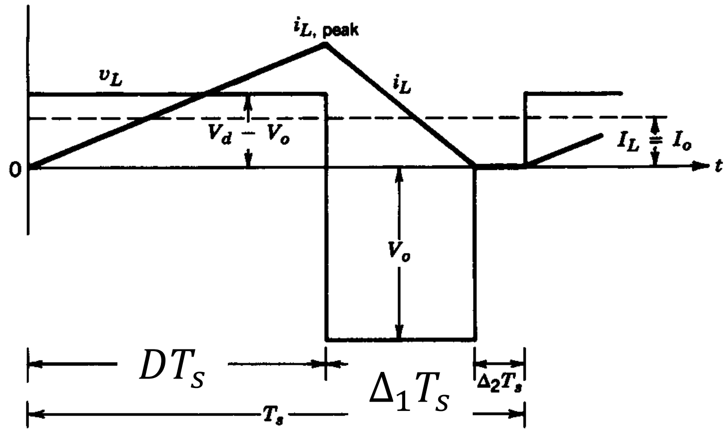

Figure 7-7 Discontinuous conduction in step-down converter.

$$\frac{V_o}{V_d} = \frac{D^2}{D^2 + I_o/(4I_{LBmax})}$$

Limits of continuous-discontinuous conduction (constant Vd)

Continuous

$$\frac{I_o}{I_{\text{LBmax}}} > 4D(1-D)$$

$$\frac{V_o}{V_d} = D$$

Discontinuous

$$\frac{I_o}{I_{\text{LBmax}}} < 4D(1-D)$$

$$\frac{V_o}{V_d} = \frac{D^2}{D^2 + \frac{I_o}{4I_{\text{LBmax}}}}$$

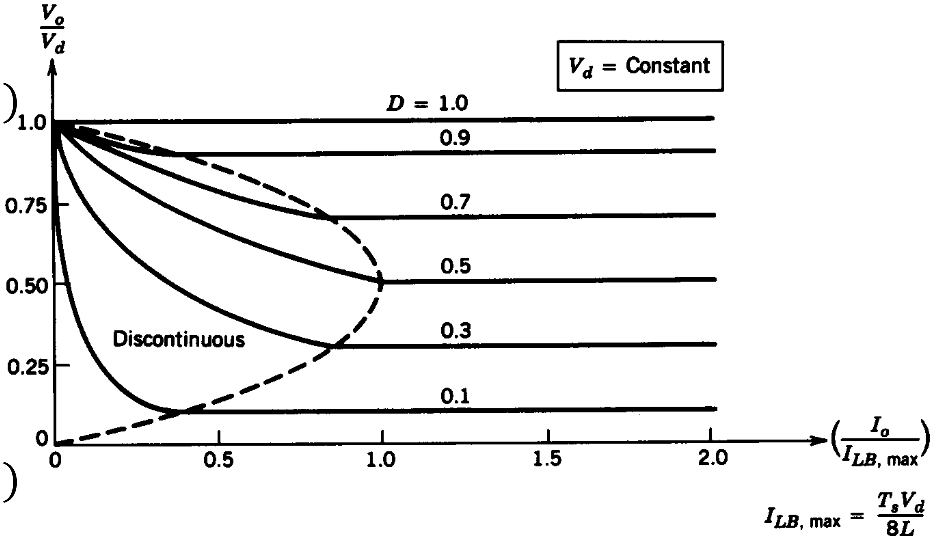

Figure 7-8 Step-down converter characteristics keeping V_d constant.

Discontinuous-conduction with constant Vo

DC voltage supply

At the limit of continuous conduction

$$I_{LB} = \frac{V_o T_s (1 - D)}{2L} = I_{\mathrm{LBmax}} (1 - D)$$

Discontinuous-conduction with constant Vo

DC voltage supply

At the limit of continuous conduction

$$I_{LB} = \frac{V_o T_s (1 - D)}{2L} = I_{\mathrm{LBmax}} (1 - D)$$

We can write D explicitly from:

$$I_{\mathrm{peak}} = \frac{V_o \Delta_1 T_s}{L} = 2 I_{\mathrm{LBmax}} \Delta_1$$

Discontinuous-conduction with constant Vo

DC voltage supply

At the limit of continuous conduction

$$I_{LB} = \frac{V_o T_s (1 - D)}{2L} = I_{\mathrm{LBmax}}(1 - D)$$

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$$I_{\mathrm{peak}} = \frac{V_o \Delta_1 T_s}{L} = 2 I_{\mathrm{LBmax}} \Delta_1$$

$$I_o = \frac{I_{\mathrm{peak}}(D + \Delta_1)}{2} = I_{\mathrm{LBmax}} \Delta_1 (D + \Delta_1) \qquad \frac{V_d}{V_o} = \frac{D + \Delta_1}{D}$$

Discontinuous-conduction with constant Vo

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At the limit of continuous conduction

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$$\frac{I_o}{I_{\mathrm{LBmax}}} = D^2 \frac{V_d}{V_o} \left(1 - \frac{V_d}{V_o} \right)$$

Discontinuous-conduction with constant Vo

DC voltage supply

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$$I_{LB}=\frac{V_oT_s(1-D)}{2L}=I_{\text{LBmax}}(1-D)$$

We can write D explicitly from:

$$I_{\text{peak}}=\frac{V_o\Delta_1T_s}{L}=2I_{\text{LBmax}}\Delta_1$$

$$I_o=\frac{I_{\text{peak}}(D+\Delta_1)}{2}=I_{\text{LBmax}}\Delta_1(D+\Delta_1)$$

$$\frac{V_d}{V_o}=\frac{D+\Delta_1}{D}$$

$$\frac{I_o}{I_{\text{LBmax}}}=D^2\frac{V_d}{V_o}\left(1-\frac{V_d}{V_o}\right)\Longrightarrow D=\left[\frac{V_o}{V_d}\frac{I_o}{I_{LBmax}}\left(1-\frac{V_d}{V_o}\right)^{-1}\right]^{\frac{1}{2}}$$

Discontinuous-conduction with constant Vo

DC voltage supply

Continuous: $I_o > I_{LB}$

$$D > 1-\frac{I_o}{I_{LBmax}}$$

$$D=\frac{V_o}{V_d}$$

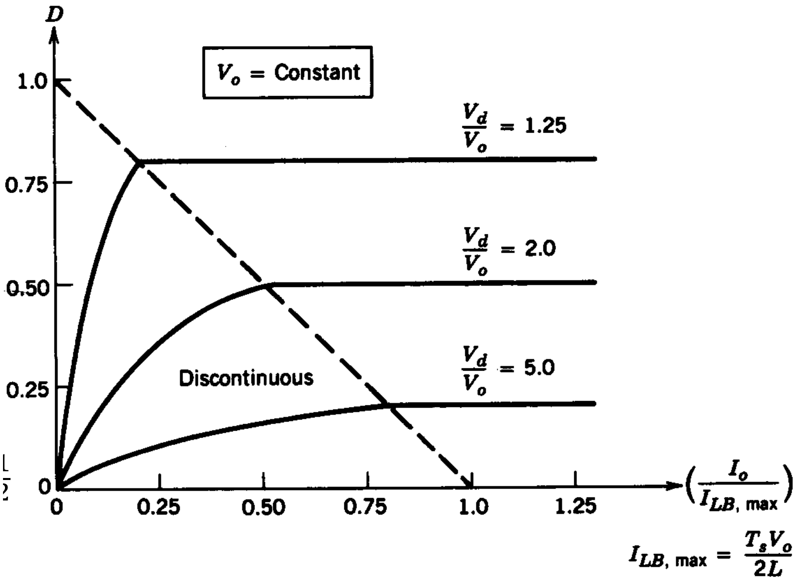


Figure 7-9 Step-down converter characteristics keeping V_o constant.

