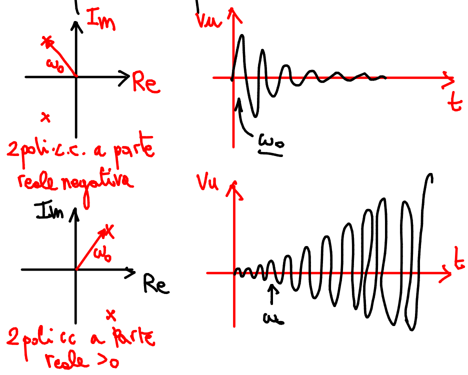
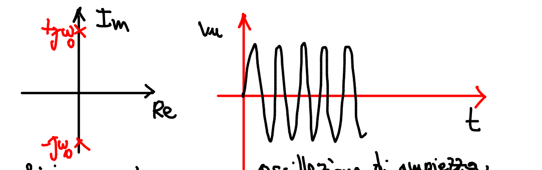
Oscillatori

Oscillatore: circuito in grado di fornire in uscita un segnale periodico a freq. prefissata

Consideriamo oscillatori ottenuti da sistemi lineari in reazione

Risposte all'impulso di un sistema lineare a 2 poli





poli imm. puri

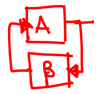
oscillazione di ampiezza costante a frequenza $f_0 = \frac{\omega_0}{2\pi}$

Abbiamo bisogno di:

All'innesco : poli c.c. a parte reale positiva

A regime : poli imm puri di modulo ω_0

Sistema in Reazione



$$A_F = \frac{A_e}{1-\beta A_e}$$

A regime : i poli di A_F devono essere $\pm j\omega_0$

$\rightarrow \; 1-\beta A_e(j\omega_0) = 0 \rightarrow \boxed{\beta A_e(j\omega_0) = 1}$

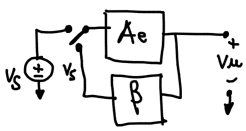
Criterio di Barkhausen a Regime
$$\begin{cases} |\beta A_e(j\omega_0)| = 1 \\ \angle \beta A_e(j\omega_0) = 0 \end{cases}$$

Criterio di Barkhausen all'innesco

$|\beta A_e(j\omega_0)| > 1$

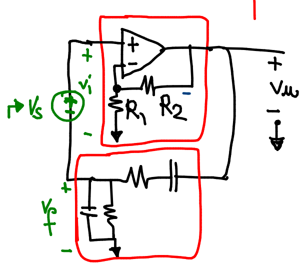
$\angle \beta A_e(j\omega_0) = 0$

[Condizione necessaria] (non sufficiente)



Vogliamo che il criterio valga soltanto per un valore di ω_0 → fase di βA_e monotona

Oscillatore a ponte di Wien

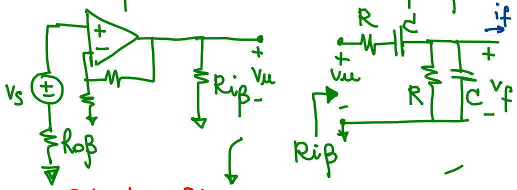


βA_e è il guadagno d'anello

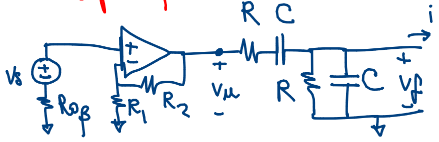
$A_e = \frac{v_u}{v_s}\Big|_{\beta=0}$

$\beta = \frac{v_f}{v_u}\Big|_{i_f=0}$

Rete per A_e | Rete per β



Rete per il βA_e



$$A_e = \left.\frac{v_u}{v_s}\right|_{\beta=0} = 1 + \frac{R_2}{R_1}$$

$$\beta = \left.\frac{v_f}{v_u}\right|_{i_f=0} = \frac{R \parallel \frac{1}{Cs}}{R \parallel \frac{1}{Cs} + R + \frac{1}{Cs}} = \frac{\frac{R}{RCs+1}}{\frac{R}{RCs+1} + R + \frac{1}{Cs}}$$

$$= \frac{RCs}{RCs + R^2C^2s^2 + RCs + RCs + 1} = \frac{RCs}{R^2C^2s^2 + 3RCs + 1}$$

