

DC-DC Converters

Power Delivery Network of a Smartphone

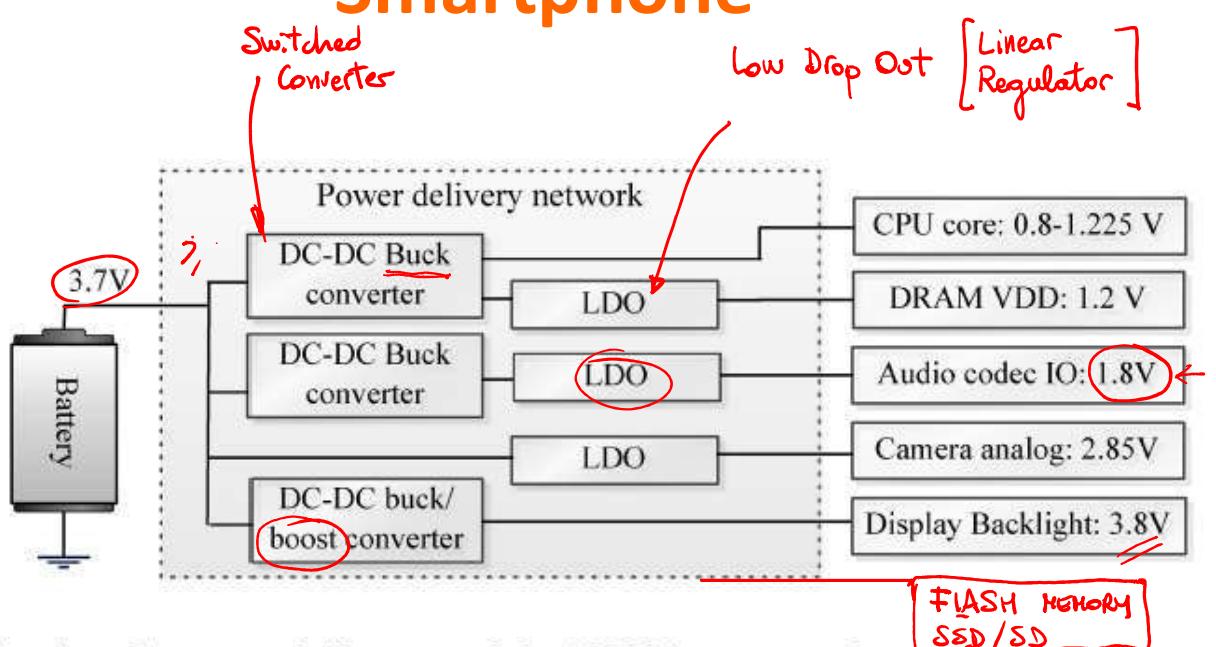


Fig. 1. Conceptual diagram of the PDN in a smartphone platform.

DC-DC Converters

Typical uses:

- DC Power supplies]
- DC Motor drives]
- Portable electronics]

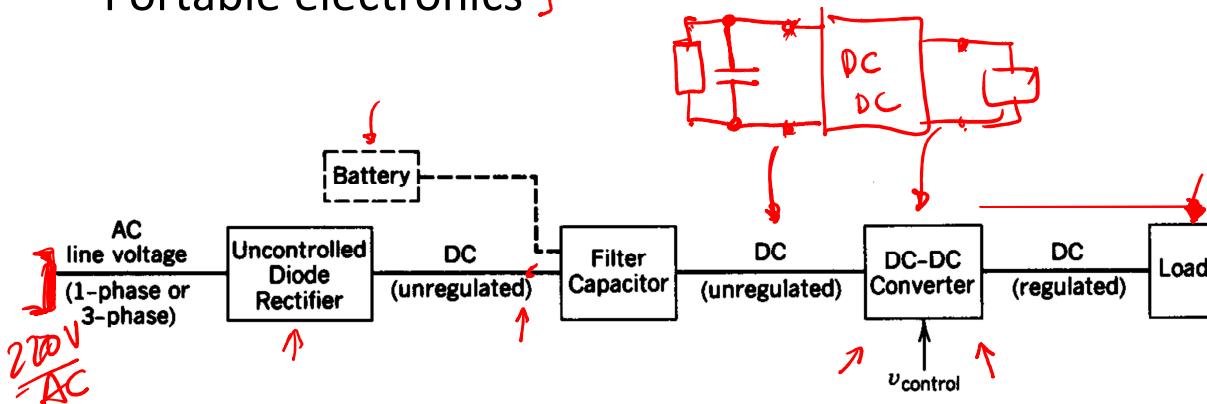


Figure 7-1 A dc-dc converter system.

DC-DC Converters

Typical uses:

- DC Power supplies
- DC Motor drives
- Portable Electronics

Types of converters

- { • Step-down (buck)
- Step-up (boost)
- Buck-boost *changes the sign*
- Cuk *MOTOR CONTROL*
- Full-Bridge

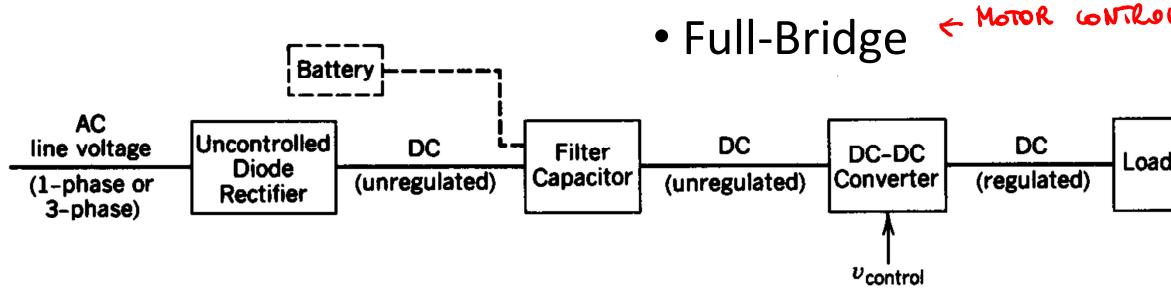


Figure 7-1 A dc-dc converter system.

Ideal concept of step-down converter with PWM* switching

v_o (* Pulse Width Modulation)

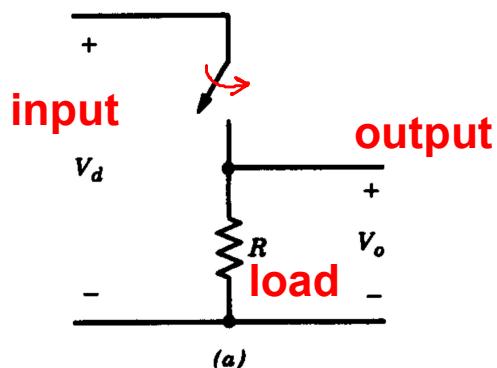


Figure 7-2 Switch-mode dc-dc

Assumptions: Switches, L, C are lossless, DC input has zero internal impedance, load is an equivalent R

Ideal concept of step-down converter with PWM* switching

(* Pulse Width Modulation)

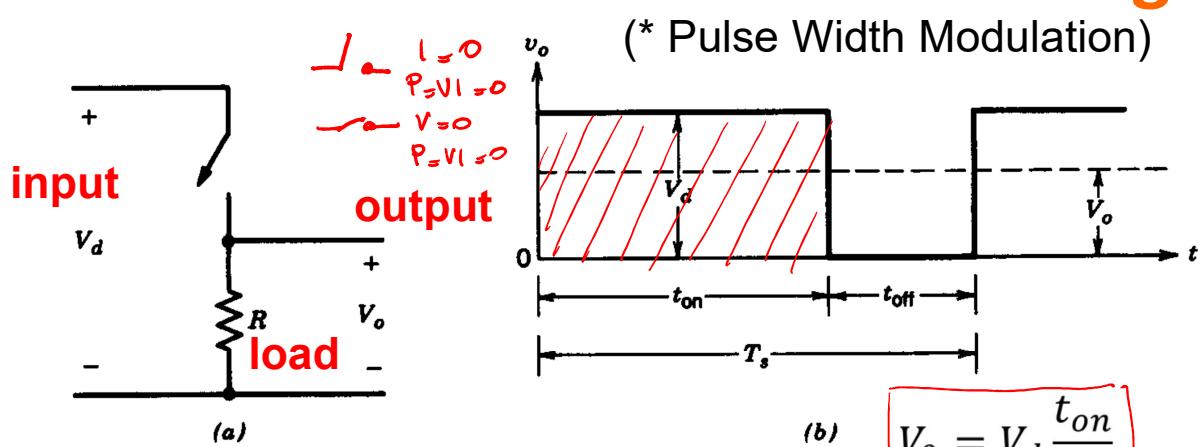


Figure 7-2 Switch-mode dc-dc conversion.

$$V_0 = V_d \frac{t_{on}}{T_s}$$

Assumptions: Switches, L, C are lossless, DC input has zero internal impedance, load is an equivalent R