Discontinuous-conduction with constant Vo

DC voltage supply

Continuous: $I_o > I_{LB}$

$$D > 1 - \frac{I_o}{I_{LBmax}}$$

$$D = \frac{V_o}{V_d}$$

Discontinuous: $I_o < I_{LB}$

$$D < 1 - \frac{I_o}{I_{LBmax}}$$

$$D = \left[\frac{V_o}{V_d}\frac{I_o}{I_{LBmax}}\left(1-\frac{V_d}{V_o}\right)^{-1}\right]^{\frac{1}{2}}$$

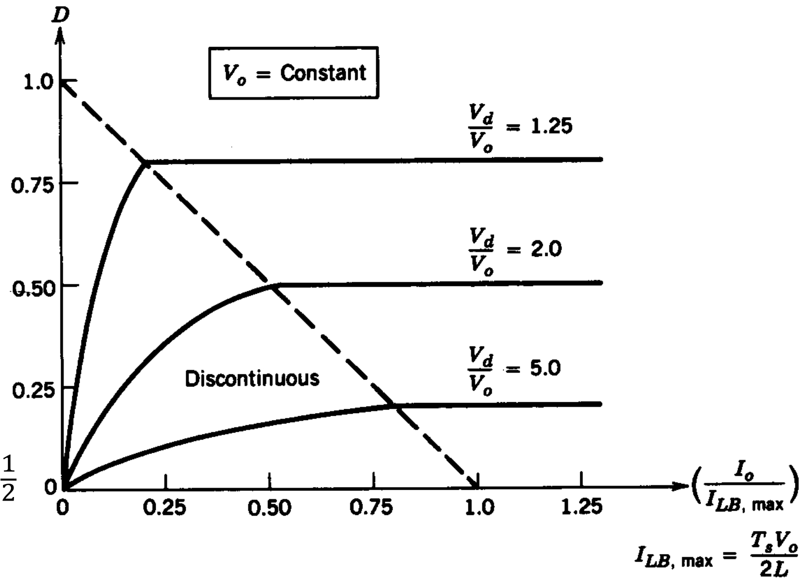


Figure 7-9 Step-down converter characteristics keeping V_o constant.

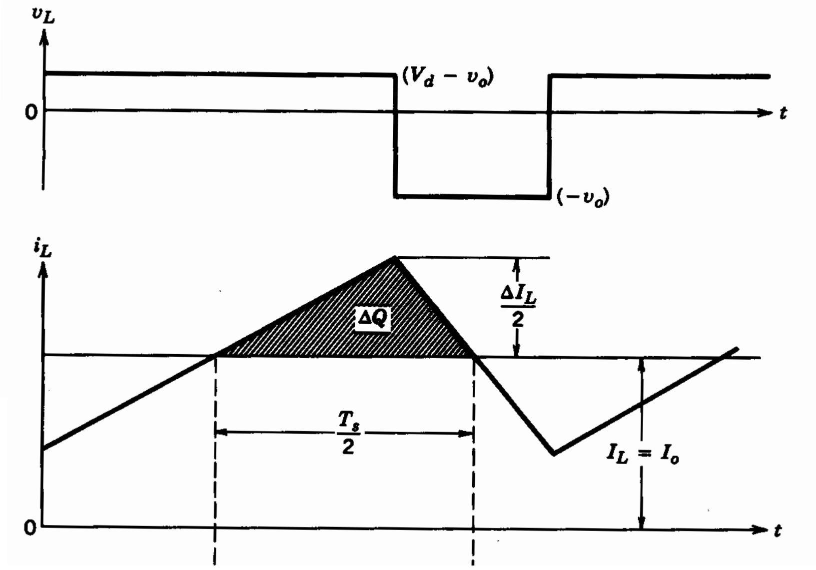
Output voltage ripple

First order calculation:

The average I_L flows in the load, and the ripple component in C.

Additional charge:

$$\Delta Q = \frac{1}{2}\frac{\Delta I_L}{2}\frac{T_s}{2}$$



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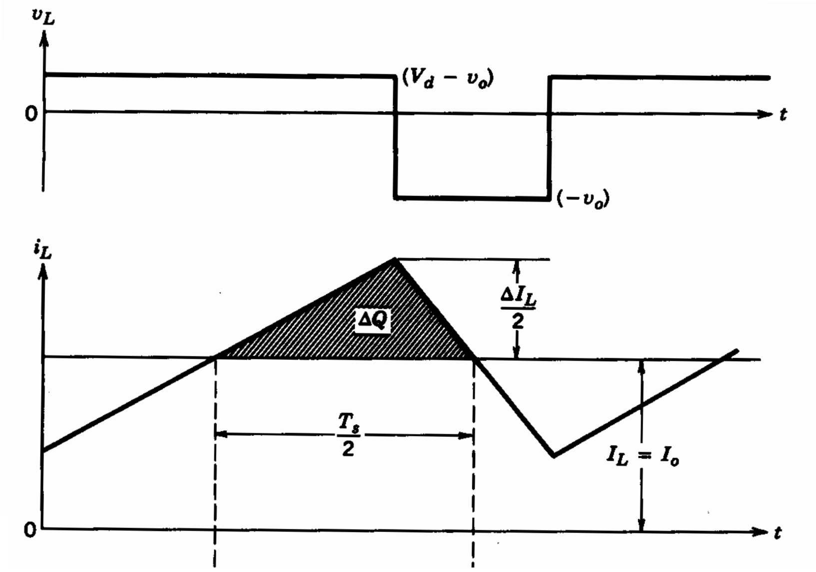
Current ripple:

$$\Delta I_L = (V_o/L)(1-D)T_s$$



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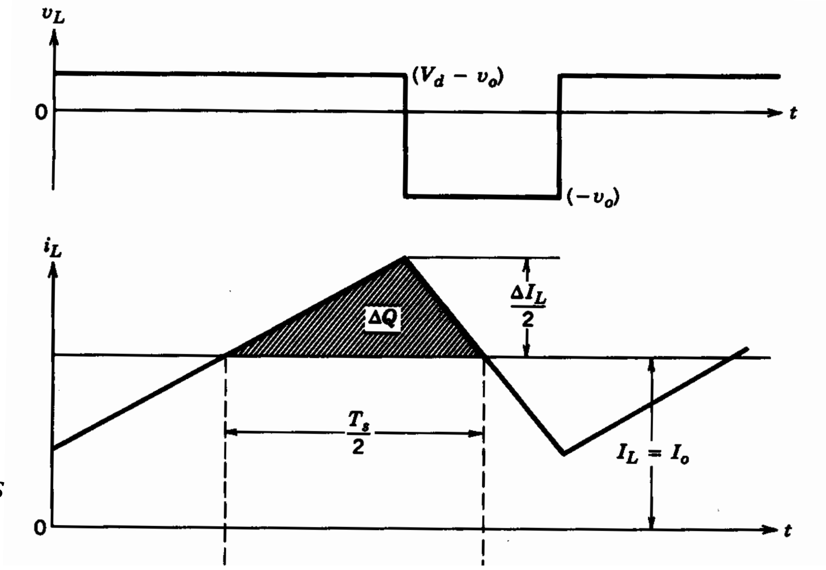
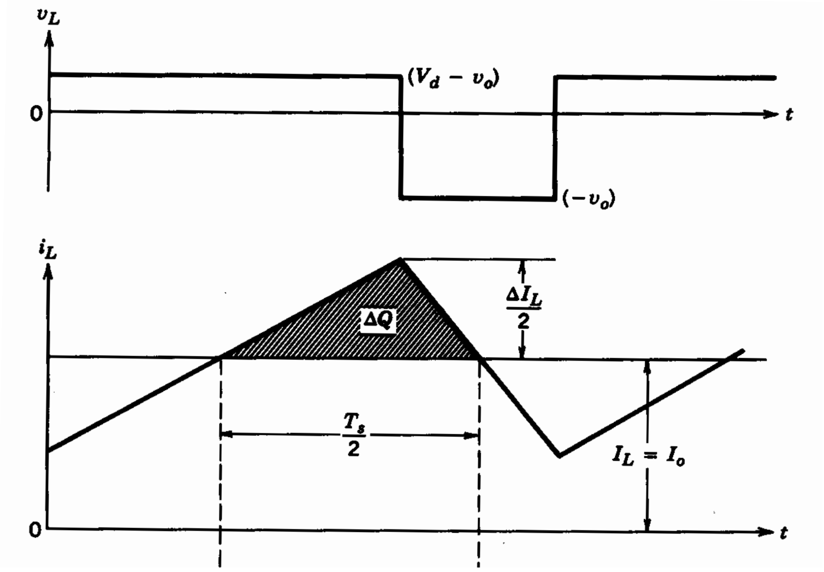
Voltage ripple:

$$\Delta V_o = \frac{\Delta Q}{C} = \frac{V_o}{8LC}T_S^2(1-D)$$



Output voltage ripple



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Voltage ripple:

$$\Delta V_o = \frac{\Delta Q}{C} = \frac{V_o}{8LC}T_s^2(1-D)$$

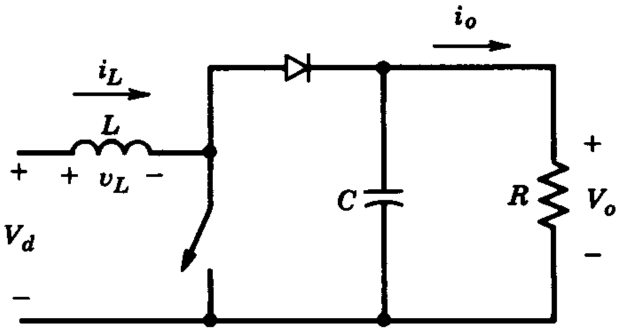
$$f_c = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$$

$$\frac{\Delta V_o}{V_o} = \frac{\pi^2}{2}(1-D)\frac{f_c^2}{f_s^2}$$

Step-up (boost) converter

- DC power supplies
- Regenerative breaking of DC motors

Output voltage always larger than the input

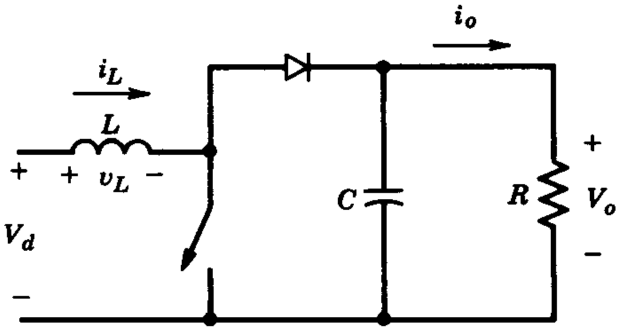


Step-up (boost) converter

- DC power supplies
- Regenerative breaking of DC motors

Output voltage always larger than the input

Switch on → diode off, output isolated, L accumulates energy from input



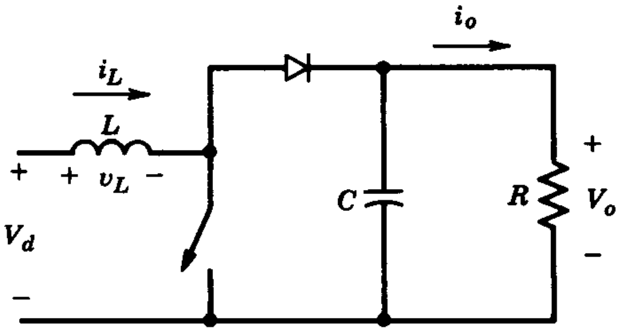
Step-up (boost) converter

- DC power supplies
- Regenerative breaking of DC motors

Output voltage always larger than the input

Switch on → diode off, output isolated, L accumulates energy from input

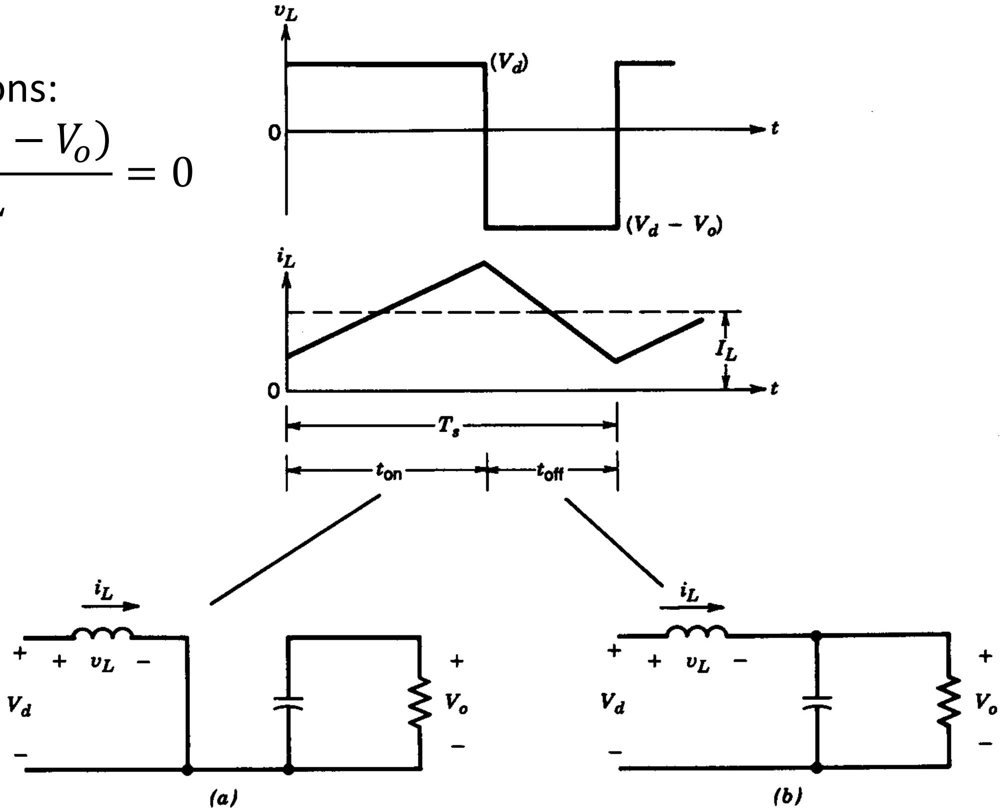
Switch off → diode on, load receives energy from input and from L



Continuous-conduction mode

Periodic conditions:

$$\frac{t_{\text{on}}V_d}{L}+\frac{t_{\text{off}}(V_d-V_o)}{L}=0$$



Continuous-conduction mode

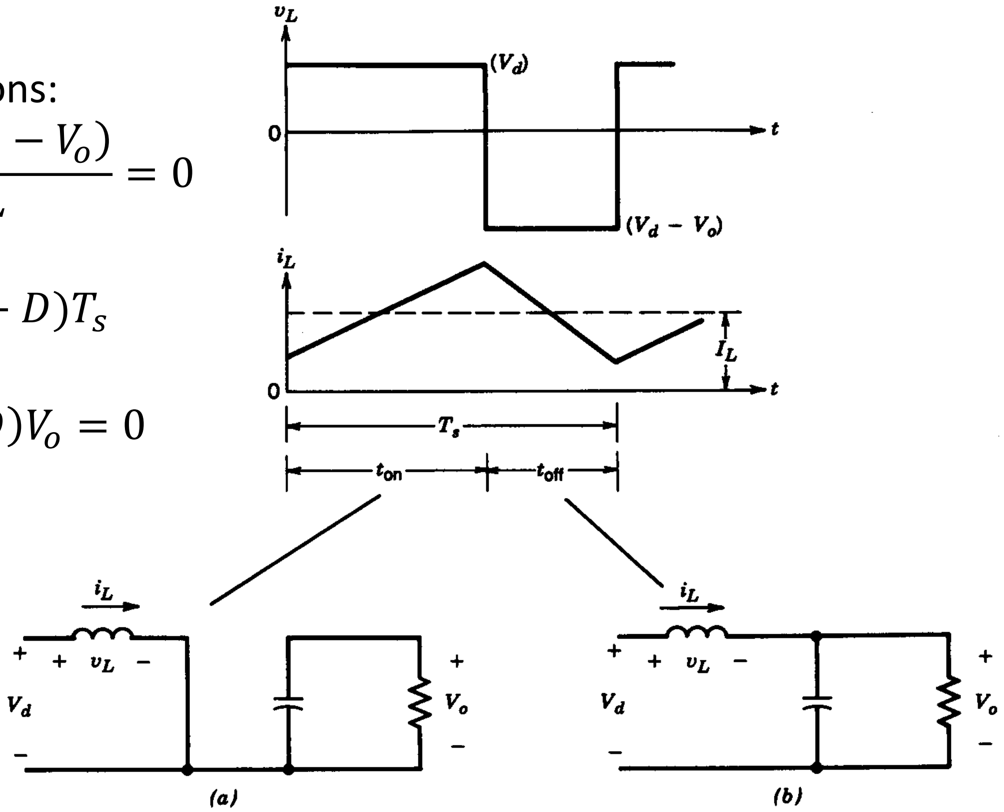
Periodic conditions:

$$\frac{t_{\text{on}}V_d}{L}+\frac{t_{\text{off}}(V_d-V_o)}{L}=0$$

if $t_{\text{on}} = DT_s$ and

$t_{\text{off}} = (1-D)T_s$

$$T_sV_d + T_s(1-D)V_o = 0$$



Continuous-conduction mode

Periodic conditions:

$$\frac{t_{\text{on}}V_d}{L} + \frac{t_{\text{off}}(V_d - V_o)}{L} = 0$$

if $t_{\text{on}} = DT_s$ and

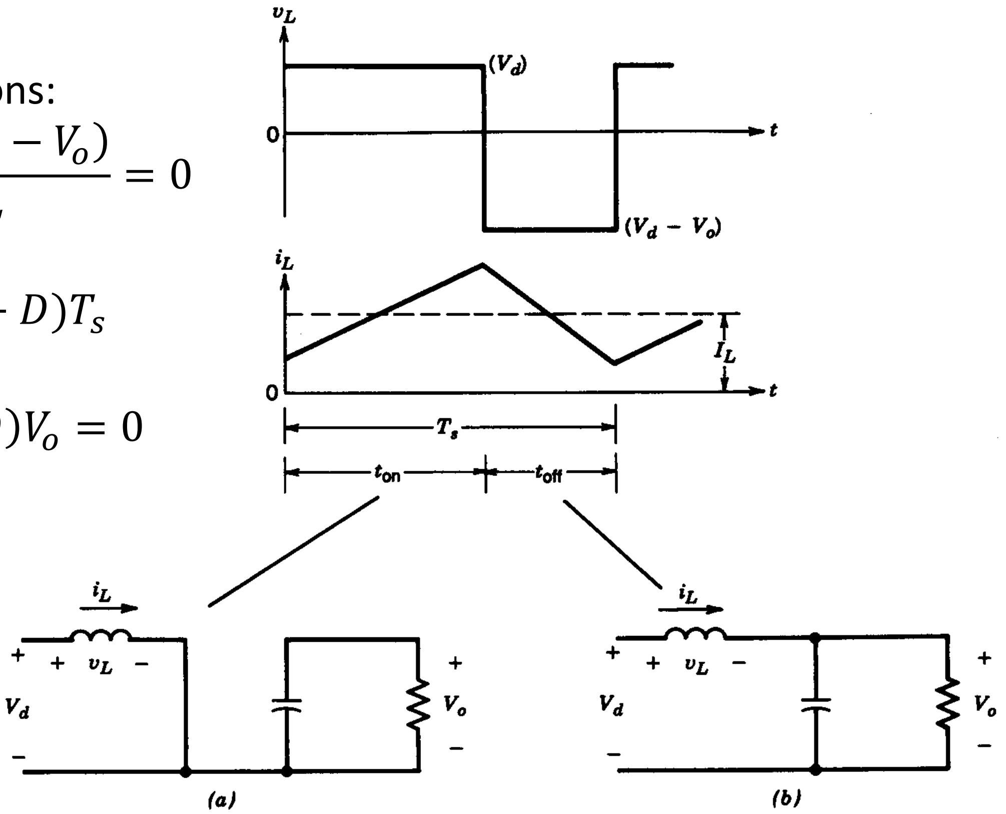
$$t_{\text{off}} = (1 - D)T_s$$

$$T_sV_d + T_s(1 - D)V_o = 0$$

$$\frac{V_o}{V_d} = \frac{1}{1 - D}$$

No losses:

$$V_oI_o = V_dI_d$$



(a) (b)

Continuous-discontinuous boundary

Average current in L = ripple :

$$I_{LB} = \frac{1}{2}\frac{V_d t_{on}}{L}$$

$$= \frac{V_o(1 - D)T_sD}{2L}$$

Continuous-discontinuous boundary

Average current in L = ripple :

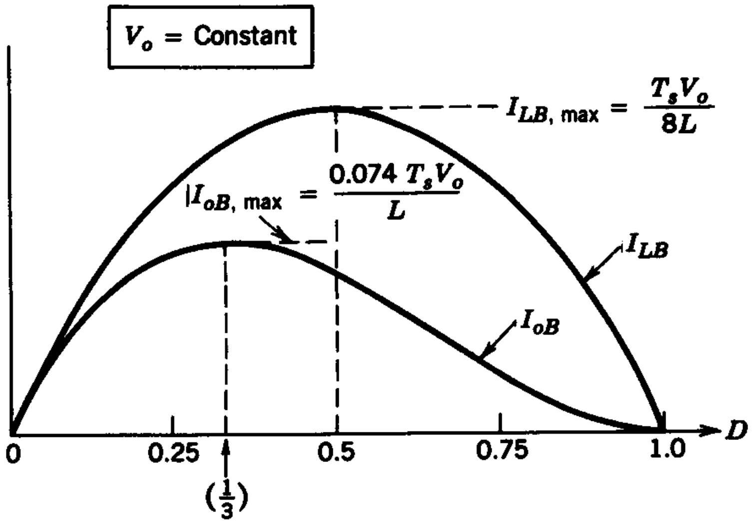
$$I_{LB} = \frac{V_d t_{on}}{2L}$$

$$= \frac{V_o(1-D)T_s D}{2L}$$

Average output current at the limit:

$$I_{oB} = I_{LB}(1-D)$$

$$= \frac{V_o T_s (1-D)^2 D}{2L}$$



Continuous-discontinuous boundary

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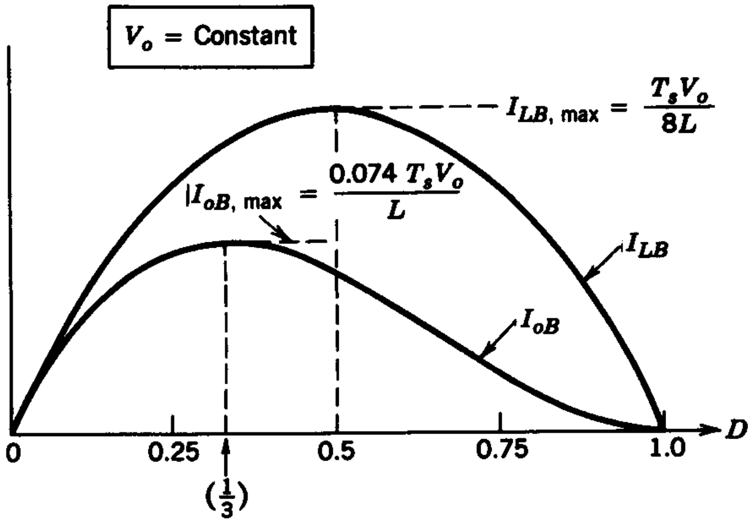