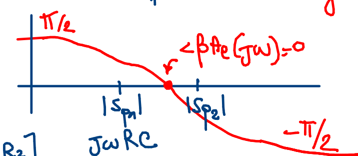
$$\beta A_e(s) = \left[1 + \frac{R_2}{R_1}\right] \frac{RCs}{(RCs)^2 + 3RCs + 1}$$

1 zero nell'origine

2 poli reali negativi

Criterio di Barkhausen all'innesco

$\angle \beta A_e(j\omega)$



$$\beta A_e(j\omega) = \left[1 + \frac{R_2}{R_1}\right] \frac{j\omega RC}{-R^2C^2\omega^2 + j\omega 3RC + 1}$$

$$1 - R^2C^2\omega^2 = 0 \longrightarrow \boxed{\omega_0 = \frac{1}{RC}}$$

soddisfa la condizione sulla fase

$$\beta A_e(j\omega_0) = \left[1 + \frac{R_2}{R_1}\right] \cdot \frac{1}{3}$$

$\beta A(j\omega_0) > 1$ se $R_2 > 2R_1$

$\beta A(j\omega_0) = 1$ se $R_2 = 2R_1$

devo avere all'innesco $R_2 > 2R_1$, e a regime $R_2 = 2R_1$

Regolazione d'ampiezza

Es: R_2 NTC Negative Temperature Coefficient

$$R_2(T) = R_2(T_0)\left[1 + T_C(T - T_0)\right]$$

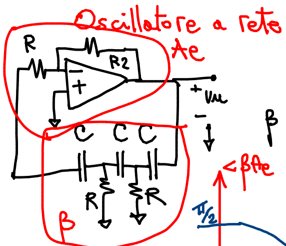
$$P(R_2) = \frac{1}{2} R_2 \left(\frac{V_u}{R_1 + R_2}\right)^2$$

V_u ampiezza della sinusoide

$P(R_2) \propto (T - T_0)$

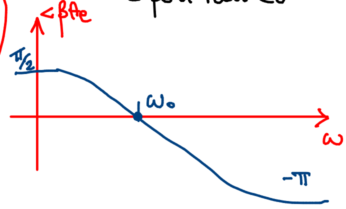
$V_u \uparrow \to P(R_2)\uparrow \to T\uparrow \to R_2\downarrow \to |\beta A_e|\downarrow$

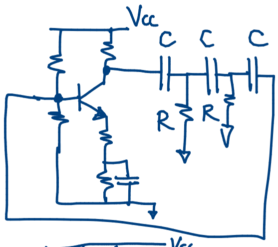
Oscillatore a rete di sfasamento



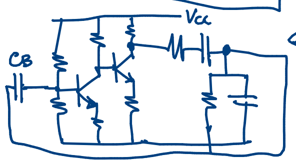
$A_e < 0$

β: 3 zeri nell'origine, 3 poli reali < 0



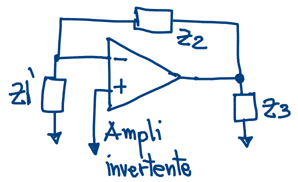


Oscillatore a rete di sfasamento



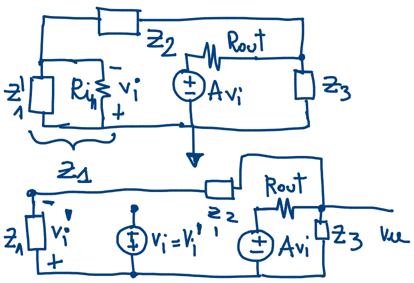
Oscillatore a ponte di Wien

Oscillatore a tre punti

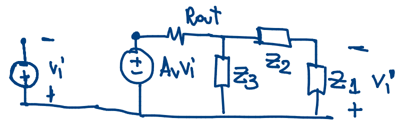


Z_1, Z_2, Z_3 impedenze reattiva (parte reale nulla)