Ideal concept of step-down converter with PWM* switching

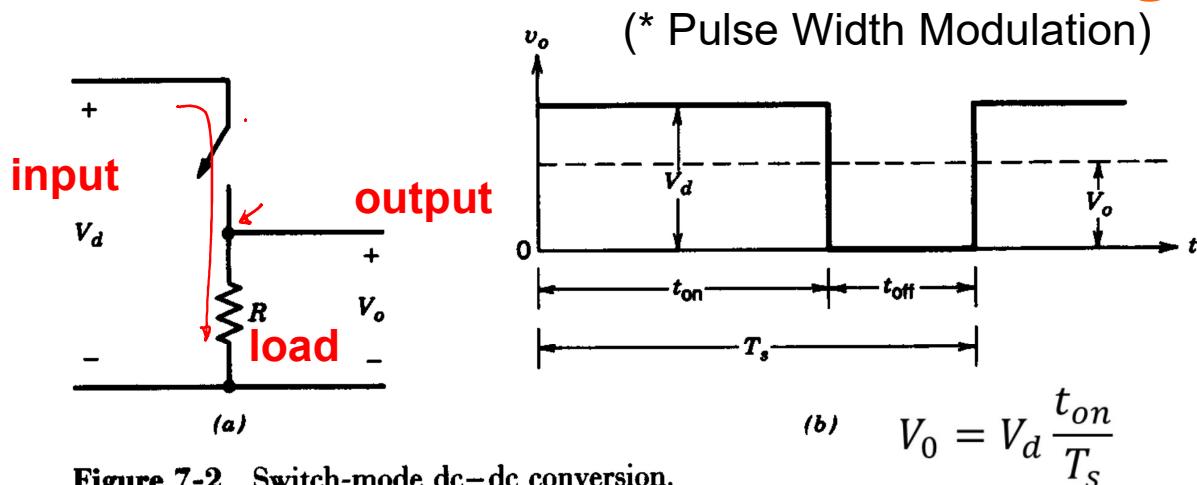


Figure 7-2 Switch-mode dc-dc conversion.

Assumptions: Switches, L, C are lossless, DC input has zero internal impedance, load is an equivalent R

This cannot work: 1. Load is inductive and can destroy switch by dissipating all stored energy, 2. output voltage must be continuous

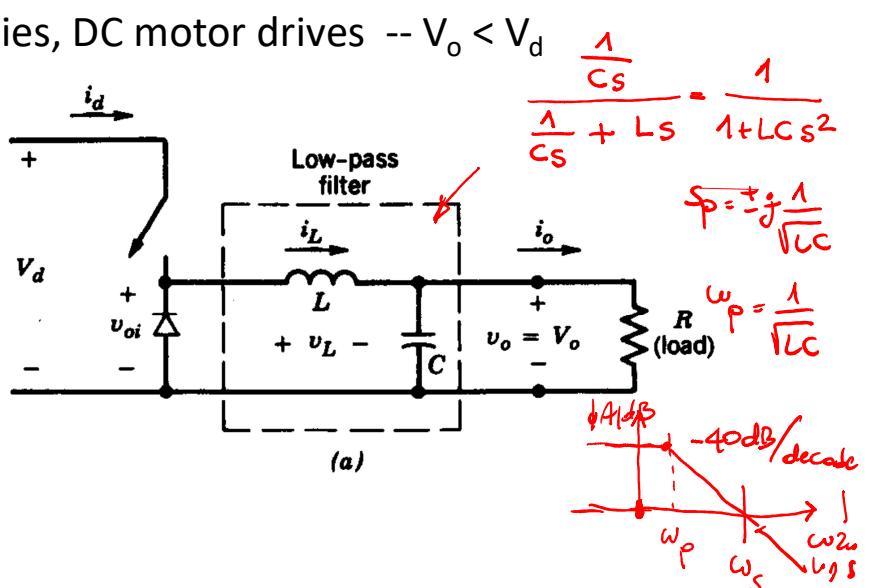
Step-down (buck) converter

DC power supplies, DC motor drives -- $V_o < V_d$

Low-pass filter keeps output voltage constant

Note: 2nd order non dissipative filter

$$\rightarrow f_c = \frac{1}{2\pi} \frac{1}{\sqrt{LC}} \ll f_s$$



Step-down (buck) converter

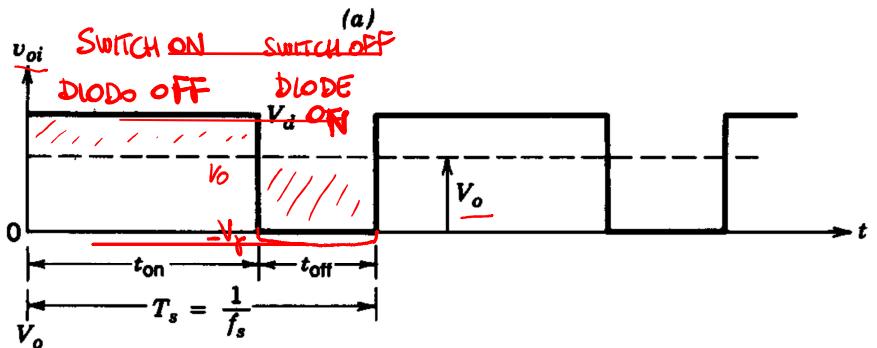
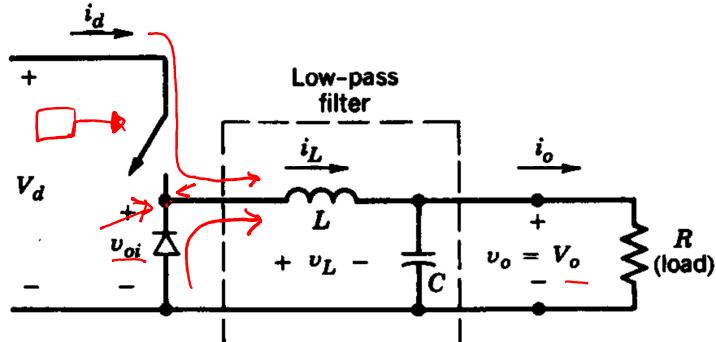
DC power supplies, DC motor drives -- $V_o < V_d$

Low-pass filter keeps output voltage constant

Note: 2nd order non dissipative filter

$$f_c = \frac{1}{2\pi} \frac{1}{\sqrt{LC}} \ll f_s$$

Diode avoids voltage spike on switch (when switch is off, diode provides current to L)



Continuous-conduction mode

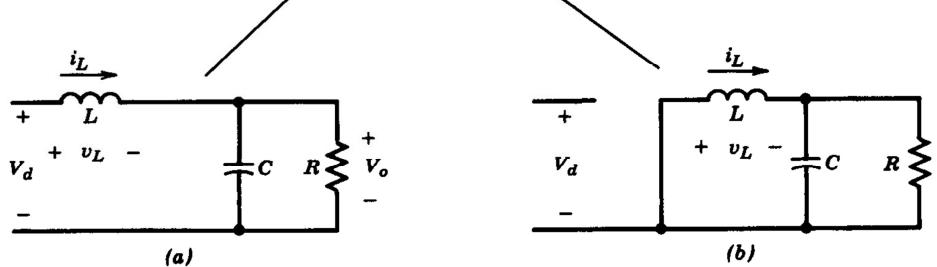
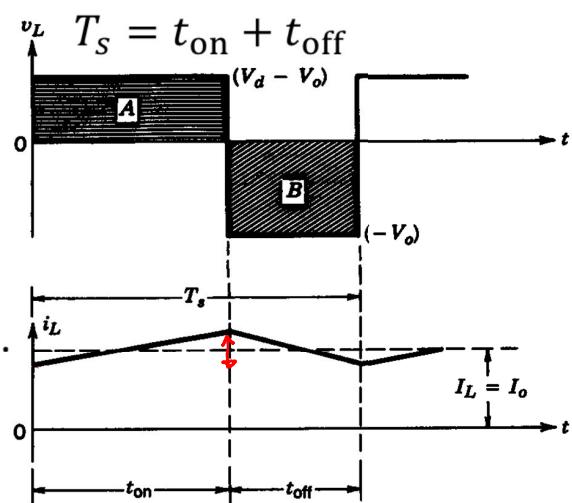
Current in L is always > 0

- t_{on} : $\frac{dI}{dt} = \frac{V_d - V_o}{L}$
- t_{off} : $\frac{dI}{dt} = -\frac{V_o}{L}$

At steady state: $I(t + T_s) = I(t)$.