I_{LB} is max if D=0.5 → $I_{LBmax} = \frac{V_o T_s}{8L}$,

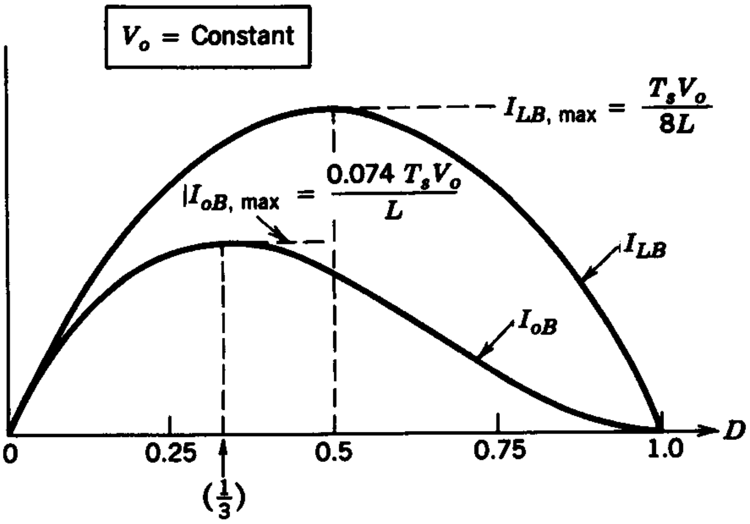
Continuous-discontinuous boundary

Average current in L = ripple :

$$I_{LB} = \frac{V_d t_{on}}{2L}$$
$$= \frac{V_o(1-D)T_s D}{2L}$$

Average output current at the limit:

$$I_{oB} = I_{LB}(1-D)$$
$$= \frac{V_o T_s (1-D)^2 D}{2L}$$



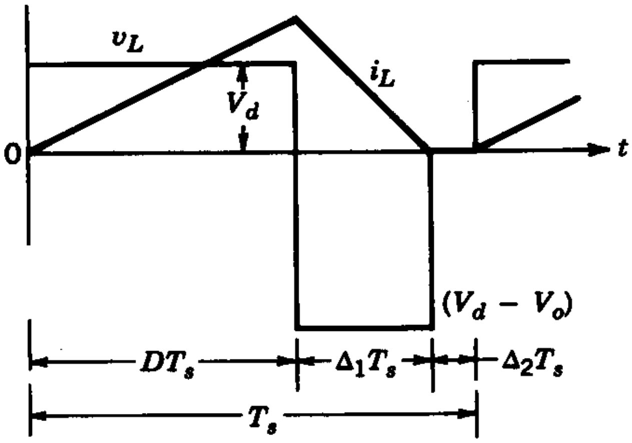
I_{LB} is max if D=0.5 → $I_{LBmax} = \frac{V_o T_s}{8L}$,

I_{oB} is max if D=1/3 → $I_{oBmax} = \frac{2V_o T_s}{27L}$ → $I_{oB} = \frac{27}{4}(1-D)^2 D I_{oBmax}$

Discontinuous conduction mode (constant V_o)

Periodic conditions:

$$\frac{DT_s V_d}{L} + \frac{\Delta_1 T_s (V_d - V_o)}{L} = 0$$



Discontinuous conduction mode (constant V_o)

Periodic conditions:

$$\frac{DT_sV_d}{L}+\frac{\Delta_1T_s(V_d-V_o)}{L}=0$$

$$\frac{V_o}{V_d}=1+\frac{D}{\Delta_1}=\frac{I_d}{I_o}$$

tions:

$$\frac{(V_d\ -\ V_o)}{L}=0$$

$$-\frac{D}{\Delta_1}=\frac{I_d}{I_o}$$

Discontinuous conduction mode (constant V_o)

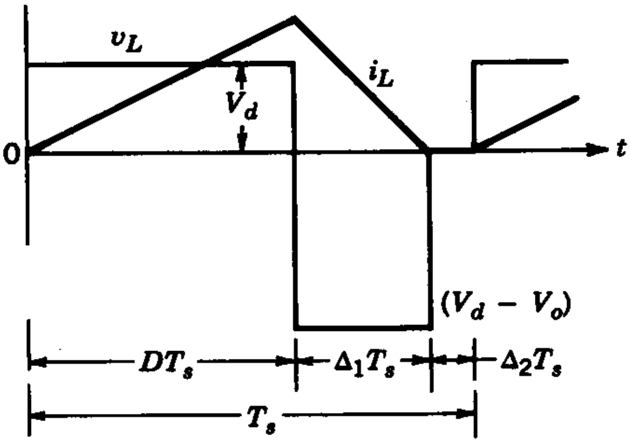
Periodic conditions:

$$\frac{DT_sV_d}{L}+\frac{\Delta_1T_s(V_d-V_o)}{L}=0$$

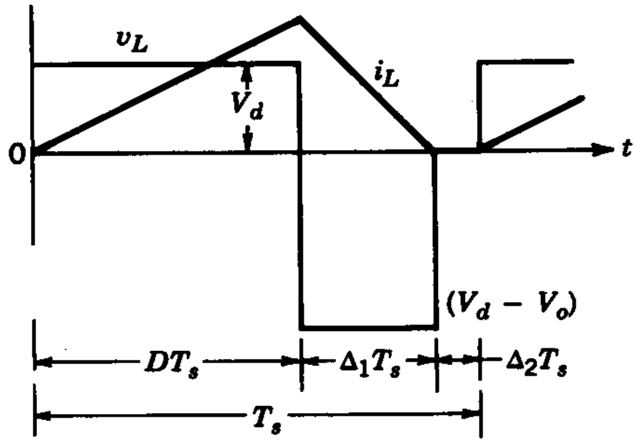
$$\frac{V_o}{V_d}=1+\frac{D}{\Delta_1}=\frac{I_d}{I_o}$$

Average current in L

$$I_dT_s=\frac{DT_sV_d}{L}\frac{(D+\Delta_1)T_s}{2}$$



Discontinuous conduction mode (constant V_o)



Periodic conditions:

$$\frac{DT_sV_d}{L}+\frac{\Delta_1T_s(V_d-V_o)}{L}=0$$

$$\frac{V_o}{V_d}=1+\frac{D}{\Delta_1}=\frac{I_d}{I_o}$$

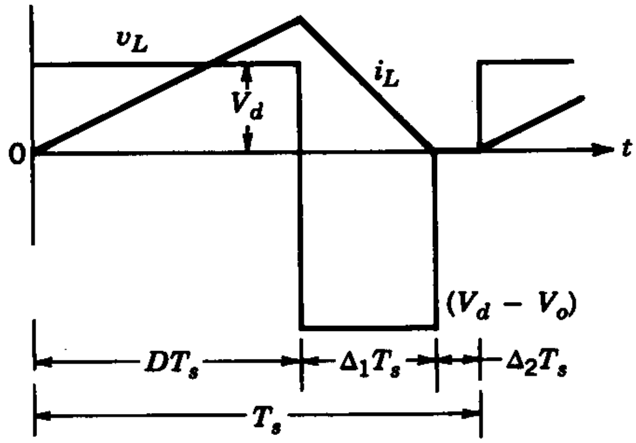
Average current in L

$$I_dT_s=\frac{DT_sV_d}{L}\frac{(D+\Delta_1)T_s}{2}$$

Average output current

$$I_o=I_d\frac{\Delta_1}{D+\Delta_1}=\frac{T_sV_d}{2L}D\Delta_1$$

Discontinuous conduction mode (constant V_o)



Periodic conditions:

$$\frac{DT_sV_d}{L}+\frac{\Delta_1T_s(V_d-V_o)}{L}=0$$

$$\frac{V_o}{V_d}=1+\frac{D}{\Delta_1}=\frac{I_d}{I_o}$$

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Average output current

$$I_o=I_d\frac{\Delta_1}{D+\Delta_1}=\frac{T_sV_d}{2L}D\Delta_1$$

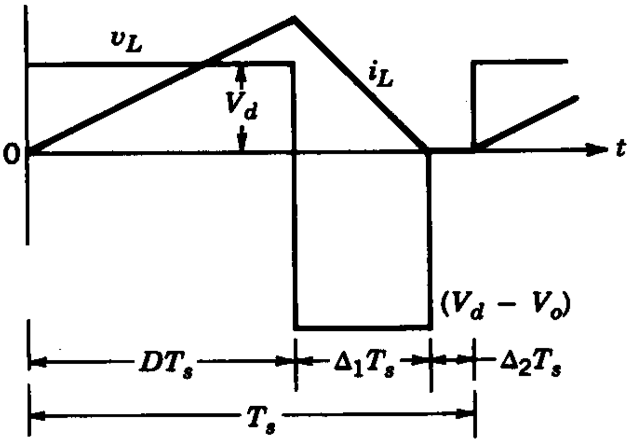
$$=\frac{27}{4}I_{oBmax}\frac{V_d}{V_o}D^2\frac{V_d}{V_o-V_d}$$

Discontinuous conduction mode (constant V_o)