$Z_1 = [Z_1' \,/\!/\, R_{in}]$



$A_e = \frac{v_u}{v_i}$ $\quad \beta = \frac{v_i'}{v_u}$ $\quad \rightarrow \beta A_e = \frac{v_i'}{v_i}$



$$\frac{v_i'}{v_i} = \beta A_e = -A_v \frac{Z_3 /\!/ [Z_1+Z_2]}{R_{out} + Z_3 /\!/ [Z_1+Z_2]} \cdot \frac{Z_1}{Z_1+Z_2} =$$

$$= -A_v \frac{Z_3 \cancel{(Z_1+Z_2)}}{R_{out}[Z_1+Z_2+Z_3] + Z_3(Z_1+Z_2)} \cdot \frac{Z_1}{\cancel{(Z_1+Z_2)}}$$

$$\beta A_e = \frac{-A_v Z_1 Z_3}{R_{out}[Z_1+Z_2+Z_3] + Z_3(Z_1+Z_2)}$$

Z_1, Z_2, Z_3 reattive : $\quad Z_1 = jX_1$, $Z_2 = jX_2$, $Z_3 = jX_3$

$$\beta A(j\omega) = \frac{+A_v X_1 X_3}{jR_{out}(X_1+X_2+X_3) - X_3(X_1+X_2)}$$

$\angle \beta A(j\omega) = 0$ se $(X_1+X_2+X_3) = 0$

Supponiamo sia per $\omega = \omega_0$

$$\beta A(j\omega_0) = \frac{-A_v X_1 \cancel{X_3}}{\cancel{X_3}(X_1+X_2)} = A_v \frac{X_1}{X_3} \boxed{\begin{matrix} >1 \\ =1 \end{matrix}}$$

All'innesco $\quad \beta A(j\omega_0) = \frac{A_v X_1(j\omega_0)}{X_3(j\omega_0)} > 1$

A regime $\quad \beta A(j\omega_0) = \frac{A_v X_1(j\omega_0)}{X_3(j\omega_0)} = 1$

2 possibilità :

- $X_1, X_3 > 0$ [L]; $X_2 < 0$ [C] $\quad$ Oscillatore di Hartley

$X_1+X_2+X_3 = 0 \rightarrow j\omega L_1 + j\omega L_3 + \frac{1}{j\omega C_2} = 0$

$\rightarrow \omega_0 = \frac{1}{\sqrt{(L_1+L_3)C_2}} \leftarrow$

$X_1, X_3 < 0$ [C] Oscillatore di Colpitts

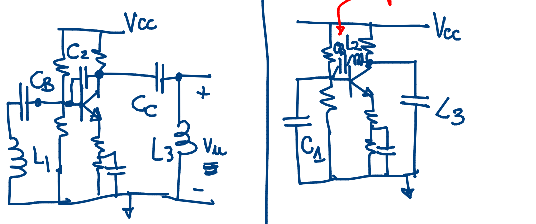
$X_2 > 0$ [L]

$$X_1 + X_2 + X_3 = 0 \rightarrow \frac{1}{j\omega C_1} + j\omega L_2 + \frac{1}{j\omega C_3} = 0$$

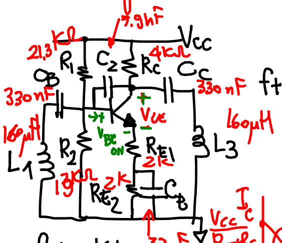
$$\omega_0 = \frac{1}{\sqrt{L_2 \frac{C_1 C_3}{C_1 + C_3}}}$$

Oscillatore di Hartley | Oscillatore di Colpitts

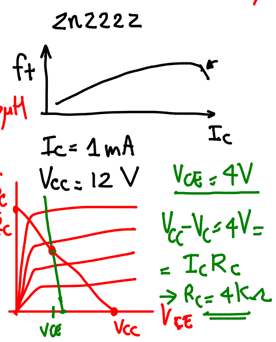
Capacità di blocco



Es. Progetto di un oscillatore di Hartley



$f = 100$ KHz



$I_C = 1$ mA

$V_{CC} = 12$ V

$V_{CE} = 4V$

$V_{CC} - V_C = 4V = I_C R_C$

$\rightarrow R_C = 4k\Omega$

$V_E = 4V \rightarrow V_E = (R_{E_1} + R_{E_2}) I_C \rightarrow R_{E_1} + R_{E_2} = 4k\Omega$ (I_C: 1mA)