Therefore

$$\frac{V_d - V_o}{L} t_{on} - \frac{V_o}{L} t_{off} = 0$$



Continuous-conduction mode

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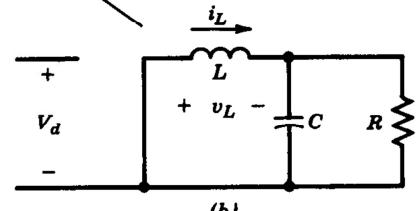
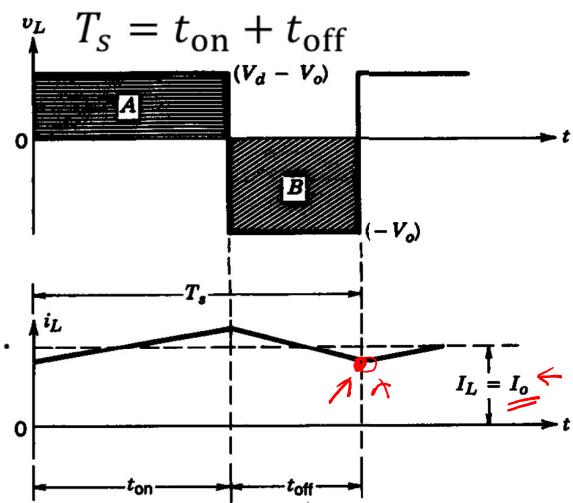
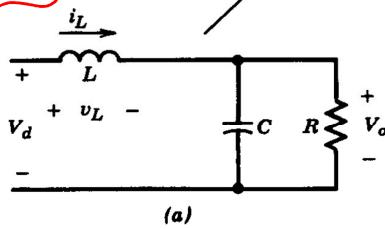
Therefore

$$\frac{V_d - V_o}{L} t_{\text{on}} - \frac{V_o}{L} t_{\text{off}} = 0$$

$$V_d t_{\text{on}} = V_o (t_{\text{on}} + t_{\text{off}})$$

$$\frac{V_o}{V_d} = \frac{t_{\text{on}}}{T_s} = D$$

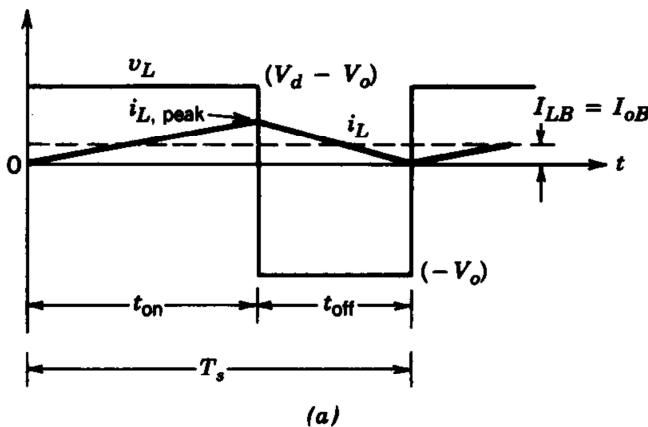
doty cycle



Limit of continuous conduction

If the ripple amplitude $I_{LB} \equiv \frac{I_{\text{peak}}}{2} = I_o$, the converter is at the limit of continuous conduction (i.e. $\min\{I_L\} = 0$)

$$I_{LB} \equiv \frac{I_{\text{peak}}}{2} = \frac{t_{\text{on}}(V_d - V_o)}{2L} = \frac{DT_s V_d (1 - D)}{2L}$$



(b)

Figure 7-6 Current at the boundary of continuous-discontinuous conduction: (a) current waveform; (b) I_{LB} versus D keeping V_d constant.

Limit of continuous conduction

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$$I_{LB} \equiv \frac{I_{peak}}{2} = \frac{t_{on}(V_d - V_o)}{2L} = \frac{DT_s V_d (1 - D)}{2L} = I_{LBmax} 4D(1 - D)$$

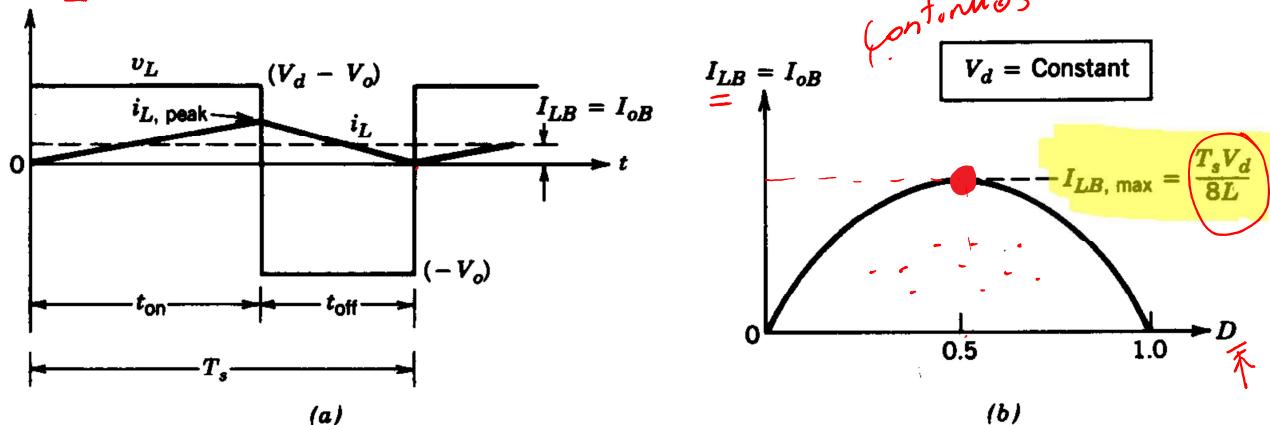


Figure 7-6 Current at the boundary of continuous-discontinuous conduction: (a) current waveform; (b) I_{LB} versus D keeping V_d constant.

Limits of continuous-discontinuous conduction (constant V_d)

Continuous

$$\frac{V_o}{V_d} = D$$

$$\frac{I_o}{I_{LBmax}} > 4D(1 - D)$$

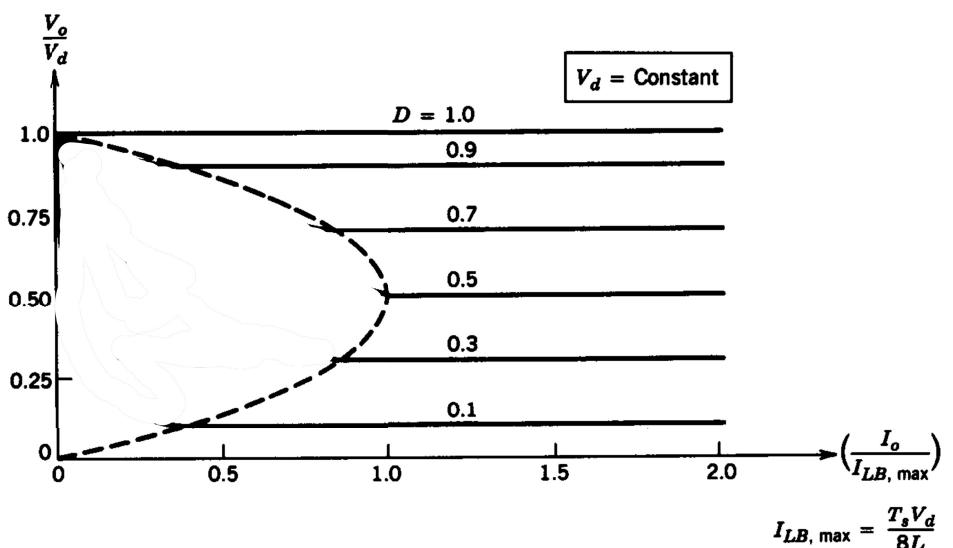


Figure 7-8 Step-down converter characteristics keeping V_d constant.

Discontinuous-conduction mode with constant V_d

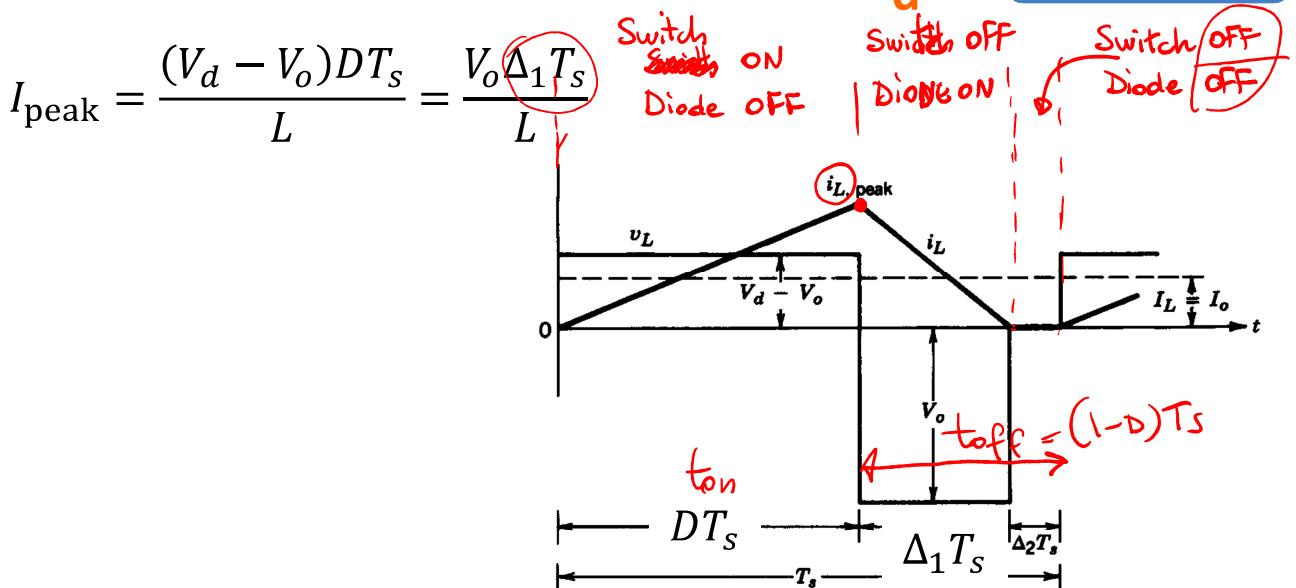


Figure 7-7 Discontinuous conduction in step-down converter.

