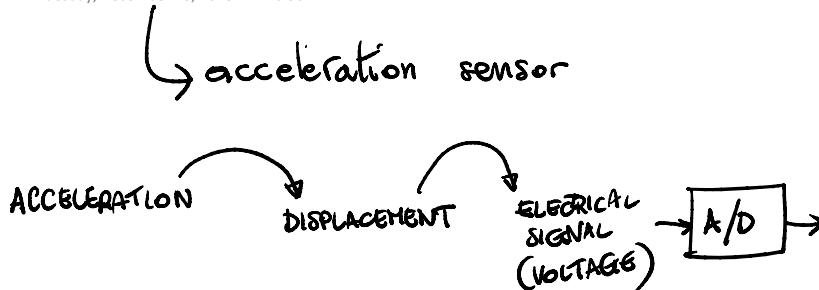


Accelerometer

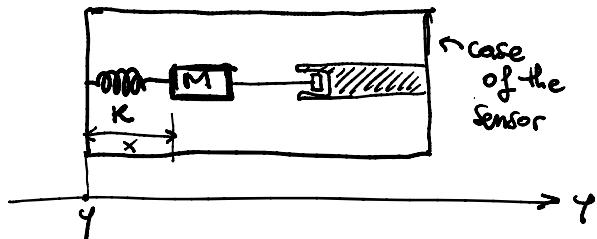
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Example : 1D accelerometer

- ▷ mass (M)
- ▷ spring (elastic constant k)
- ▷ damping term (b)

We want to measure $\ddot{a} = \ddot{y}$



$$M(\ddot{x} + \ddot{y}) = -Kx - b\dot{x}$$

\ddot{a} ↑
displacement

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in the Laplace domain

$$M[s^2X + A] = -KX - bSX$$

$$X[Ms^2 + bS + K] = -MA$$

$$X = A \cdot \frac{-1}{s^2 + \frac{b}{M}s + \frac{K}{M}}$$

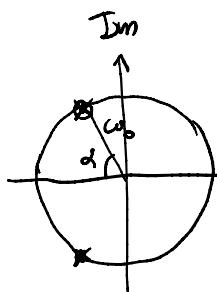
TRANSFER FUNCTION
WITH TWO POLES

$$\rightarrow \frac{-1}{s^2 + 2\xi\omega_0 s + \omega_0^2}$$

where

$$\omega_0 = \sqrt{\frac{K}{M}}$$

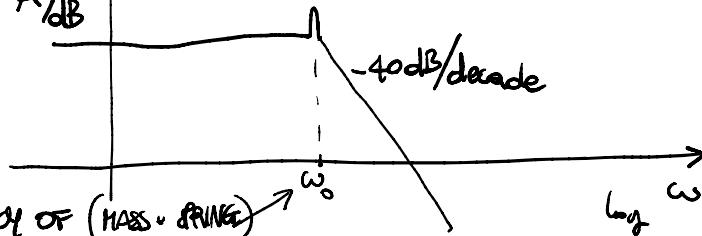
$$\xi = \frac{b}{2\sqrt{MK}} \rightarrow \xi = \frac{b}{2\sqrt{MK}}$$



$$\xi = \cos(\phi)$$

$$\left(\frac{X}{A}\right)_{dB}$$

RESONANT FREQUENCY OF (MASS + SPRING)



Capacitive accelerometer

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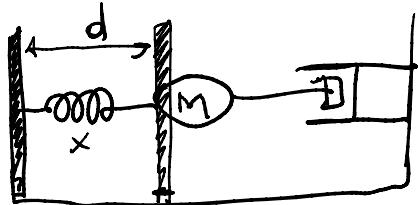
USING MEMS TECHNOLOGY

↓
Micro ElectroMechanical Systems

2 plates →) 1 STATIONARY PLATE [Connected to the package (case)]
) MOVING PLATE (Connected to the MASS)