$V_B = V_E + V_{BE_{ON}} = 4.7V$

$I_B = \frac{I_C}{h_{FE}} = \frac{1}{150} = 0{,}0067 = 6{,}7\,\mu A$

$\rightarrow$ Scegliamo I_2 (corrente in R_2) $= 350\,\mu A$

$R_2 I_2 = V_B \rightarrow R_2 = \frac{V_B}{I_2} = \frac{4{,}7}{0{,}35} = 13\,K\Omega$

$I_1 = I_2 - I_B = 343.3\,\mu A$

$R_1 I_1 = V_{CC} - V_B \rightarrow R_1 = \frac{V_{CC} - V_B}{I_1} = \frac{7{,}3}{0{,}343} = 21{,}3k\Omega$

$I_C = 1mA$ $V_{CE} = 4V$

$h_{ie} = 5K\Omega$

$h_{fe} = 175$

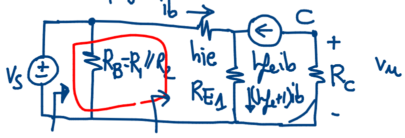
$h_{oe} = 20 \times 10^{-6}\,\Omega^{-1} \rightarrow r_o = \frac{1}{h_{oe}} = 50k\Omega > R_C, R_E$ quindi nei succ. calcoli trascuriamo r_o

$C_{b'e}$, $C_{b'c} = 5pF$

$$f_T = 90\,MHz = \frac{g_m}{2\pi(C_{b'e} + C_{b'c})} \qquad g_m = I_C/V_T$$

$$C_{b'e} = \frac{I_C}{2\pi f_T V_T} - C_{b'c} = 68 - 5 = 63pF$$

Amplificatore a centro banda



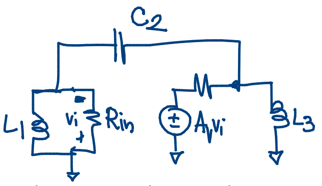
$$V_S = h_{ie} i_b + R_{E_1}(1 + h_{fe}) i_b \rightarrow \frac{V_S}{i_b} = h_{ie} + R_{E_1}(1 + h_{fe})$$

$$V_u = -R_C h_{fe} i_b$$

$$A_V = \frac{V_u}{V_S} = \frac{-R_C h_{fe}}{h_{ie} + R_{E_1}(1 + h_{fe})}$$

$$R_{in} = R_B /\!/ [h_{ie} + R_{E_1}(1 + h_{fe})]$$

(R_B: 8kΩ)



C_B, C_C, C_E in modo che siano cc a 100kHz

Scelgo L_1 in modo che $j\omega_0 L_1 \parallel R_{in} \sim j\omega_0 L_1 \sim 100\Omega$

Dobbiamo avere in $\omega_0 = 2\pi \cdot 100\, 10^3$ rad/s

$$X_1(\omega_0) + X_2(\omega_0) + X_3(\omega_0) = 0 \Rightarrow \underset{100}{j\omega_0 L_1} + \underset{-200}{\frac{1}{j\omega_0 C_2}} + \underset{100}{j\omega_0 L_3} = 0$$

$$\beta A_e(\omega_0) = A_v \frac{X_1(\omega_0)}{X_3(\omega_0)} = A_v \frac{L_1}{L_3}$$

$$\omega_0 L_1 = 100\Omega \Rightarrow L_1 = \frac{100}{2\pi \cdot 10^5} = 159\,\mu H$$

$$\underline{160\,\mu H}$$

$L_2 = 160\,\mu H$

$$\frac{1}{\omega_0 C_2} = \omega_0 (L_1 + L_2)$$

$$\hookrightarrow C_2 = \frac{1}{\omega_0^2 (L_1 + L_2)} = \underline{\underline{7.9\,nF}} \gg C_{bc}$$

Se $R_{E1} = 2k\Omega \rightarrow A_v \simeq 2\ [1{,}96] \leftarrow$ Criterio di Barkhausen all'innesco è soddisfatto

$$\left| \frac{1}{j\omega_0 C_B} \right| \sim 5\Omega \Rightarrow C_B = \frac{1}{5 \cdot \omega_0} = 318\,nF\ \left[\underline{\underline{330\,nF}}\right]$$

$$\parallel$$

$$C_C$$

$$C_E = \underline{33\,nF}$$

Es. 2 SPICE

Progettare un oscillatore di Colpitts a frequenza 1MHz con ampiezza minima dell'oscillazione (picco-picco) di 1V.
Si usi un 2n2222 con $I_C = 2mA$.
Tensione di alimentazione $V_C = 15V$