Discontinuous-conduction mode with constant V_d

$$I_{\text{peak}} = \frac{(V_d - V_o)DT_s}{L} = \frac{V_o \Delta_1 T_s}{L} \rightarrow \frac{V_o}{V_d} = \frac{D}{(D + \Delta_1)}$$

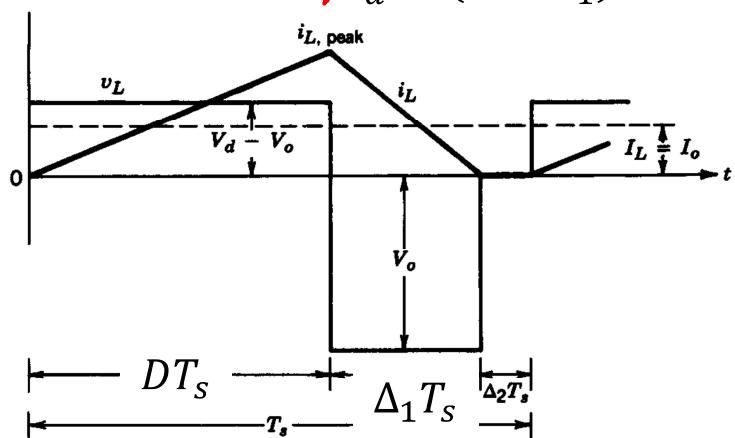


Figure 7-7 Discontinuous conduction in step-down converter.

Discontinuous-conduction mode with constant V_d

Motor drives

$$I_{\text{peak}} = \frac{(V_d - V_o)DT_s}{L} = \frac{V_o\Delta_1 T_s}{L} \quad \rightarrow \quad \frac{V_o}{V_d} = \frac{D}{(D + \Delta_1)}$$

$$I_{\text{peak}} = \frac{V_d T_s}{L} \frac{D\Delta_1}{D + \Delta_1}$$

$$I_{\text{peak}} = 8I_{LB\max} \frac{D\Delta_1}{D + \Delta_1}$$

$$\underline{\underline{I_o T_s = \frac{I_{\text{peak}}(D + \Delta_1)T_s}{2}}} \quad \text{area of the triangle}$$

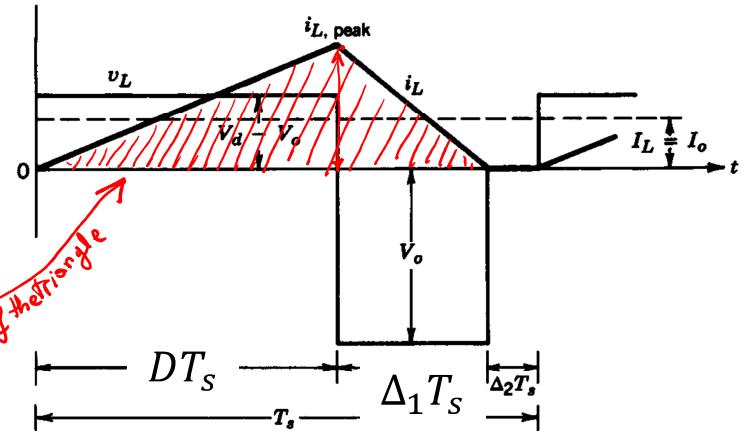


Figure 7-7 Discontinuous conduction in step-down converter.

Discontinuous-conduction mode with constant V_d

Motor drives

$$I_{\text{peak}} = \frac{(V_d - V_o)DT_s}{L} = \frac{V_o\Delta_1 T_s}{L} \quad \rightarrow \quad \frac{V_o}{V_d} = \frac{D^2}{(D^2 + D\Delta_1)}$$

$$I_{\text{peak}} = \frac{V_d T_s}{L} \frac{D\Delta_1}{D + \Delta_1}$$

$$I_{\text{peak}} = 8I_{LB\max} \frac{D\Delta_1}{D + \Delta_1}$$

$$\underline{\underline{I_o T_s = \frac{I_{\text{peak}}(D + \Delta_1)T_s}{2}}}$$

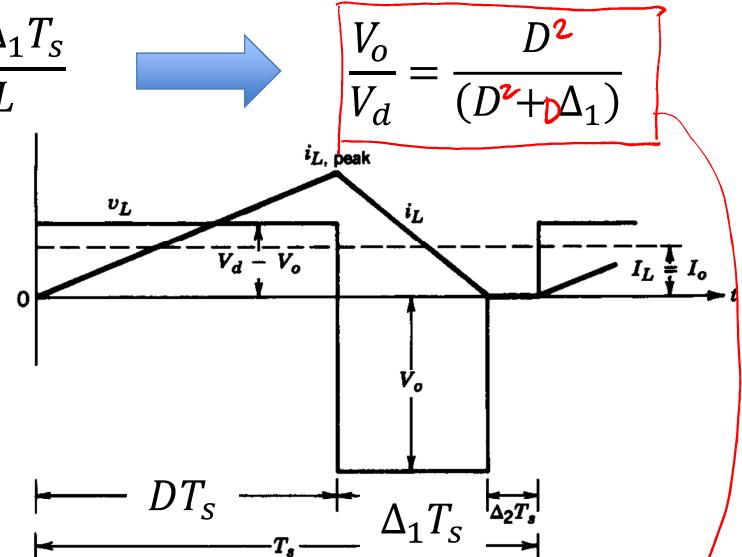


Figure 7-7 Discontinuous conduction in step-down converter.

$$I_o = 4I_{LB\max} D\Delta_1$$

$$\frac{V_o}{V_d} = \frac{D^2}{D^2 + I_o / (4I_{LB\max})} \quad \text{with } I_{LB\max} = \frac{V_d T_s}{8L}$$

Limits of continuous-discontinuous conduction (constant V_d)

Continuous

$$\frac{I_o}{I_{LB\max}} > 4D(1 - D)$$

$$\frac{V_o}{V_d} = D$$

Discontinuous

$$\frac{I_o}{I_{LB\max}} < 4D(1 - D)$$

$$I_{LB, \max} = \frac{T_s V_d}{8L}$$

$$\frac{V_o}{V_d} = \frac{D^2}{D^2 + \frac{I_o}{4I_{LB\max}}}$$

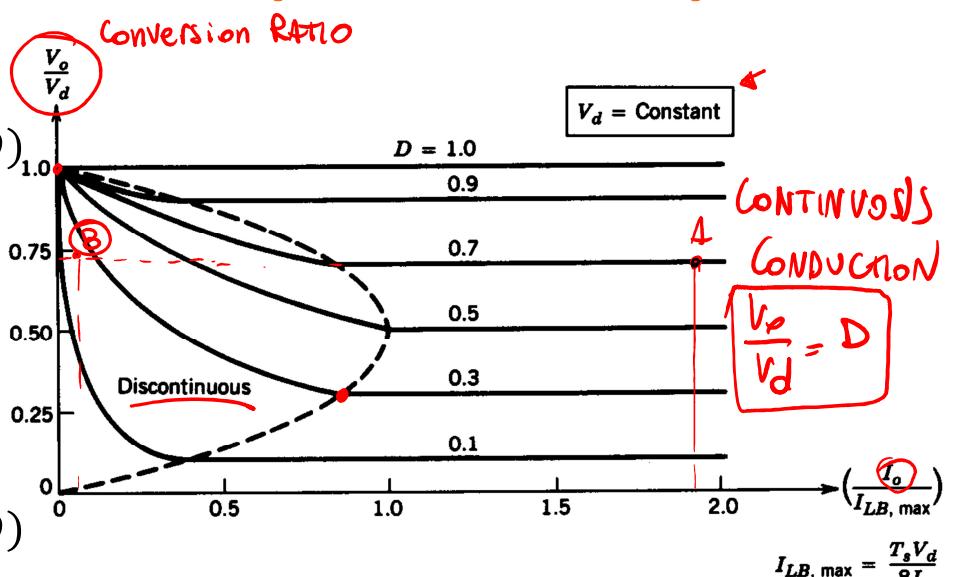
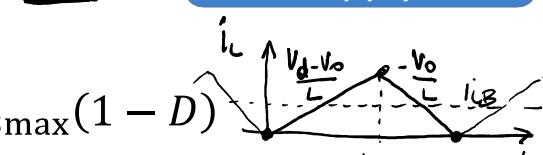


Figure 7-8 Step-down converter characteristics keeping V_d constant.

