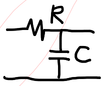
Filtri

→ Sistemi lineari a 2 porte

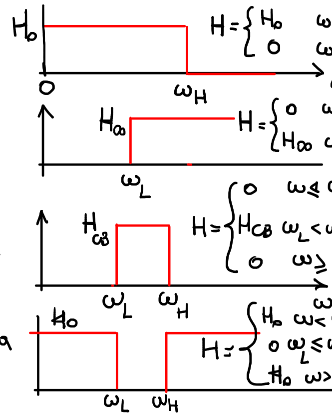
$$X(f) \to \boxed{H(f)} \to Y(f) = H(f)\,X(f)$$

- Filtri passivi e.g.



- Filtri attivi

$$H(j\omega) = \frac{N(j\omega)}{D(j\omega)}$$

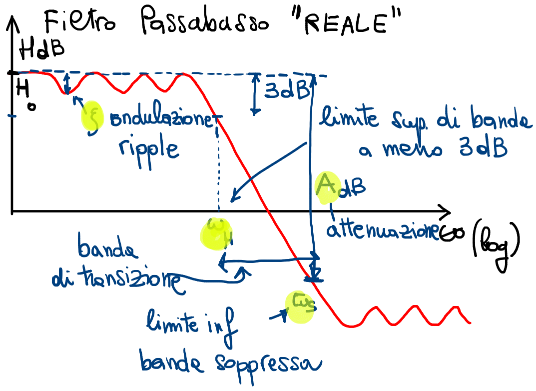


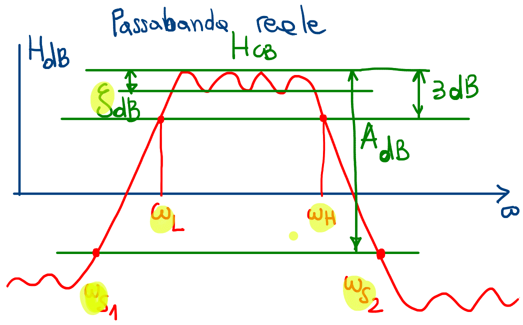
1) Passabasso $H = \begin{cases} H_0 & \omega < \omega_H \\ 0 & \omega \geq \omega_H \end{cases}$

2) Passa alto $H = \begin{cases} 0 & \omega \leq \omega_L \\ H_\infty & \omega > \omega_L \end{cases}$

3) Passabanda $H = \begin{cases} 0 & \omega \leq \omega_L \\ H_{CB} & \omega_L < \omega < \omega_H \\ 0 & \omega \geq \omega_H \end{cases}$

4) Eliminabanda $H = \begin{cases} H_0 & \omega < \omega_L \\ 0 & \omega_L \leq \omega \leq \omega_H \\ H_0 & \omega > \omega_H \end{cases}$





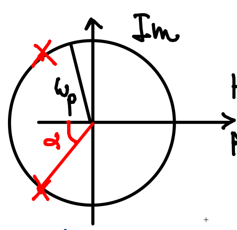
Filtri biquadratici

$$H(s) = \frac{b_2 s^2 + b_1 s + b_0}{a_2 s^2 + a_1 s + a_0} = \frac{\left(\frac{s}{\omega_z}\right)^2 + \frac{s}{Q_z \omega_z} + 1}{\left(\frac{s}{\omega_p}\right)^2 + \frac{s}{Q_p \omega_p} + 1} H_0$$

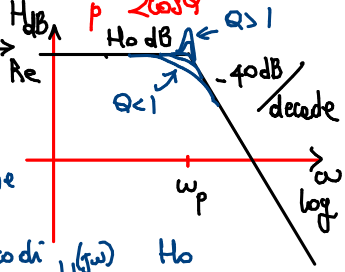
Passa basso $\leftrightarrow b_2, b_1 = 0$