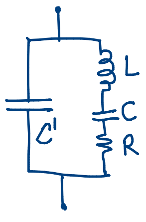
Oscillatori al quarzo

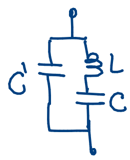
$\underline{SiO_2}$ cristallino

materiale piezoelettrico

XTal

Circuito equiv. elettrico





$$Z = \frac{1}{j\omega C'} \parallel \left[j\omega L + \frac{1}{j\omega C} \right] =$$

$$= \frac{\left(j\omega L + \frac{1}{j\omega C}\right) \frac{1}{j\omega C'}}{j\omega L + \frac{1}{j\omega C} + \frac{1}{j\omega C'}} =$$

$$Z = \frac{(1 - \omega^2 LC) \frac{1}{j\omega C'}}{1 - \omega^2 LC + \frac{C}{C'}} = \frac{1}{j\omega (C' + C)} \frac{1 - \omega^2 LC}{1 - \omega^2 L \frac{CC'}{(C+C')}}$$

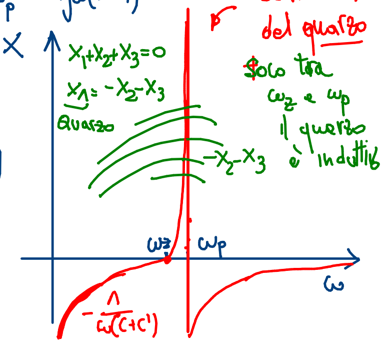
$$Z^{(j\omega)} = \frac{1-\omega^2/\omega_z^2}{1-\omega^2/\omega_p} \cdot \frac{1}{j\omega(C+C_1)} = jX$$

$$\omega_z^2 = \frac{1}{LC}$$

$$\omega_p^2 = \frac{1}{L\frac{CC'}{(C+C')}}$$

$\omega_z \lesssim \omega_p$

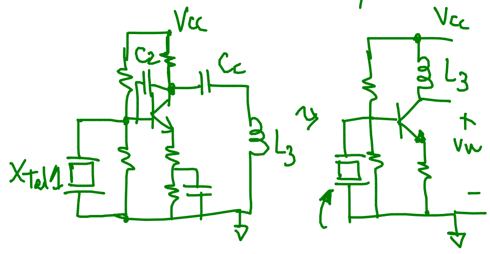
$\underline{C' \gg C}$



reattanza del quarzo

Solo tra ω_z e ω_p il quarzo è induttivo

Oscillatore di Hartley



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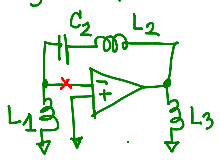
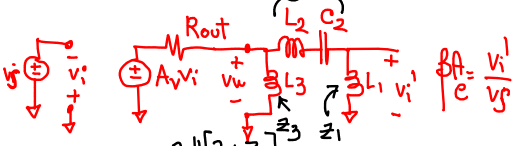


$L_1 = L_2 = L_3 = 1\ \mu H$

$C_2 = 0.47\ nF$

$A_V = 10,\ R_{in}\infty,\ R_{out} = 1K\Omega$



$$\beta A_e = \frac{v_i'}{v_s}$$

$$v_i' = \frac{Z_1}{Z_1+Z_2} \cdot A_v v_i \frac{Z_3 \| [Z_1+Z_2]}{Z_3\|[Z_1+Z_2] + R_{out}}$$

$$\beta A_e = \frac{v_i'}{v_s} = -A_v \frac{Z_1}{\cancel{Z_1+Z_2}} \cdot \frac{Z_3\cancel{(Z_1+Z_2)}}{Z_3(Z_1+Z_2) + R_{out}(Z_1+Z_2+Z_3)}$$