Discontinuous-conduction with constant V_o

At the limit of continuous conduction

$$I_{LB} = \frac{V_o T_s (1 - D)}{2L} = I_{LB\max} (1 - D)$$



We can write D explicitly from:

$$I_{peak} = \frac{V_o \Delta_1 T_s}{L} = 2I_{LB\max} \Delta_1$$

$$I_o = \frac{I_{peak}(D + \Delta_1)}{2} = I_{LB\max} \Delta_1 (D + \Delta_1)$$

$$\frac{I_o}{I_{LB\max}} = D - \frac{\Delta_1}{D} \left(1 + \frac{\Delta_1}{D} \right)$$

$$\frac{I_o}{I_{LB\max}} = D^2 \frac{V_d}{V_o} \left(-1 + \frac{V_d}{V_o} \right) \rightarrow D = \left[\frac{V_o}{V_d} \frac{I_o}{I_{LB\max}} \left(-1 + \frac{V_d}{V_o} \right)^{-1} \right]^{\frac{1}{2}}$$

$$\frac{V_d}{V_o} = \frac{D + \Delta_1}{D}$$

$$\frac{\Delta_1}{D} = \frac{V_d}{V_o} - 1$$

Discontinuous-conduction with constant V_o

DC voltage supply

$$\text{Continuous: } I_o > I_{LB} = I_{LB\max} (1-D) \rightarrow \frac{I_o}{I_{LB\max}} > 1-D \rightarrow D > 1 - \frac{I_o}{I_{LB\max}}$$

$$D > 1 - \frac{I_o}{I_{LB\max}}$$

$$D = \frac{V_o}{V_d}$$

$$\text{Discontinuous: } I_o < I_{LB}$$

$$D < 1 - \frac{I_o}{I_{LB\max}}$$

$$D = \left[\frac{V_o}{V_d} \frac{I_o}{I_{LB\max}} \left(1 - \frac{V_d}{V_o} \right)^{-1} \right]^{\frac{1}{2}}$$

$$I_{LB, \max} = \frac{T_s V_o}{2L}$$

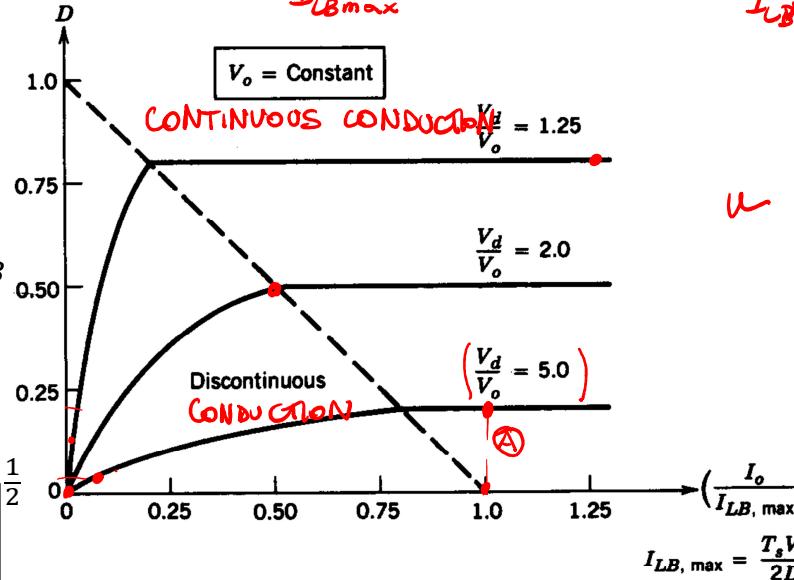


Figure 7-9 Step-down converter characteristics keeping V_o constant.

Output voltage ripple

First order calculation:

The average i_L flows in the load, and the ripple component in C .

Additional charge:

$$\Delta Q = \frac{1}{2} \frac{\Delta I_L}{2} \frac{T_s}{2}$$

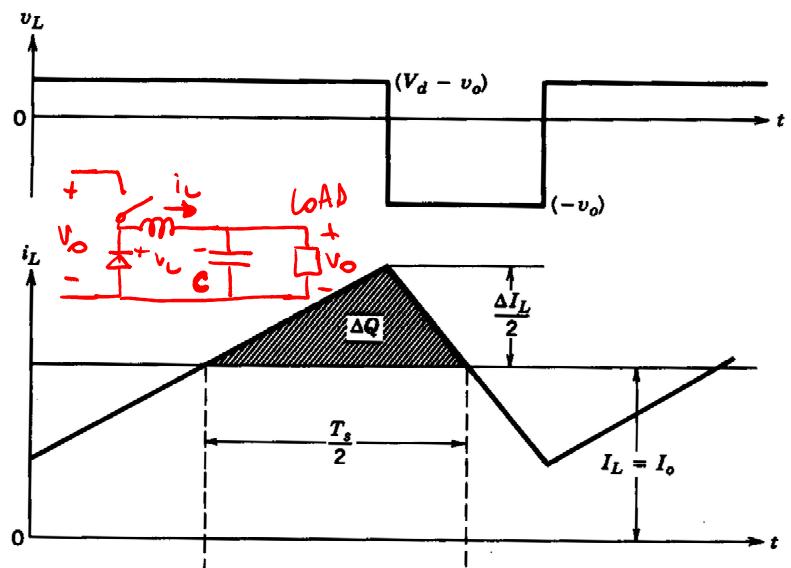
Current ripple:

$$\Delta I_L = (V_o/L)(1 - D)T_s$$

Voltage ripple:

$$\Delta V_o = \frac{\Delta Q}{C} = \frac{V_o}{8LC} T_s^2 (1 - D)$$

$$f_c = \frac{1}{2\pi} \frac{1}{\sqrt{LC}} \frac{1}{f_s^2}$$



$$\frac{\Delta V_o}{V_o} = \frac{\pi^2}{2} (1 - D) \frac{f_c^2}{f_s^2}$$

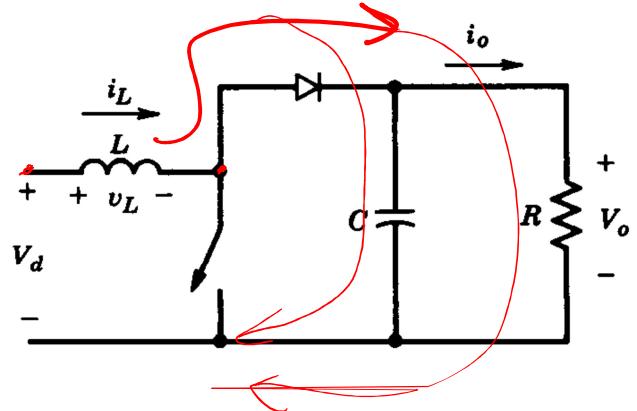
Step-up (boost) converter

- DC power supplies
- Regenerative breaking of DC motors

Output voltage always larger than the input

Switch on → diode off, output isolated, L accumulates energy from input

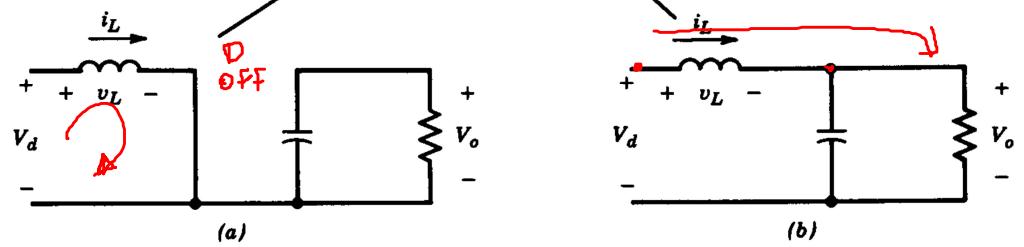
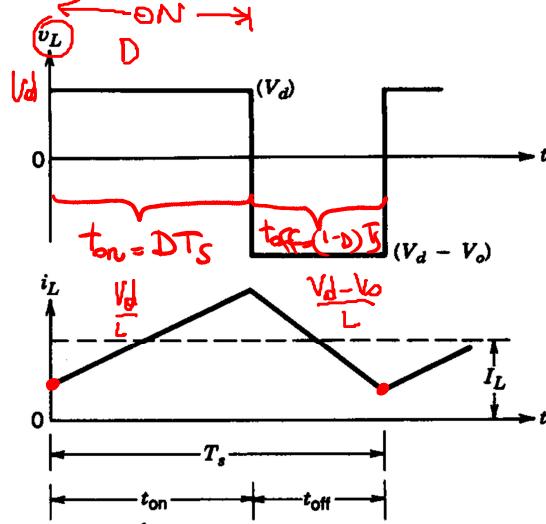
Switch off → diode on, load receives energy from input and from L