2 poli, nessuno zero $[\omega_z \to \infty]$

$$\omega_p = \begin{cases} \text{media geom dei poli [se i poli sono reali]} \\ \text{modulo dei poli [se i poli sono c.c.]} \end{cases}$$



$$Q_p = \frac{1}{2\cos\alpha}$$



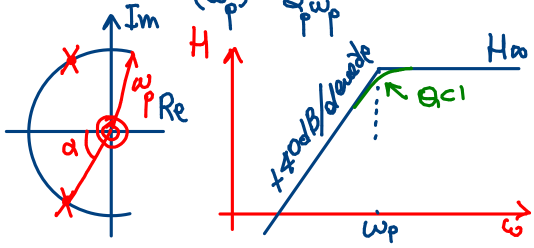
si fa in modo che sia $Q < 1$ per evitare il picco di risonanza

$$H(j\omega) = \frac{H_0}{\left(\frac{j\omega}{\omega_p}\right)^2 + \frac{j\omega}{Q_p \omega_p} + 1}$$

Passa alto: $b_1, b_0 = 0$

2 zeri nell'origine

$$H = \frac{b_2 s^2}{a_2 s^2 + a_1 s + a_0} = \frac{\left(\frac{s}{\omega_p}\right)^2}{\left(\frac{s}{\omega_p}\right)^2 + \frac{s}{Q_p \omega_p} + 1} H_\infty$$

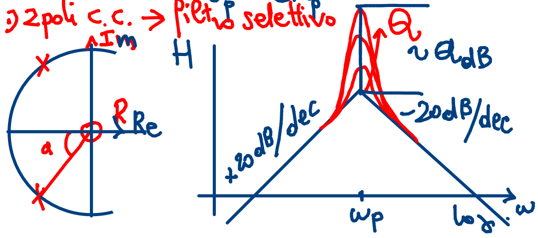


Passa banda: $b_2 = 0$, $b_0 = 0$

1 zero nell'origine, 2 poli

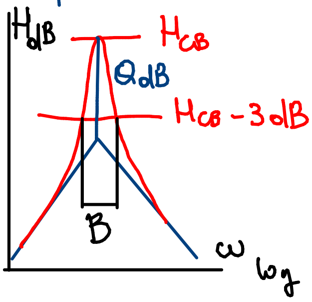
$$H = \frac{b_1 s}{a_2 s^2 + a_1 s + a_0} = \frac{\frac{s}{Q\omega_p}}{\frac{s^2}{\omega_p^2} + \frac{s}{Q\omega_p} + 1} H_{CB}$$

2 poli c.c. → filtro selettivo



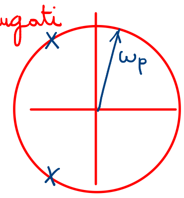
altezza del picco di risonanza: Q dB

ampiezza della banda a -3dB: $B = \frac{\omega_p}{Q}$



2 poli complessi coniugati

$$H = \frac{1}{\left(\frac{s}{\omega_p}\right)^2 + \frac{s}{Q\omega_p} + 1}$$



$s = j\omega_p$

$$H = \frac{1}{\underbrace{\cancel{\left(\frac{j\omega_p}{\omega_p}\right)^2}}_{= -1} + \frac{j\cancel{\omega_p}}{Q\cancel{\omega_p}} + \cancel{1}}$$

→ • $|H(j\omega_p)| = Q$

• $|H(j\omega)|_{\omega \ll \omega_p} = 1$

$\omega = \Delta\omega + \omega_p$

$$H(j\omega) = \frac{1}{\left[j\frac{(\Delta\omega + \omega_p)}{\omega_p}\right]^2 + j\frac{\Delta\omega + \omega_p}{Q\omega_p} + 1}$$

$\Delta\omega$ piccoli

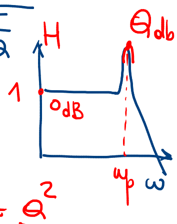
$$H(j\omega) = \frac{1}{-\cancel{\left(\frac{\Delta\omega}{\omega_p}\right)^2} - \underbrace{\frac{2\Delta\omega}{\omega_p}} - \cancel{1} + \cancel{j\frac{\Delta\omega}{Q\omega_p}} + \frac{j}{Q} + \cancel{1}}$$

se $Q \gg 1$

per $\Delta\omega$ piccoli

$$H(\omega_p + \Delta\omega) \cong \frac{1}{-\frac{2\Delta\omega}{\omega_p} + \frac{j}{Q}}$$