Periodic conditions:

$$\frac{DT_sV_d}{L}+\frac{\Delta_1T_s(V_d-V_o)}{L}=0$$

$$\frac{V_o}{V_d}=1+\frac{D}{\Delta_1}=\frac{I_d}{I_o}$$



Average current in L

$$I_dT_s=\frac{DT_sV_d}{L}\frac{(D+\Delta_1)T_s}{2}$$

Average output current

$$I_o=I_d\frac{\Delta_1}{D+\Delta_1}=\frac{T_sV_d}{2L}D\Delta_1$$

$$=\frac{27}{4}I_{oBmax}\frac{V_d}{V_o}D^2\frac{V_d}{V_o-V_d}$$

$$\Rightarrow D=\left[\frac{4}{27}\frac{V_o}{V_d}\left(\frac{V_o}{V_d}-1\right)\frac{I_o}{I_{oBmax}}\right]^{\frac{1}{2}}$$

Continuous-discontinuous mode (constant V_o)

Continuous mode:

$$I_o>I_{oB}$$

$$=I_{oBmax}\frac{27(1-D)^2D}{4}$$

$$D=1-\frac{V_d}{V_o}$$

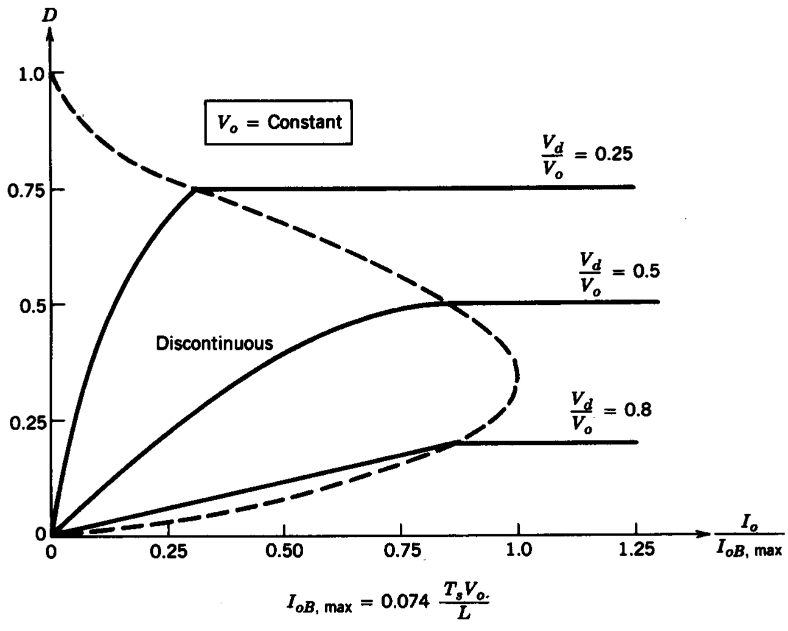


Figure 7-15 Step-up converter characteristics keeping V_o constant.

Continuous-discontinuous mode (constant V$_o$)

Continuous mode:

$$I_o > I_{oB} = I_{oBmax}\frac{27(1-D)^2D}{4}$$

$$D = 1 - \frac{V_d}{V_o}$$

Discontinuous mode:

$$I_o < I_{oB}$$

$$D = \left[\frac{4}{27}\frac{V_o}{V_d}\left(\frac{V_o}{V_d} - 1\right)\frac{I_o}{I_{oBmax}}\right]^{\frac{1}{2}}$$

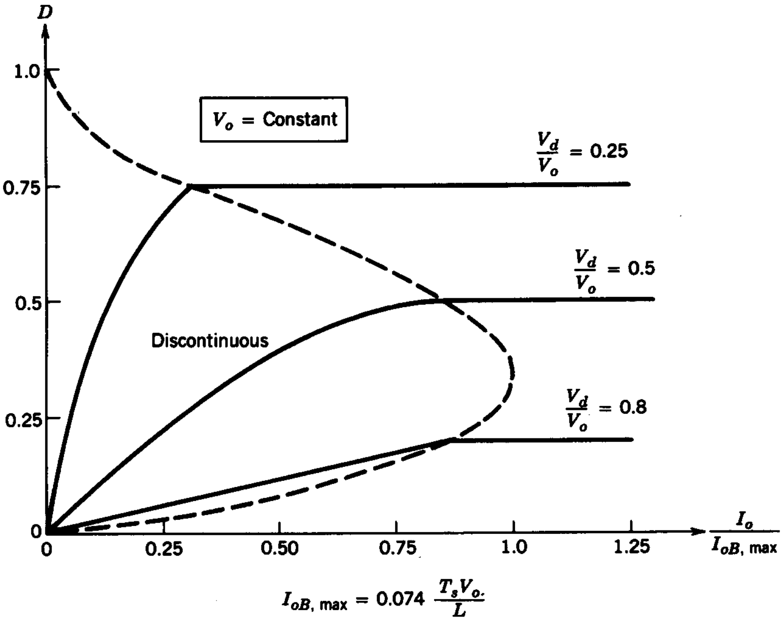


Figure 7-15 Step-up converter characteristics keeping V_o constant.

Losses and ripple

Losses: inductor, capacitor, switch, diode

Ripple: first order assumption: when the switch is on the C is discharged through the load

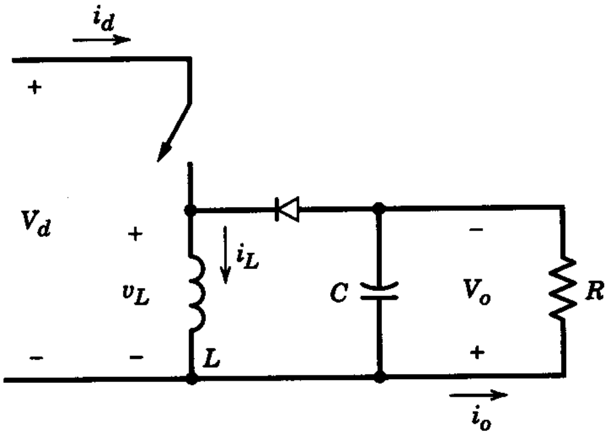
$$\Delta V_o = \frac{\Delta Q}{C} = \frac{I_oDT_s}{C} = \frac{V_oDT_s}{RC}$$

$$\frac{\Delta V_o}{V_o} = D\frac{T_s}{\tau}$$

Buck-boost converter

Negative DC power supply

Switch on: inductance accumulates energy, diode off, C supplies the load

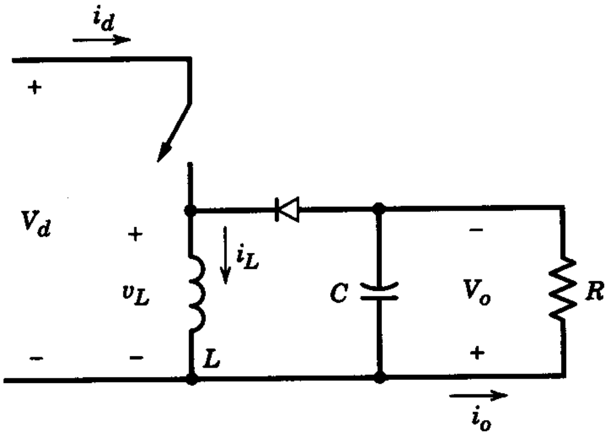


Buck-boost converter

Negative DC power supply

Switch on: inductance accumulates energy, diode off, C supplies the load

Switch off: diode on, inductance transfers energy to the capacitance and to the load



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Switch on: inductance accumulates energy, diode off, C supplies the load

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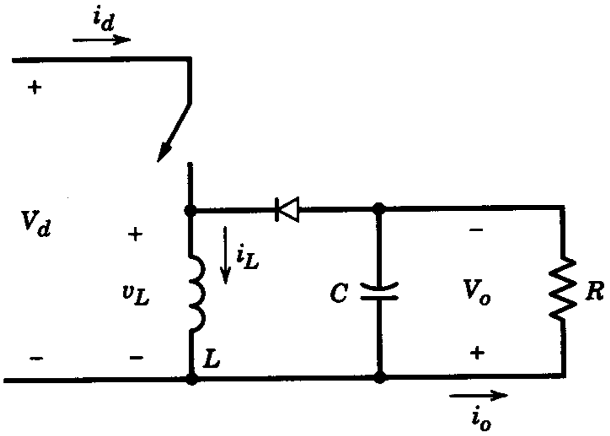