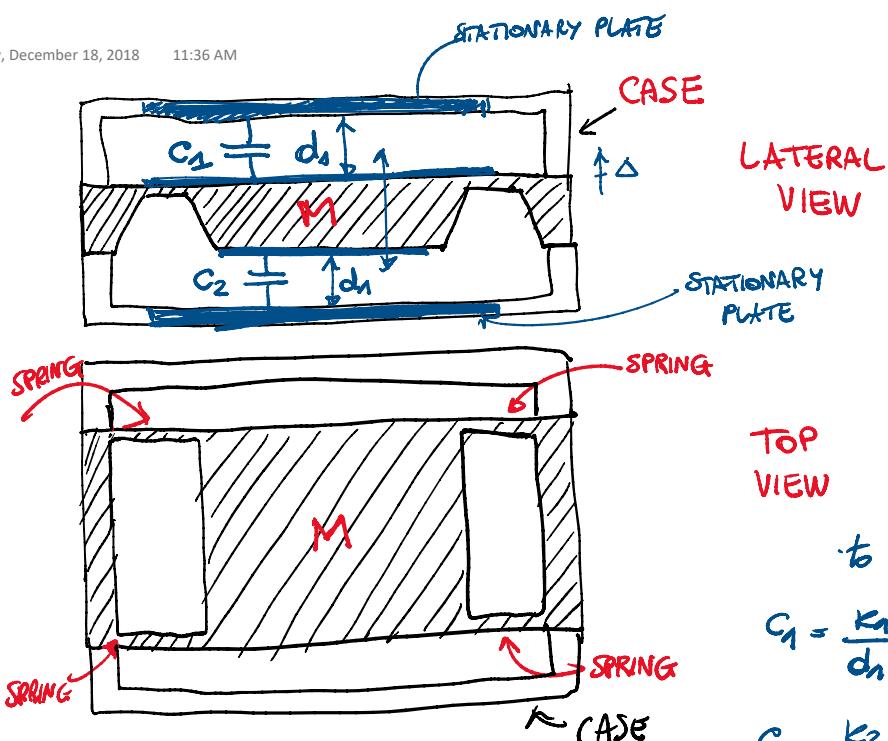
if d is the distance between the two plates

$$C \propto \frac{1}{d}$$



$$\Delta d \approx 10 \mu\text{m}$$

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$$C_1 \propto \frac{1}{d_1}$$

$$C_2 \propto \frac{1}{d_2}$$

displacement Δ

$$C_1 \left[\propto \frac{1}{d_1 - \Delta} \right] = \frac{k_1}{d_1 - \Delta}$$

$$C_2 \left[\propto \frac{1}{d_2 + \Delta} \right] = \frac{k_2}{d_2 + \Delta}$$

to first order in Δ

$$C_1 = \frac{k_1}{d_1} \left(\frac{1}{1 - \frac{\Delta}{d_1}} \right) \sim \frac{k_1}{d_1} \left(1 + \frac{\Delta}{d_1} \right)$$

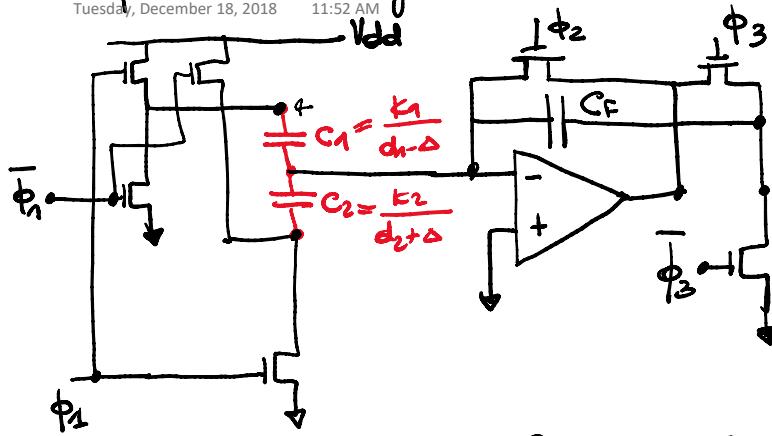
$$C_2 = \frac{k_2}{d_2} \left(\frac{1}{1 + \frac{\Delta}{d_2}} \right) \sim \frac{k_2}{d_2} \left(1 - \frac{\Delta}{d_2} \right)$$

$$\Delta C = C_1 - C_2 = \frac{k_1}{d_1} - \frac{k_2}{d_2} + \Delta \left[\frac{k_1}{d_1^2} + \frac{k_2}{d_2^2} \right] \Rightarrow \text{if } \frac{k_1}{d_1} = \frac{k_2}{d_2} \Rightarrow \text{then } \Delta C \propto \Delta$$

Capacitance - Voltage Transducer

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$$V = \frac{Q}{C} \quad CV = Q$$

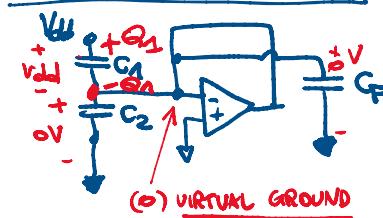
$$V_F = \frac{-Q_1 + Q_2}{C_F}$$

$$V_{out} = -V_F \cdot \frac{Q_1 - Q_2}{C_F} = \frac{V_{dd}}{C_F} (C_1 - C_2) \propto \Delta \propto a$$

Switched-Capacitor Circuit

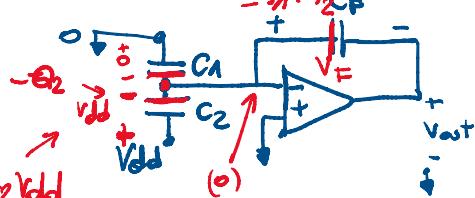
$$\textcircled{1} \quad \phi_1 = 1 \quad \bar{\phi}_1 = 0$$

$$\phi_2 = 1 \quad \bar{\phi}_2 = 0$$



$$Q_1 = C_1 V_{dd}$$

$$\textcircled{2} \quad \phi_1 = 0, \phi_2 = 0, \phi_3 = 1 \quad \text{the charge on } C_1 \text{ is } -Q_1 + Q_2 C_F \quad \leftarrow C_F \text{ is } -Q_1 + Q_2$$