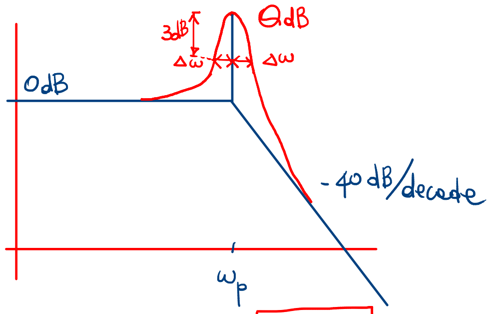
$$|H|^2 \cong \frac{1}{\frac{1}{Q^2} + \left(\frac{4\Delta\omega^2}{\omega_p^2}\right)}$$



limite a meno 3 dB

$|H(\omega_p + \Delta\omega)| = \frac{Q}{\sqrt{2}}$ → $|H|^2 = \frac{Q^2}{2}$

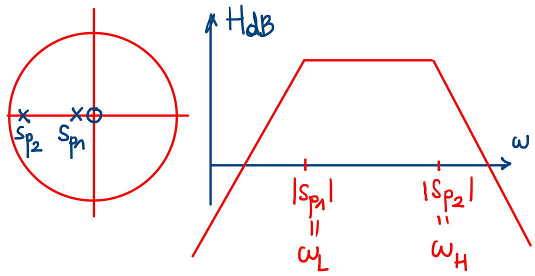
→ $\frac{4\Delta\omega^2}{\omega_p^2} = \frac{1}{Q^2}$ → $\boxed{\Delta\omega = \frac{\omega_p}{2Q}}$



$$B = 2\Delta\omega = \frac{\omega_p}{Q} \rightarrow \boxed{Q = \frac{\omega_p}{B}}$$

Filtro passabanda NON selettivo

→ 2 poli reali, 1 zero nell'origine



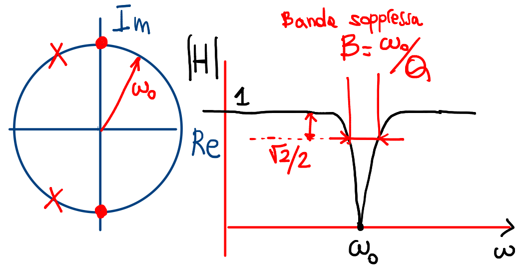
Filtri elimina banda

$$H = \frac{\left(\frac{s}{\omega_0}\right)^2 + 1}{\left(\frac{s}{\omega_0}\right)^2 + \frac{s}{Q\omega_0} + 1}$$

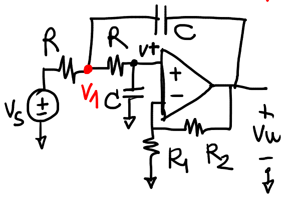
2 poli, 2 zeri immaginari $j\omega_0$

poli e zeri devono avere lo stesso modulo

$H(j\omega)$:
- $\omega \ll \omega_0 \rightarrow H = 1$
- $\omega = \omega_0 \rightarrow H = 0$
- $\omega \gg \omega_0 \rightarrow H = 1$



Cella di Sallen-Key . Passa basso



Ampli. non inv.

$$\frac{V_u}{V_+} = 1 + \frac{R_2}{R_1} = A$$

$$V^+ = V_u / A$$

$$V_1\left[\frac{1}{R} + \frac{1}{R} + Cs\right] - \frac{V_s}{R} - V_u Cs - \frac{V_u}{AR} = 0$$

$$\frac{V_u}{A}\left[\frac{1}{R} + Cs\right] - \frac{V_1}{R} = 0 \rightarrow V_1 = \frac{1+RCs}{A} V_u$$

$$\left(\frac{1+RCs}{A}\right) V_u \left(2 + RCs\right) - V_s - V_u RCs - \frac{V_u}{A} = 0$$

$$\boxed{\frac{V_u}{V_s} = \frac{A}{R^2C^2s^2 + (3-A)RCs + 1}}$$

FdT 2 poli

$A = 1 + \frac{R_2}{R_1} \geq 1$

poli sempre complessi coniugati

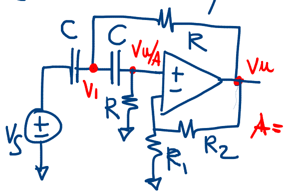
$$\frac{H_0}{\left(\frac{s}{\omega_p}\right)^2 + \frac{s}{Q\omega_p} + 1}$$