↳ Continuous-conduction mode

Periodic conditions:

$$\frac{t_{\text{on}} V_d}{L} + \frac{t_{\text{off}}(V_d - V_o)}{L} = 0$$



Continuous-conduction mode

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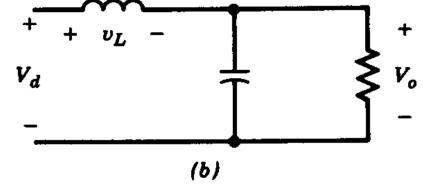
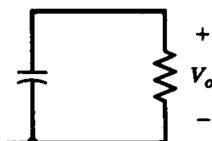
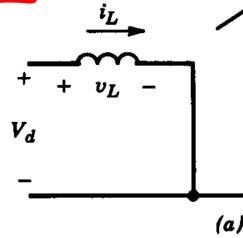
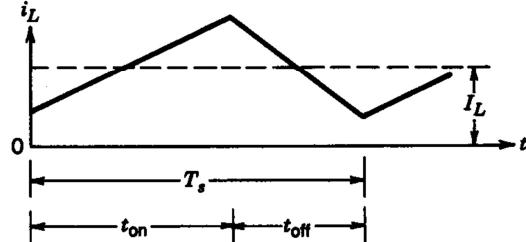
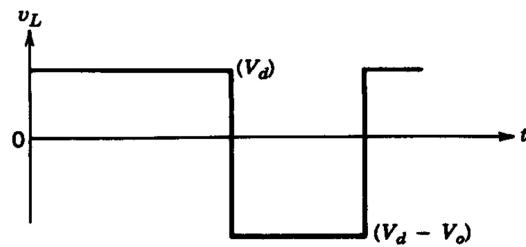
if $t_{\text{on}} = DT_s$ and

$$t_{\text{off}} = (1 - D)T_s$$

$$\cancel{DT_s V_d + (1-D)T_s (V_d - V_o) = 0}$$

$$\cancel{T_s V_d + T_s (1 - D)V_o = 0}$$

$$V_o = \frac{V_d}{1 - D}$$



Continuous-conduction mode

Periodic conditions:

$$\frac{t_{\text{on}} V_d}{L} + \frac{t_{\text{off}}(V_d - V_o)}{L} = 0$$

if $t_{\text{on}} = DT_s$ and

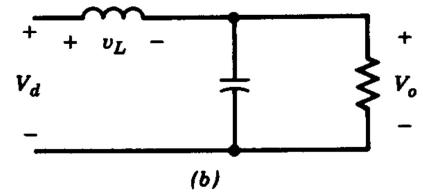
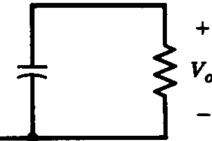
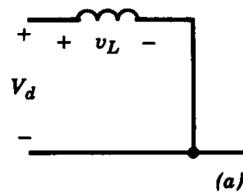
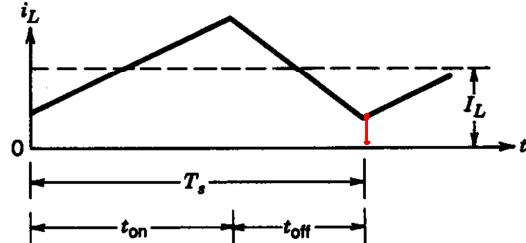
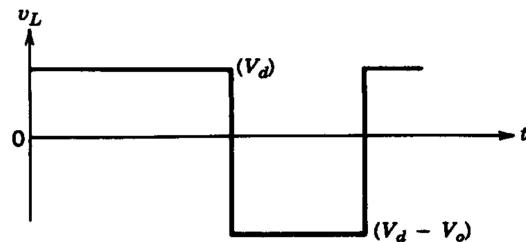
$$t_{\text{off}} = (1 - D)T_s$$

$$\cancel{T_s V_d + T_s (1 - D)V_o = 0}$$

$$\frac{V_o}{V_d} = \frac{1}{1 - D} \Rightarrow \frac{V_o}{V_d} = \frac{I_o}{I_d}$$

No losses:

$$\cancel{V_o I_o = V_d I_d}$$



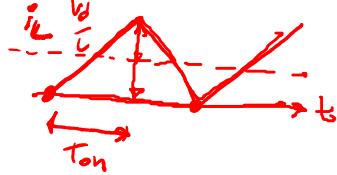
Continuous-discontinuous boundary

Average current in L

= ripple :

$$I_{LB} = \frac{1}{2} \frac{V_d t_{on}}{L}$$

$$= \frac{V_o (1 - D) T_s D}{2L}$$



$$\frac{V_o}{V_d} = \frac{1}{1-D}$$

Continuous-discontinuous boundary

Average current in L

= ripple :

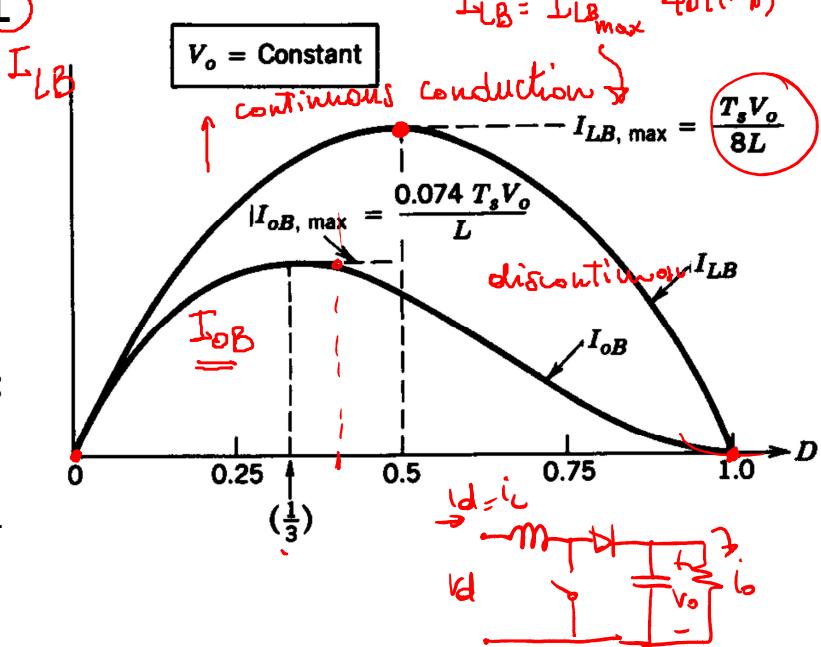
$$I_{LB} = \frac{V_d t_{on}}{2L}$$

$$= \frac{V_o (1 - D) T_s D}{2L}$$

Average output current at the limit:

$$I_{oB} = I_{LB} (1 - D)$$

$$= \frac{V_o T_s (1 - D)^2 D}{2L}$$



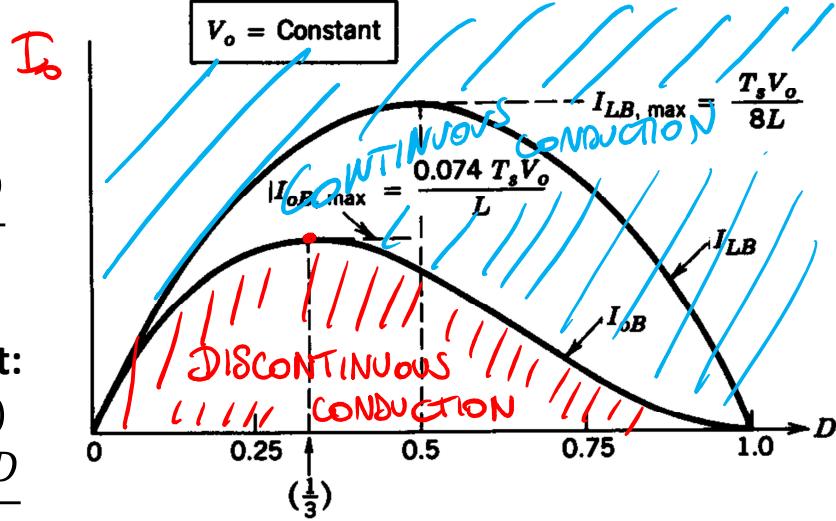
Continuous-discontinuous boundary

Average current in L

= ripple :

$$I_{LB} = \frac{V_d t_{on}}{2L}$$

$$= \frac{V_o(1-D)T_s D}{2L}$$



Average output current at the limit:

$$I_{oB} = I_{LB}(1-D)$$

$$= \frac{V_o T_s (1-D)^2 D}{2L}$$

I_{LB} is max if $D=0.5 \rightarrow I_{LBmax} = \frac{V_o T_s}{8L}$,

I_{oB} is max if $D=1/3 \rightarrow I_{oBmax} = \frac{2V_o T_s}{27L} \rightarrow I_{oB} = \frac{27}{4} (1-D)^2 D I_{oBmax}$

Discontinuous conduction mode (constant V_o)

Periodic conditions:

$$\frac{DT_s V_d}{L} + \frac{\Delta_1 T_s (V_d - V_o)}{L} = 0$$

$$\frac{V_o}{V_d} = 1 + \frac{D}{\Delta_1} = \frac{I_d}{I_o} \quad (\frac{V_o - V_d}{V_d}) \cdot \frac{D}{\Delta_1}$$

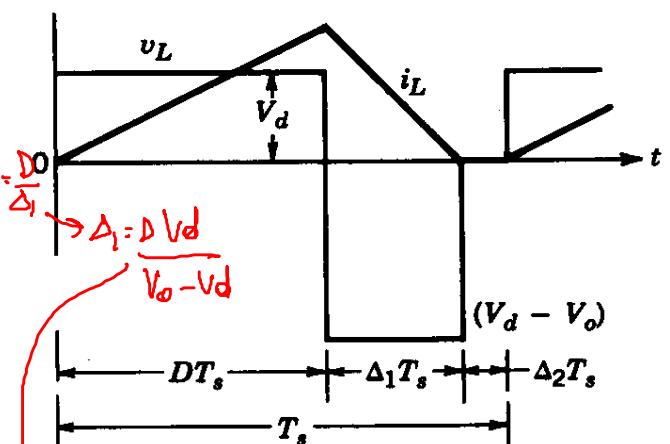
Average current in L

$$I_d T_s = \frac{DT_s V_d}{L} \frac{(D + \Delta_1) T_s}{2}$$

Average output current

$$I_o = I_d \frac{\Delta_1}{D + \Delta_1} = \frac{T_s V_d}{2L} D \Delta_1$$

$$= \frac{27}{4} I_{oBmax} \frac{V_d}{V_o - V_d} D^2 \frac{V_d}{V_o - V_d}$$



$$\frac{V_o}{V_d} = \frac{\Delta_1}{D} + \frac{D}{\Delta_1} \Rightarrow \frac{V_o - V_d}{V_d} = \frac{D}{\Delta_1}$$

$$\Delta_1 = \frac{V_d}{V_o - V_d} D$$

$$I_o = \frac{T_s}{2L} \frac{V_d}{V_o - V_d} D^2 \frac{V_d}{V_o - V_d}$$

$$D \propto \sqrt{I_o}$$

Discontinuous conduction mode (constant V_o)