$$\beta A_e = \frac{+A_v \quad X_1 X_3}{-X_3(X_1+X_2) + jR_{out}(X_1+X_2+X_3)}$$

in ω_0: $X_1+X_2+X_3=0$

$$j\omega L_1 + j\omega L_2 + \frac{1}{j\omega C_2} + j\omega L_3 = 0$$

$$j\omega[L_1+L_2+L_3] + \frac{1}{j\omega C_2} = 0$$

$$\omega_0 = \frac{1}{\sqrt{(L_1+L_2+L_3)C_2}} = 26.6\cdot 10^6\ rad/s$$

$\underline{\underline{f_0 = 4.24\ MHz}}$

$$\beta A_e(j\omega_0) = \frac{A_v X_1}{X_3} = \frac{A_v L_1}{L_3} = 10 > 1$$

l'oscillazione si innesca

Criterio di Barkhausen verificato per $\underline{f_0 = 4.2\ MHz}$

15.2.04 A1

$A_{vo} = 10^4$

$R_{in} = 100\ K\Omega$

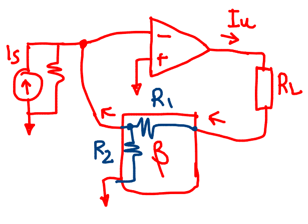
$R_{out} = 200\ \Omega$

$R_{IF} < 100\ \Omega$

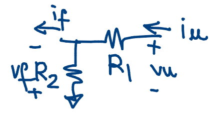
$R_{OF} > 20\ K\Omega$

INSERZIONE DI CORRENTE

PRELIEVO DI CORRENTE



β



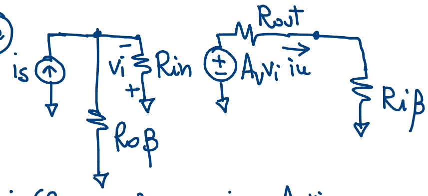
$$\begin{bmatrix} i_f \\ v_u \end{bmatrix} = \begin{bmatrix} \beta & \frac{1}{R_{o\beta}} \\ R_{i\beta} & \times \end{bmatrix} \begin{bmatrix} i_u \\ v_f \end{bmatrix}$$

$$\beta = \left.\frac{i_f}{i_u}\right|_{v_f=0} = 1 \; ; \quad R_{o\beta} = \left.\frac{v_f}{i_f}\right|_{i_u=0} = R_2 \; ; \quad R_{i\beta} = \left.\frac{v_u}{i_u}\right|_{v_f=0} = R_1$$

A_e

$R_s \to \infty$

$R_L = 0$



$$v_i = -i_s\left(R_{o\beta} \parallel R_{in}\right) \qquad i_u = \frac{A_v v_i}{R_{out} + R_{i\beta}}$$

$$A_e = \left.\frac{i_u}{i_s}\right|_{\beta=0} = \frac{-A_v\left(R_{o\beta} \parallel R_{in}\right)}{R_{out} + R_{i\beta}} = \frac{-A_v\left(R_2 \parallel R_{in}\right)}{R_{out} + R_1}$$

$$R_{IF} = \frac{R_{in} \parallel R_{o\beta}}{(1-\beta A_e)} = \frac{R_{in} \parallel \overset{100\,k\Omega}{R_2}}{1 + \dfrac{A_v\left(R_{in} \parallel R_2\right)}{R_{out} + R_1}} < 100\,\Omega$$

$$R_{OF} = \left(R_{i\beta} + R_{out}\right)\left(1 - \beta A_e\right) = \left(R_1 + \underset{200\,\Omega}{R_{out}}\right)\left[1 + \frac{A_v\left(R_{in} \parallel R_2\right)}{R_{out} + R_1}\right] > 20\,k\Omega$$

Poniamo $-\beta A_e \gg 1$

$$R_{IF} \sim \frac{R_1 + \overset{200}{R_{out}}}{\underset{10^4}{A_V}} < 100\,\Omega \;\rightarrow\; R_1 = 100\,k\Omega$$

$\parallel$

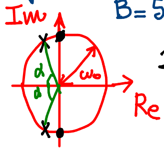
$$R_{OF} \sim \underset{10^4}{A_v}\left(\underset{100\,k\Omega}{R_{in}} \parallel R_2\right) > 20\,k\Omega \;\rightarrow\; R_2 = 100\,k\Omega$$

$$\boxed{R_{IF} = 10\,\Omega \,,\; R_{OF} = 500\,M\Omega}$$

Es 3 15.2.04

filtro eliminabanda selettivo (Notch)