modulo dei poli → $\omega_p = \frac{1}{RC}$

fattore di qualità → $Q = \frac{1}{3-A}$

$H_0 = A$

Cella di Sallen-Key Passa alto



2 zeri nell'origine

2 poli

$A = 1 + \frac{R_2}{R_1}$

$$V_1\left[Cs + Cs + \frac{1}{R}\right] - V_s Cs - \frac{V_u}{A} Cs - V_u \frac{1}{R} = 0$$

$$\frac{V_u}{A}\left[Cs + \frac{1}{R}\right] - V_1 Cs = 0 \rightarrow V_1 = \frac{RCs+1}{RCs}\frac{V_u}{A}$$

$$\frac{RCs+1}{RCs}(1+2RCs)V_u - V_s ARCs - V_u RCs - V_u A = 0$$

$$\left[1 + 3RCs + \cancel{2}R^2C^2s^2\right]V_u - V_s A\left[RCs\right]^2 - V_u(\cancel{R^2C^2s^2}) - V_u A RCs = 0$$

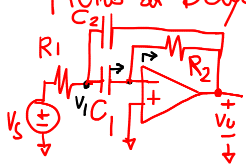
$$\boxed{\frac{V_u}{V_s} = \frac{A R^2C^2s^2}{R^2C^2s^2 + (3-A)RCs + 1}}$$

2 zeri nell'origine

2 poli complessi coniugati

$\omega_p = \frac{1}{RC}$ $\quad Q = \frac{1}{3-A}$

Filtro di Delyannis



1 zero nell'origine

2 poli.

$$V_1\left[\frac{1}{R_1} + C_1 s + C_2 s\right] - \frac{V_s}{R_1} - V_u C_2 s = 0$$

$$V_1 C_1 s = -\frac{V_u}{R_2} \rightarrow V_1 = -\frac{V_u}{R_2 C_1 s}$$

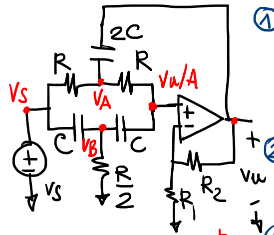
$$-\frac{V_u}{R_2C_1s}(1 + R_1C_1s + R_1C_2s) - V_s - V_u R_1 C_2 s = 0$$

$$\frac{V_u}{V_s} = -\frac{R_2C_1s}{\underbrace{R_1R_2C_1C_2}_{\frac{1}{\omega_p^2}}s^2 + \underbrace{R_1(C_1+C_2)}_{\frac{1}{Q\omega_p}}s + 1}$$

$$\omega_p = \frac{1}{\sqrt{R_1R_2C_1C_2}}$$

$$Q = \frac{\sqrt{R_1R_2C_1C_2}}{R_1(C_1+C_2)} = \sqrt{\frac{R_2}{R_1}} \cdot \frac{\sqrt{C_1C_2}}{C_1+C_2}$$

Filtro eliminabanda a doppio T



$A = 1 + \frac{R_2}{R_1}$

① $V_A\left[\frac{2}{R} + 2Cs\right] - \frac{V_s}{R} - \frac{V_u}{AR} - V_u 2Cs = 0$

② $V_B\left[\frac{2}{R} + 2Cs\right] - V_s Cs - \frac{V_u}{A}Cs = 0$

③ $\frac{V_u}{A}\left[Cs + \frac{1}{R}\right] - \frac{V_A}{R} - V_B Cs = 0$

$$\frac{V_u}{A}(1+RCs) = V_A + V_B RCs$$

① + RCs ②

$$(V_A + RCs V_B)(2+2RCs) - V_s(1+R^2C^2s^2) - \frac{V_u}{A}(1+R^2C^2s^2) - V_u 2RCs = 0$$

$$\frac{V_u}{A}(1+RCs)2(1+RCs) - V_s(1+R^2C^2s^2) - \frac{V_u}{A}(1+R^2C^2s^2) - V_u 2RCs = 0$$

$$\frac{V_u}{V_s} = \frac{(1+R^2C^2s^2)}{R^2C^2s^2 + (4-2A)RCs + 1}A$$

$$\frac{V_u}{V_s} = \frac{1+\left(\frac{s}{\omega_0}\right)^2}{\left(\frac{s}{\omega_0}\right)^2 + \frac{1}{Q\omega_0} + 1}H_0$$

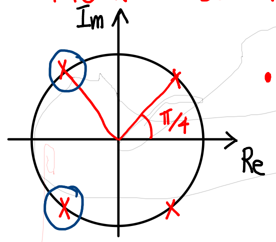
$\rightarrow \omega_0 = \frac{1}{RC}$

$Q = \frac{1}{4-2A}$

Filtri con più di due poli

Problema
- ORDINE DEL FILTRO ?
- POSIZIONI DI POLI E ZERI ?