Periodic conditions:

$$\frac{DT_s V_d}{L} + \frac{\Delta_1 T_s (V_d - V_o)}{L} = 0$$

$$\frac{V_o}{V_d} = 1 + \frac{D}{\Delta_1} = \frac{I_d}{I_o}$$

Average current in L

$$I_d T_s = \frac{DT_s V_d}{L} \frac{(D + \Delta_1)T_s}{2}$$

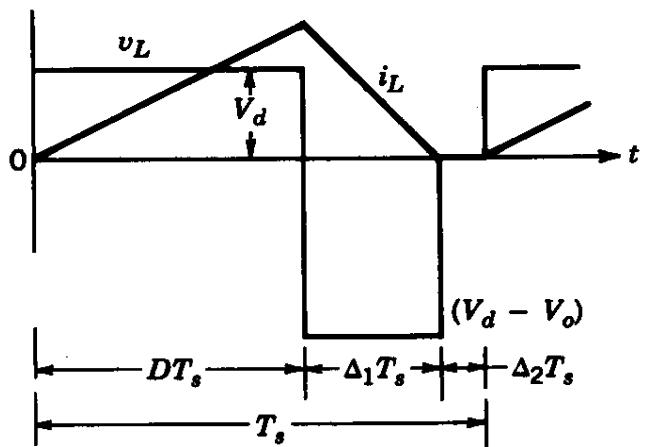
Average output current

$$I_o = I_d \frac{\Delta_1}{D + \Delta_1} = \frac{T_s V_d}{2L} D \Delta_1$$

$$= \frac{27}{4} I_{oBmax} \frac{V_d}{V_o} D^2 \frac{V_d}{V_o - V_d}$$

$D = \left[\frac{4}{27} \frac{V_o}{V_d} \left(\frac{V_o}{V_d} - 1 \right) \frac{I_o}{I_{oBmax}} \right]^{\frac{1}{2}}$

$D \propto \sqrt{I_o}$



Continuous-discontinuous mode (constant V_o)

$$\frac{V_o}{V_d} = \frac{1}{1-D} \rightarrow \frac{V_d}{V_o} = \frac{1-D}{1}$$

$$D = 1 - \frac{V_d}{V_o}$$

Continuous mode:

$$I_o > I_{oB}$$

$$= I_{oBmax} \frac{27(1-D)^2 D}{4}$$

$$D = 1 - \frac{V_d}{V_o}$$

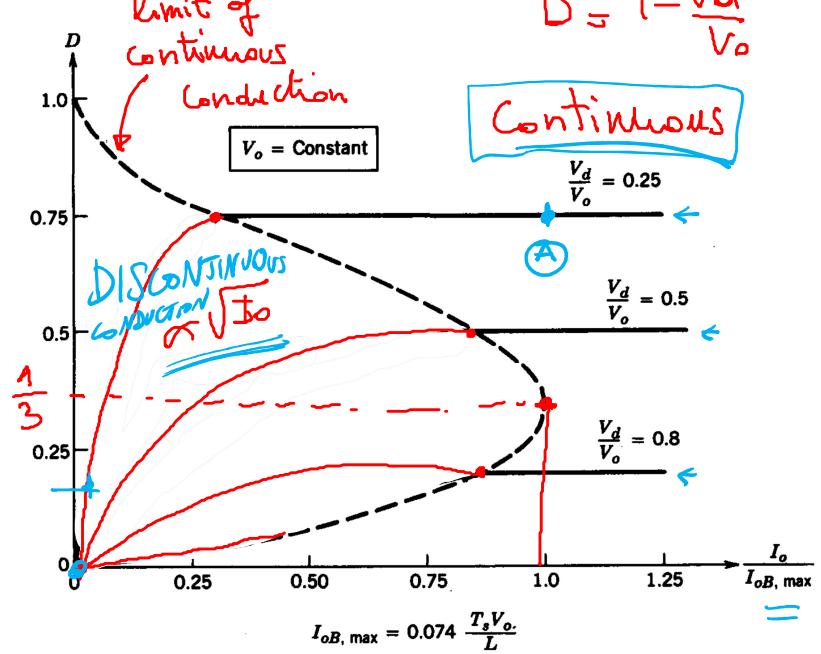


Figure 7-15 Step-up converter characteristics keeping V_o constant.

Continuous-discontinuous mode (constant V_o)

Continuous mode:

$$I_o > I_{oB} \\ = I_{oBmax} \frac{27(1 - D)^2 D}{4}$$

$$D = 1 - \frac{V_d}{V_o}$$

Discontinuous mode:

$$I_o < I_{oB}$$

$$D = \left[\frac{4}{27} \frac{V_o}{V_d} \left(\frac{V_o}{V_d} - 1 \right) \frac{I_o}{I_{oBmax}} \right]^{\frac{1}{2}}$$

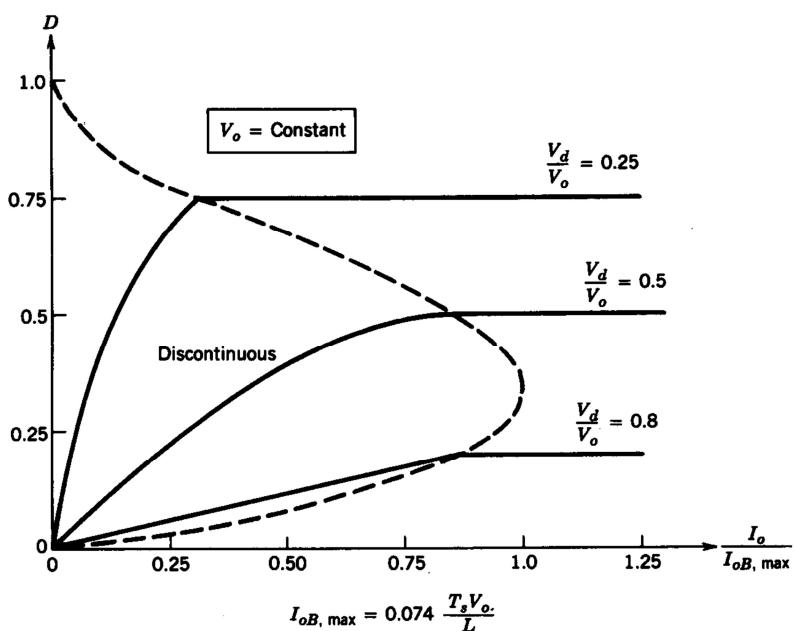
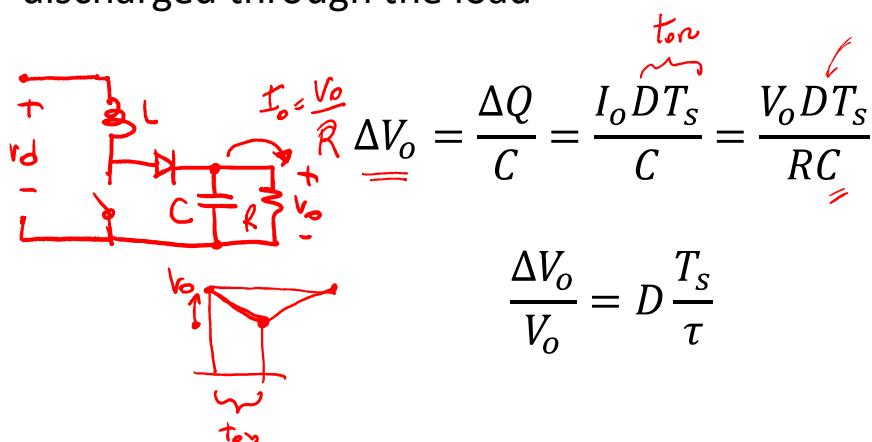


Figure 7-15 Step-up converter characteristics keeping V_o constant.