Periodic conditions in continuous conduction mode:

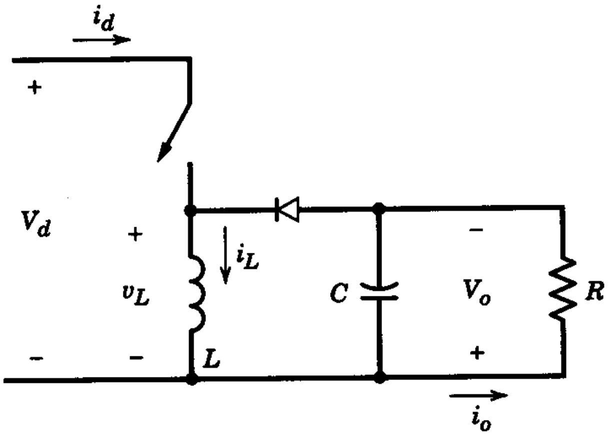
$$\frac{DT_sV_d}{L} - \frac{V_o(1-D)T_s}{L} = 0$$

Buck-boost converter

Negative DC power supply

Switch on: inductance accumulates energy, diode off, C supplies the load

Switch off: diode on, inductance transfers energy to the capacitance and to the load



Periodic conditions in continuous conduction mode:

$$\frac{DT_sV_d}{L} - \frac{V_o(1-D)T_s}{L} = 0$$

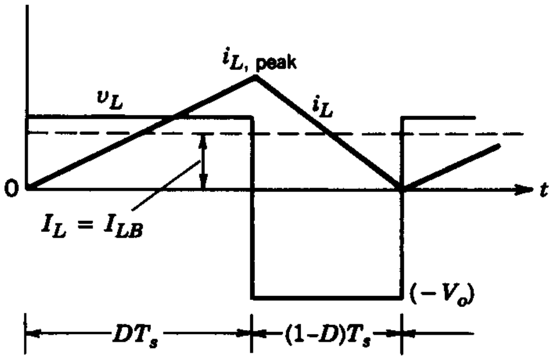
$$\frac{V_o}{V_d} = \frac{D}{1-D} = \frac{I_d}{I_o}$$

$$I_L = I_o + I_d = \frac{I_o}{1-D}$$

Continuous-discontinuous boundary

Current in L at the boundary

$$I_{LB} = \frac{DT_s V_d}{2L}$$



(a)

(b)

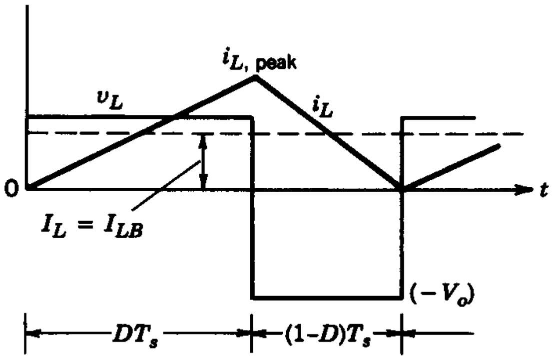
Continuous-discontinuous boundary

Current in L at the boundary

$$I_{LB} = \frac{DT_s V_d}{2L}$$

Output current at the boundary:

$$I_{oB} = I_{LB}(1-D) = \frac{T_s V_o}{2L}(1-D)^2$$



(a)

(b)

Continuous-discontinuous boundary

Current in L at the boundary

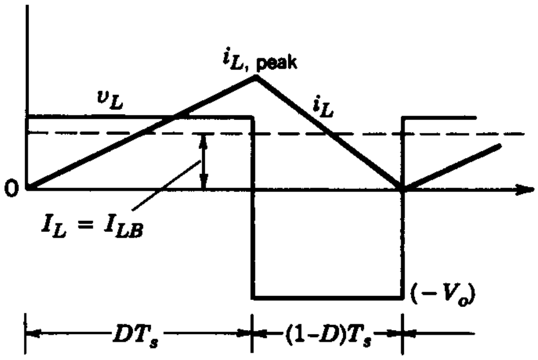
$$I_{LB} = \frac{DT_sV_d}{2L}$$

Output current at the boundary:

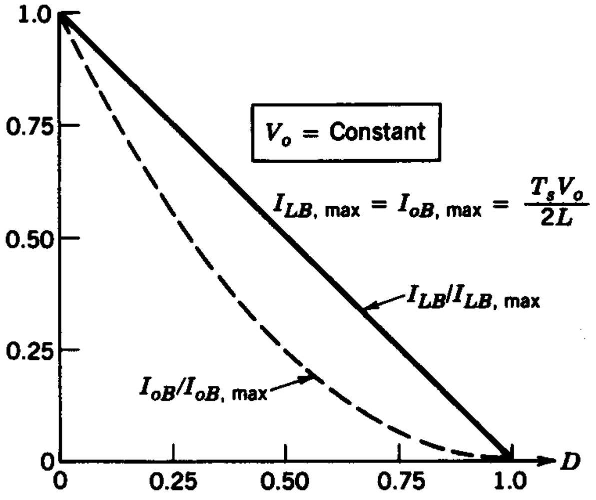
$$I_{oB} = I_{LB}(1-D) = \frac{T_sV_o}{2L}(1-D)^2$$

$$I_{LB} = I_{LBmax}(1-D)$$

$$I_{oB} = I_{oBmax}(1-D)^2$$



(a)

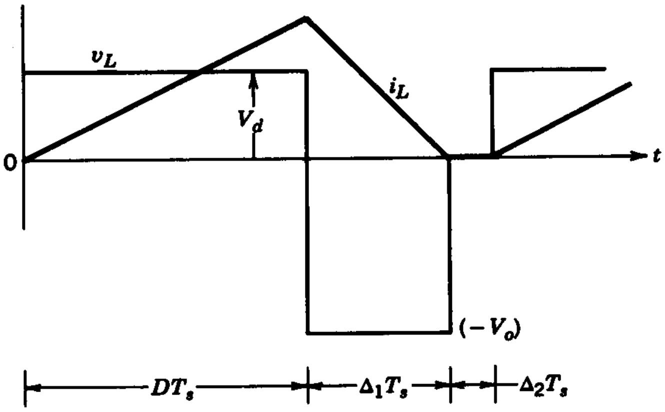


(b)

Discontinuous conduction

Periodic conditions:

$$\frac{DV_dT_s}{L} - \frac{V_o\Delta_1T_s}{L} = 0$$



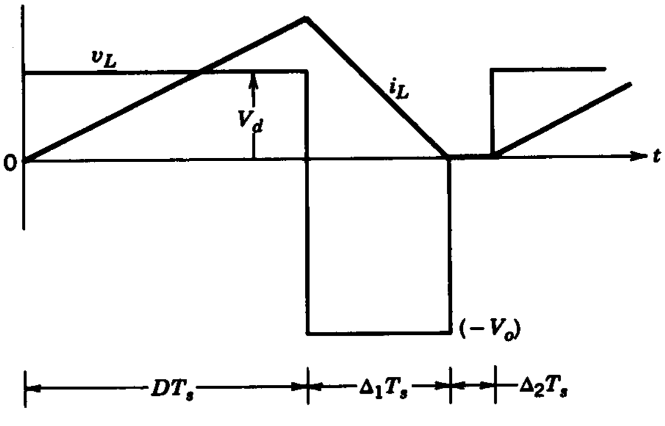
ure 7-21 Buck–boost converter waveforms in a ontinuous-conduction mode.