Filtri di Butterworth



Polinomi di Butterworth
- grado n = pari

radici: radici 2n-esime di −1 a parte reale neg.

$S_p^{2n} = -1$

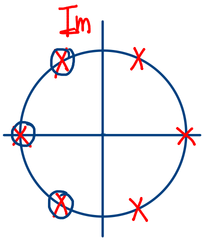
Es $n=2$

$S_p^4 = -1 \rightarrow$ fase $= \frac{\pi}{2n} + \frac{2\pi}{2n} i \quad (i = 0, \dots, 2n-1)$

Polinomio di Butterworth

grado n = dispari

↳ radici → radici 2n esime di 1 (a parte reale negativa)



Es. $n=3$

↳ $S_p^{2n} = 1$

fase: $0 + \frac{2\pi}{2n} i$

$(i = 0, \dots, 2n-1)$

$B^n(x)$ Polinomio di Butterworth di ordine n

$$|B^n(j\omega)|^2 = 1 + \omega^{2n}$$

n pari

$|B^n(x)|^2 = 1 + x^{2n}$ $\quad$ [se $x = j\omega$: $x^{2n} = j^{2n}\omega^{2n} = -1^n \omega^{2n}$]

radici: $1 + x^{2n} = 0$

$\rightarrow x^{2n} = -1$

n dispari

$|B^n(x)|^2 = 1 - x^{2n}$ $\quad$ [se $x = j\omega$: $x^{2n} = j^{2n}\omega^{2n} = -1^n \omega^{2n}$]

radici: $1 - x^{2n} = 0$

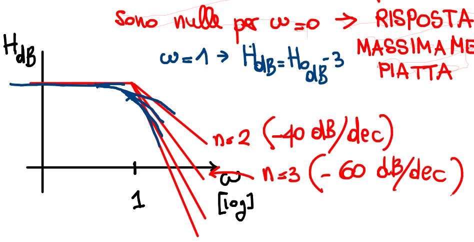
↳ $x^{2n} = 1$

Filtro passabasso di Butterworth

$$H(s) = \frac{H_0}{B(s)} \rightarrow |H(j\omega)|^2 = \frac{|H_0|^2}{1+\omega^{2n}}$$

tutte le derivate di $|H|^2$ fino alla $2n-1$ sono nulle per $\omega = 0$ → RISPOSTA MASSIMAMENTE PIATTA

$\omega = 1 \rightarrow H_{dB} = H_{0\,dB} - 3$



Se voglio lim sup bande = modulo dei poli = ω_0

$$H(s) = \frac{H_0}{B^n\left(\frac{s}{\omega_0}\right)}$$

es. $B^4(x)$

$(x^2 + 0{,}7654x + 1)(x^2 + 1{,}8478x + 1)$

$$H(s) = \frac{H_0}{\left[\left(\frac{s}{\omega_0}\right)^2 + 0.7654\left(\frac{s}{\omega_0}\right) + 1\right]\left[\left(\frac{s}{\omega_0}\right)^2 + 1.8478\left(\frac{s}{\omega_0}\right) + 1\right]}$$

Es. SPECIFICHE

→ Nessun ripple in banda passante

→ lim sup = 20 KHz

→ $f_B = 40\,KHz$ → Attenuazione banda bloccata >1000 (60dB)

1. Butterworth

2. $|H(j\omega)|^2 = \dfrac{|H_0|^2}{1+\left(\frac{\omega}{\omega_0}\right)^{2n}}$ $\qquad \omega_0 = 2\pi\cdot 20\,KHz$ $\qquad \omega_B = 2\omega_0$