Losses and ripple

Losses: inductor, capacitor, switch, diode

Ripple: first order assumption: when the switch is on the C is discharged through the load

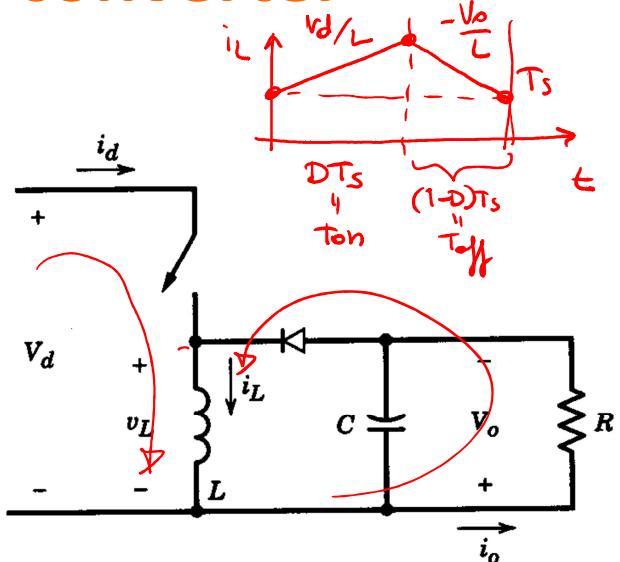


Buck-boost converter

Negative DC power supply

Switch on: inductance accumulates energy, diode off, C supplies the load

Switch off: diode on, inductance transfers energy to the capacitance and to the load



Periodic conditions in continuous conduction mode:

$$\frac{DT_s V_d}{L} - \frac{V_o(1-D)T_s}{L} = 0$$

$$\frac{V_o}{V_d} = \frac{D}{1-D} = \frac{I_d}{I_o}$$

$$I_L = I_o + I_d = \frac{I_o}{1-D}$$

Continuous-discontinuous boundary

Current in L at the boundary

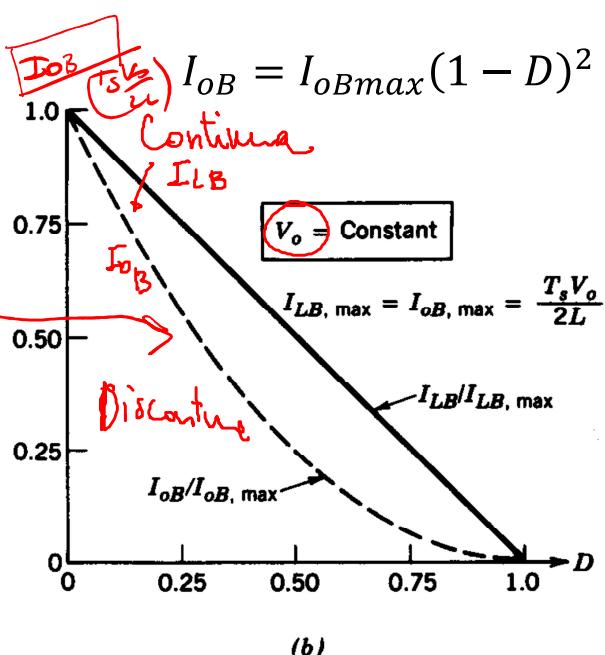
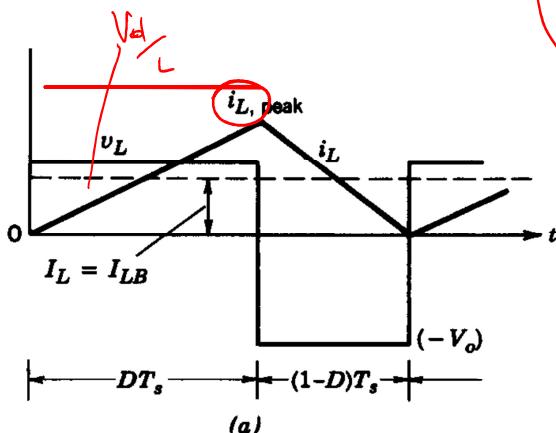
$$I_{LB} = \frac{DT_s V_d}{2L} = \frac{DT_s V_o (1-D)}{2L}$$

$$I_{LB} = I_{LBmax}(1-D)$$

Output current at the boundary:

$$I_{oB} = I_{LB} (1-D) = \frac{T_s V_o}{2L} (1-D)^2$$

$$I_{oB} = I_{oBmax} (1-D)^2$$



Discontinuous conduction

Periodic conditions:

$$\frac{DV_d T_s}{L} - \frac{V_o \Delta_1 T_s}{L} = 0$$

$$\frac{V_o}{V_d} = \frac{D}{\Delta_1} = \frac{I_o}{I_d}$$

Average current in L: $\frac{\text{base}}{\text{height}}$

$$I_L T_s = \frac{V_d D T_s}{L} \frac{(D + \Delta_1) T_s}{2}$$

Therefore: $\frac{\text{height}}{\text{base}}$

$$I_L = I_o \left(1 + \frac{D}{\Delta_1} \right) = \frac{V_d T_s}{2L} D (D + \Delta_1) \rightarrow I_o = \frac{V_d T_s}{2L} \frac{D \Delta_1}{D + \Delta_1} = \frac{V_d T_s}{2L} \frac{D^2}{D + \frac{1}{D}} = \frac{V_d T_s}{2L} D^2 \left(\frac{V_d}{V_o} \right)^2$$

$$I_o = I_{oB} + I_D = I_o \left(1 + \frac{D}{\frac{T_s}{I_o}} \right) \frac{I_o}{I_{oBmax}} = D \Delta_1 \frac{V_d}{V_o} = D^2 \left(\frac{V_d}{V_o} \right)^2 \rightarrow D = \frac{V_o}{V_d} \sqrt{\frac{I_o}{I_{oBmax}}} \quad \boxed{D \propto \sqrt{I_o}}$$

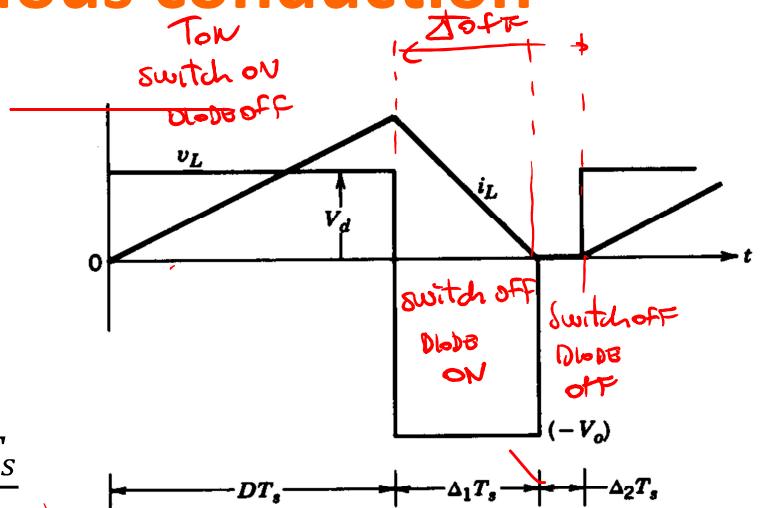


Figure 7-21 Buck-boost converter waveforms in a discontinuous-conduction mode.

Continuous-discontinuous mode

Continuous operation

$$I_o > I_{oB} = I_{oBmax} (1 - D)^2$$

$$D = \frac{V_o}{V_d - V_o}$$

Discontinuous operation

$$I_o > I_{oB}$$

$$D = \frac{V_o}{V_d} \sqrt{\frac{I_o}{I_{oBmax}}}$$

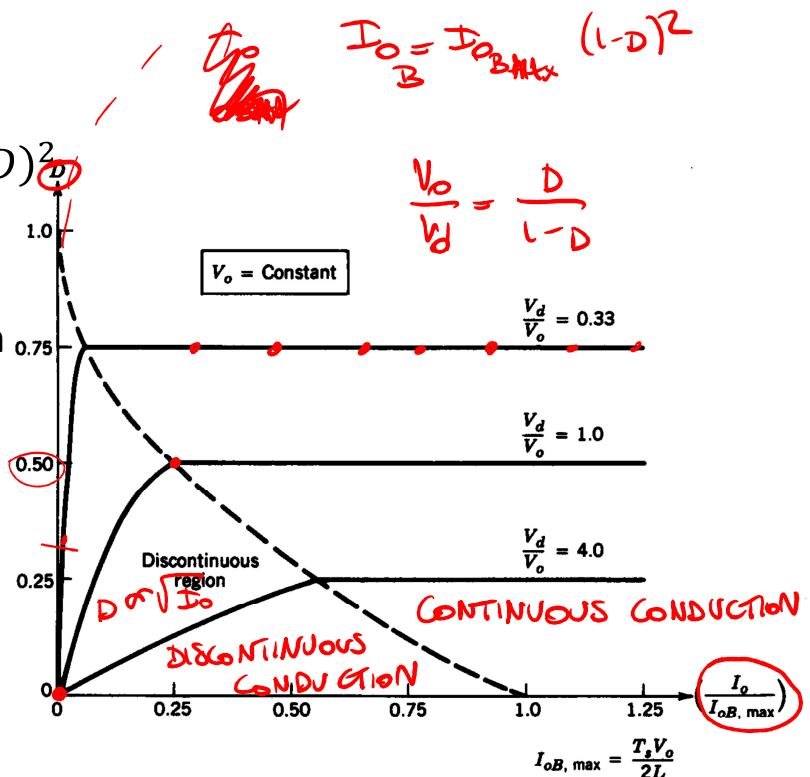


Figure 7-22 Buck-boost converter characteristics keeping V_o constant.

Output voltage ripple

When the switch is ON, C is discharged through the load

$$\Delta V_o = \frac{\Delta Q}{C} = \frac{DT_s V_o}{RC} \rightarrow \frac{\Delta V_o}{V_o} = D \frac{T_s}{RC}$$