Discontinuous conduction

Periodic conditions:

$$\frac{DV_dT_s}{L} - \frac{V_o\Delta_1T_s}{L} = 0$$

$$\frac{V_o}{V_d} = \frac{D}{\Delta_1} = \frac{I_d}{I_o}$$



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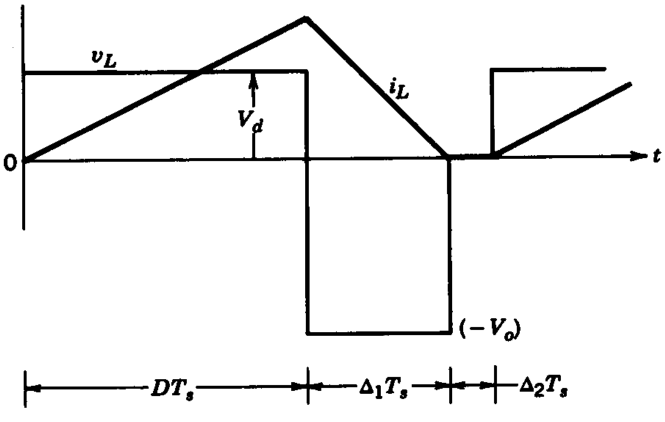
$$\frac{V_o}{V_d} = \frac{D}{\Delta_1} = \frac{I_d}{I_o}$$

Average current in L:

$$I_LT_s = \frac{V_dDT_s}{L}\frac{(D+\Delta_1)T_s}{2}$$



ure 7-21 Buck–boost converter waveforms in a ontinuous-conduction mode.

Discontinuous conduction

Periodic conditions:

$$\frac{DV_dT_s}{L}-\frac{V_o\Delta_1T_s}{L}=0$$

$$\frac{V_o}{V_d}=\frac{D}{\Delta_1}=\frac{I_d}{I_o}$$

Average current in L:

$$I_LT_s=\frac{V_dDT_s}{L}\frac{(D+\Delta_1)T_s}{2}$$

Therefore:

$$I_L=I_o\left(1+\frac{D}{\Delta_1}\right)=\frac{V_dT_s}{2L}D(D+\Delta_1)$$

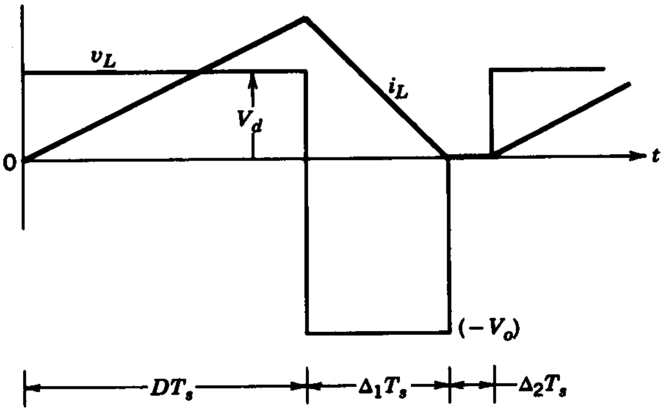


Figure 7-21 Buck–boost converter waveforms in a discontinuous-conduction mode.

Discontinuous conduction

Periodic conditions:

$$\frac{DV_dT_s}{L}-\frac{V_o\Delta_1T_s}{L}=0$$

$$\frac{V_o}{V_d}=\frac{D}{\Delta_1}=\frac{I_d}{I_o}$$

Average current in L:

$$I_LT_s=\frac{V_dDT_s}{L}\frac{(D+\Delta_1)T_s}{2}$$

Therefore:

$$I_L=I_o\left(1+\frac{D}{\Delta_1}\right)=\frac{V_dT_s}{2L}D(D+\Delta_1)$$

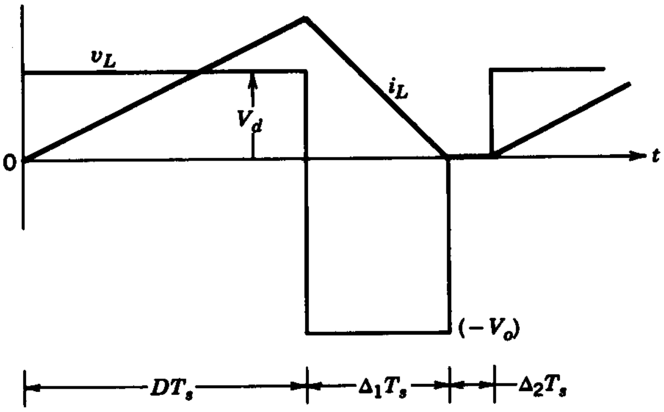


Figure 7-21 Buck–boost converter waveforms in a discontinuous-conduction mode.

$$\frac{I_o}{I_{oBmax}}=D\Delta_1\frac{V_d}{V_o}=D^2\left(\frac{V_d}{V_o}\right)^2\rightarrow D=\frac{V_o}{V_d}\sqrt{\frac{I_o}{I_{oBmax}}}$$

Continuous-discontinuous mode

Continuous operation

$$I_o > I_{oB} = I_{oBmax}(1-D)^2$$

$$D = \frac{V_o}{V_d - V_o}$$

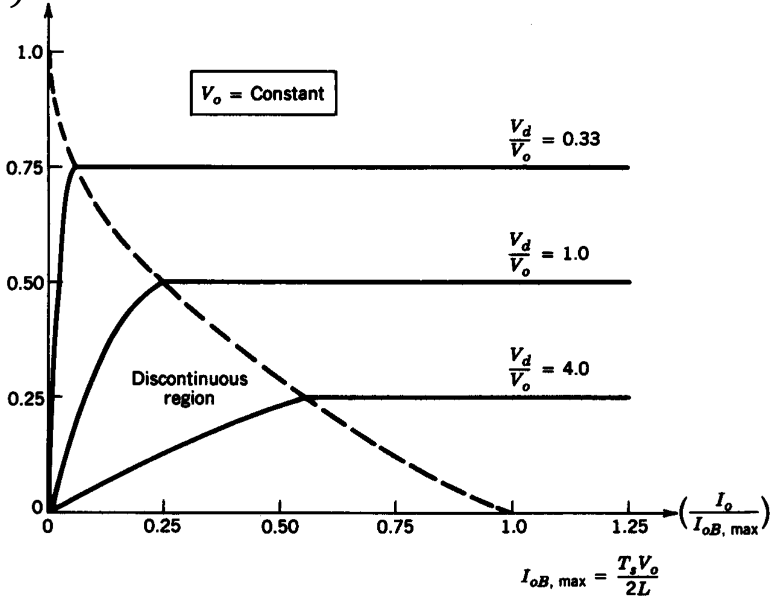


Figure 7-22 Buck–boost converter characteristics keeping V_o constant.

Continuous-discontinuous mode

Continuous operation

$$I_o > I_{oB} = I_{oBmax}(1-D)^2$$

$$D = \frac{V_o}{V_d - V_o}$$

Discontinuous operation

$$I_o > I_{oB}$$

$$D = \frac{V_o}{V_d}\sqrt{\frac{I_o}{I_{oBmax}}}$$

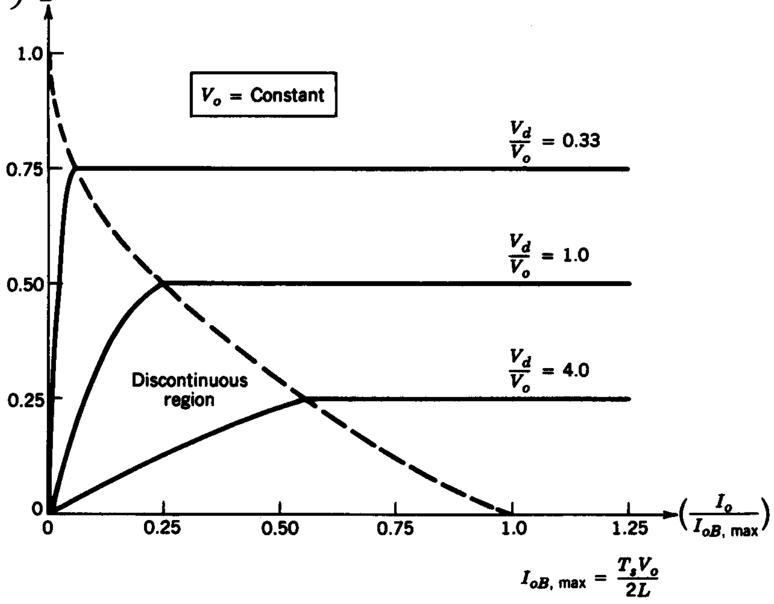


Figure 7-22 Buck–boost converter characteristics keeping V_o constant.