$|H(j\omega_B)|^2 = \dfrac{|H_0|^2}{1+2^{2n}} \leqslant \dfrac{|H_0|^2}{10^6} \rightarrow 2^{2n} \geqslant 10^6 - 1$

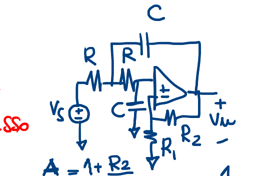
$n \geqslant 10$

den. 5 termini del 2° ordine

$$H(s) = \frac{H_0}{\left[\left(\frac{s}{\omega_0}\right)^2 + 1.974\left(\frac{s}{\omega_0}\right) + 1\right]\left[\cdots\cdots\right]\left[\cdots\right]\left[\cdot\right]\left[\cdot\right]}$$

$$H(s) = \frac{H_0}{B^{10}\left(\frac{s}{\omega_0}\right)}$$

1° filtro Biquad: celle SK passabasso



$$\frac{A}{\left(\frac{s}{\omega_0}\right)^2 + \frac{s}{Q\omega_0} + 1}$$

$A = 1 + \dfrac{R_2}{R_1}$ $\qquad \omega_0 = \dfrac{1}{RC}$ $\qquad Q = \dfrac{1}{3-A}$

1ª cella $\quad \omega_0 = 2\pi\cdot 20\cdot 10^3 = \dfrac{1}{RC}$

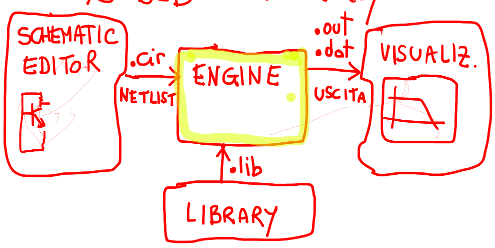
$1.974 = 3 - A \rightarrow A = 1.026 = 1 + \dfrac{R_2}{R_1}$

$\dfrac{R_2}{R_1} = 0{,}026$

SPICE

Simulation Program with Integrated Circuit Emphasys

'70 UCB Berkeley



NETLIST

→ Prima riga vuota . TITOLO

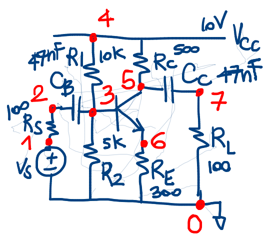
→ * ← commenti

→ COMANDI

→ CIRCUITO

→ MODELLI ←

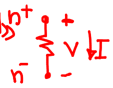
CIRCUITO



Rs	2	1	100
R1	4	3	10K
R2	3	0	5K
RC	4	5	500
RE	6	0	300
RL	7	0	100
CB	3	2	47N
CC	5	7	47N
Vcc	4	0	DC 10
Vs	1	0	AC 1

Resistenza

Rxxxxxxx n^+ n^- val <TC1=val <TC2=val>>



SUFFISSI	
K	10^3
MEG	10^6
G	10^9
T	10^{12}
M	10^{-3}
U	10^{-6}
N	10^{-9}
P	10^{-12}

$$R(T) = R(T_0)\left[1 + TC_1(T-T_0) + TC_2(T-T_0)^2\right]$$

$T_0 = 290\,K$

TC_1 coeff. di temp di ord 1

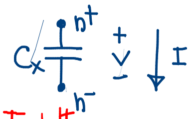
TC_2 coeff di temp di ord 2

R1 3 0 100 TC1=0.01

Capacità

Cxxxxxx n^+ n^- val < IC = val >

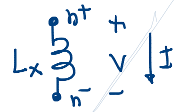
Initial Conditions (V)



Induttanza

Lxxxxxx n^+ n^- val < IC = val >

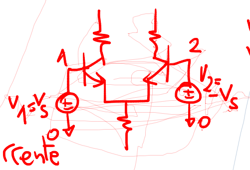
(I)



Generatori indipendenti

→ di Tensione

Vxxxxx n^+ n^- <DC val><AC val <phase>>



V1 1 0 AC 1 0

V2 2 0 AC 1 180

→ di corrente

Ixxxxx n^+ n^- <DC val><AC val <phase>>

Generatori indipendenti x i transitori

Vxxx n^+ n^- PULSE(V_1, V_2, t_{delay}, t_{rise}, t_{pulse}, t_{fall}, T_{period})