MEMS Gyroscope

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measures the orientation in space in three dimensions

$\textcircled{1}$ NAVIGATION when one cannot use a compass

[when the geomagnetic field cannot be used
(space, planes, tunnels)]

$\textcircled{2}$ ROBOTICS

$\textcircled{3}$ STABILIZATION

typically sold on a System on Package with 3D accelerometers

[6-axis accelerometer/gyroscope]

CONSERVATION OF ANGULAR MOMENTUM

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IF NO EXTERNAL FORCE ACTS ON A SYSTEM, THE TOTAL ANGULAR MOMENTUM OF THE SYSTEM (WITH RESPECT TO A GIVEN POINT) IS CONSERVED

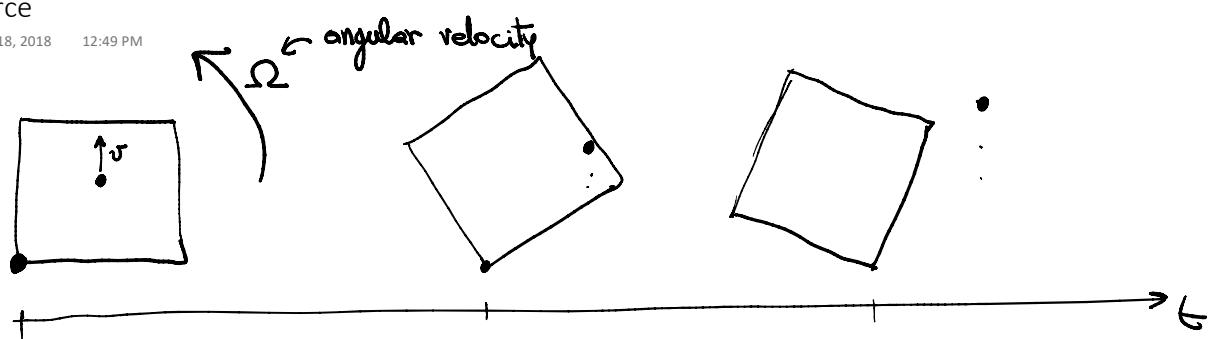
↳ VIBRATING MASS (membrane)

↳ CORIOLIS FORCE

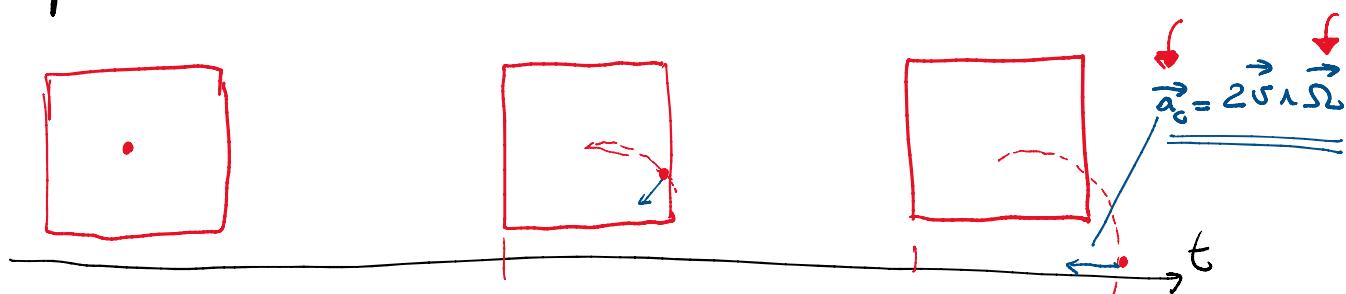
→ it is a "inertial force"
emerges in a reference system rotating
with respect to a fixed reference system

Coriolis force

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with respect to the rotating REFERENCE SYSTEM



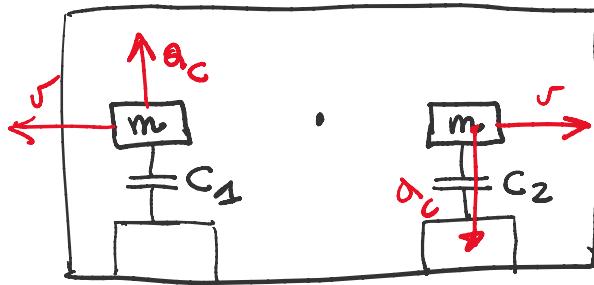
Differential implementation

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$\uparrow \alpha$



$$a_c = 2\dot{\theta} \times \Omega$$



$$C_1 - C_2 \propto a_c \propto \underline{\underline{\Omega}}$$

IF THERE IS A LINEAR ACCELERATION

C_1 and C_2 vary in the same direction, therefore $C_1 - C_2 \approx 0$