$B = 500\,Hz \cdot f_0 = 5\,kHz$

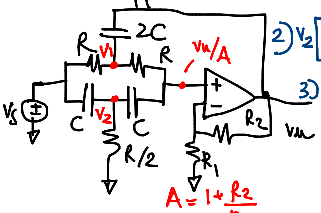


2 Poli e 2 zeri immaginari di modulo $\omega_0 = 2\pi f_0 = 31.4\,krad/s$

Q dei poli $= \dfrac{f_0}{B} = 10$

$\hookrightarrow \dfrac{1}{2\cos\varphi} \Rightarrow \cos\varphi = \dfrac{1}{2Q} = 0{,}05$

filtro a doppio T



$A = 1 + \dfrac{R_2}{R_1}$

1) $v_1\left[2Cs + \dfrac{2}{R}\right] - \dfrac{v_s}{R} - \dfrac{v_u}{AR} - 2Cs\,v_u = 0$

2) $v_2\left[2Cs + \dfrac{2}{R}\right] - v_s Cs - \dfrac{v_u Cs}{A} = 0$

3) $\dfrac{v_u}{A}\left[Cs + \dfrac{1}{R}\right] - \dfrac{v_1}{R} - v_2 Cs = 0$

$$\frac{v_u}{A}\left[1 + RCs\right] - \frac{v_s + \frac{v_u}{A} + 2RCs\,v_u}{2(RCs+1)} - RCs\,\frac{v_s RCs + \frac{v_u}{A} RCs}{2(RCs+1)} = 0$$

$$v_u\left[(1+RCs)^2\, 2 - 1 - 2ARCs - (RCs)^2\right] = v_s\left[A + A R^2C^2s^2\right]$$

$$\frac{v_u}{v_s} = \frac{A\left(1 + R^2C^2s^2\right)}{1\,\cancel{2} + 4RCs + \cancel{2}R^2C^2s^2 - \cancel{1} - 2ARCs - \cancel{R^2C^2s^2}}$$

$$\frac{v_u}{v_s} = \frac{A\left(1 + R^2C^2s^2\right)}{R^2C^2s^2 + (4-2A)RCs + 1}$$

$$= \frac{A\left[1 + s^2/\omega_0^2\right]}{s^2/\omega_0^2 + \dfrac{s}{Q\omega_0} + 1}$$

$$\omega_0 = \frac{1}{RC} \quad (31.4\,krad/s)$$

$$Q = \frac{1}{4-2A} \quad (10)$$

$C = 1\,nF$

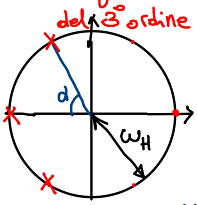
$R = \dfrac{1}{\omega_0 C} = 31.8\,k\Omega$

$PGB \gg A f_0$

$$4 - 2A = \frac{1}{10} \;\Rightarrow\; A = \left(4 - \frac{1}{10}\right)\frac{1}{2} = 1.95 = 1 + \frac{R_2}{R_1}$$

$R_1 = 10\,k\Omega \quad R_2 = 9{,}5\,k\Omega$

Progettare un filtro di Butterworth del 3° ordine con $f_H = 2KHz$

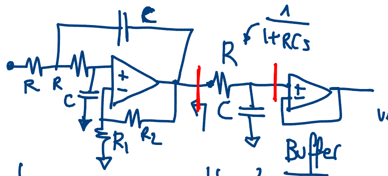


$\omega_H = 2\pi f_H = 12.56\ Krad/s$

$\alpha = \pi/3$

$Q = \frac{1}{2\cos\alpha} = 1$

Celle S&K $H = \frac{H_0}{\left(\frac{s}{\omega_H}\right)^2 + \frac{s}{Q\omega_H} + 1}$



SK

polo reale

Buffer

Inseguitore

$H = \frac{A}{R^2C^2s^2 + (3-A)RCs + 1}$

$\downarrow$

$\omega_H = \frac{1}{RC}$

$\omega_p = \frac{1}{RC} = \omega_H$

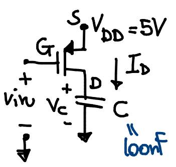
$Q = \frac{1}{3-A} \rightarrow A = 3 - \frac{1}{Q} = 2 = 1 + \frac{R_2}{R_1}$

$\omega_H = 12.56\ Krad/s$

$C = 10nF$

$R = \frac{1}{\omega_H C} = 7.96\ k\Omega$

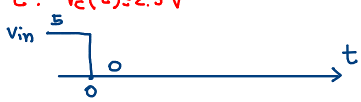
$R_1 = R_2 = R$



$t < 0 \quad vin = 5V \quad V_c = 0V$

$t \geq 0 \quad vin = 0V$

t^*? $V_c(t^*) = 2.5V$



pMOS:

$V_T = -2V$

$K = 0.2\ mA/V^2$

sat: $I_D = \frac{K}{2}(V_{GS} - V_T)^2 \quad V_{GS} < V_T,\ V_{DS} < V_{GS} - V_T$

lin: $I_D = K\left[(V_{GS} - V_T)V_{DS} - \frac{V_{DS}^2}{2}\right] \quad V_{GS} < V_T,\ V_{DS} > V_{GS} - V_T$

$t = 0^+ \quad V_{GS} = -5V < V_T$

$V_{DS} = V_D - V_S = V_C - V_{DD} = 0 - 5 = -5V < \underbrace{V_{GS} - V_T}_{-3}$

il pMOS è in sat

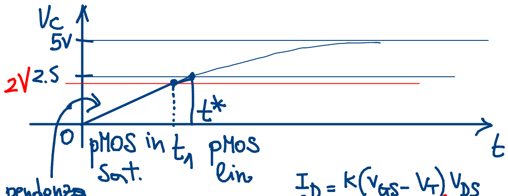
$I_D = \frac{K}{2}(V_{GS} - V_T)^2 = \frac{0.2}{2}\cdot(5-2)^2 = 0.9mA$

$V_C = \frac{1}{C}\int_0^t I_D(t')\,dt'$ finché $I_D = 0.9mA \Rightarrow V_C = \frac{I_D t}{C}$

il pMOS è in SAT finché $V_{DS} \leq V_{GS} - V_T = -3$

$V_C - V_{DD} < -3 \Rightarrow V_C < 2V$

finché $V_C < 2V$ abbiamo $V_C = \frac{I_D t}{C}$, poi il pMOS va in zona lineare



pendenza $\frac{I_D}{C}$

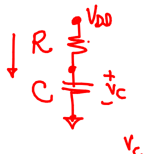
$\frac{I_D t_1}{C} = 2V$

$t_1 = \frac{2C}{I_D} = \frac{2\cdot 100\cdot 10^{-9}}{0.9\cdot 10^{-3}} = 0.22ms$

$I_D = \underbrace{K(V_{GS} - V_T)}_{\text{costante}} V_{DS}$

il pMOS si comporta come une resistenza di valore

$R = \frac{1}{|K(V_{GS} - V_T)|} = 1.6\ k\Omega$



$t_1: V_c(t_1) = 2V$

$V_c(t) = A + Be^{-\frac{t}{RC}}$

$\lim_{t\to\infty} V_C = V_{DD} = A$

$V_c(t_1) = A + Be^{-\frac{t_1}{RC}}$

$B = [V_c(t_1) - A]\,e^{t_1/RC} = -3e^{t_1/RC}$

$V_c(t) = 5 - 3e^{(t_1 - t)/RC}$

$t^* \quad V_c(t^*) = 2.5 = 5 - 3\exp\left[\frac{t_1 - t^*}{RC}\right] \Rightarrow 3\exp\left[\frac{t_1 - t^*}{RC}\right] = \frac{2.5}{3}$

$$\frac{t_1 - t^*}{RC} = \ln\left(\frac{1}{6}\right) \Rightarrow t^* = t_1 - RC \ln\left(\frac{5}{6}\right)$$

$$t^* = t_1 + RC \ln(6/5)$$

$$= 0.22\,\text{ms} + \underbrace{1.6\,\text{k}\Omega \cdot 100\,\text{nF} \cdot 0.18}_{=\,0.51\,\text{ms}}$$

$$\rightarrow \boxed{t^* = 0.25\,\text{ms}} \leftarrow$$