DC-DC Converters

Typical uses:

- DC Power supplies
- DC Motor drives
- Portable Electronics

Types of converters

- **Step-down (buck)**
- **Step-up (boost)**
- Buck-boost
- Cuk
- Full-Bridge

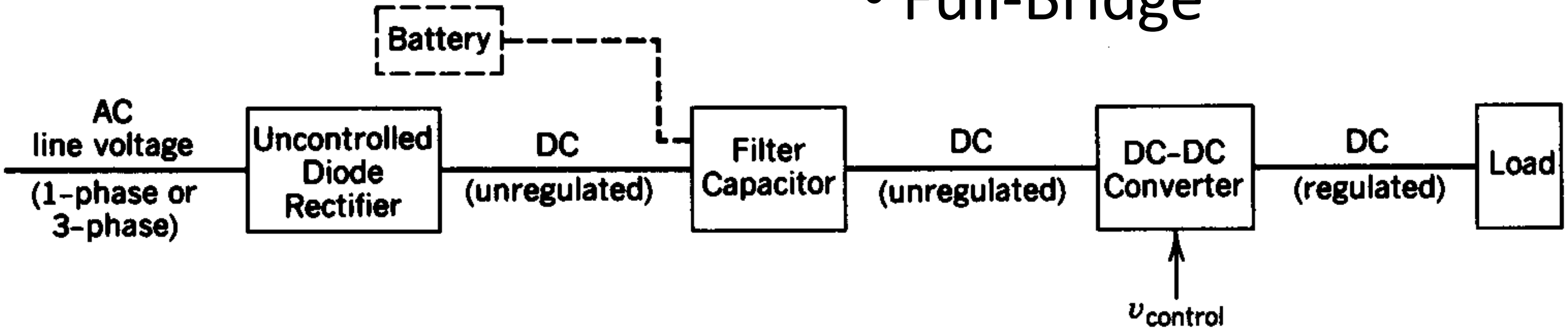


Figure 7-1 A dc–dc converter system.

Ideal concept of step-down converter with PWM* switching

(* Pulse Width Modulation)

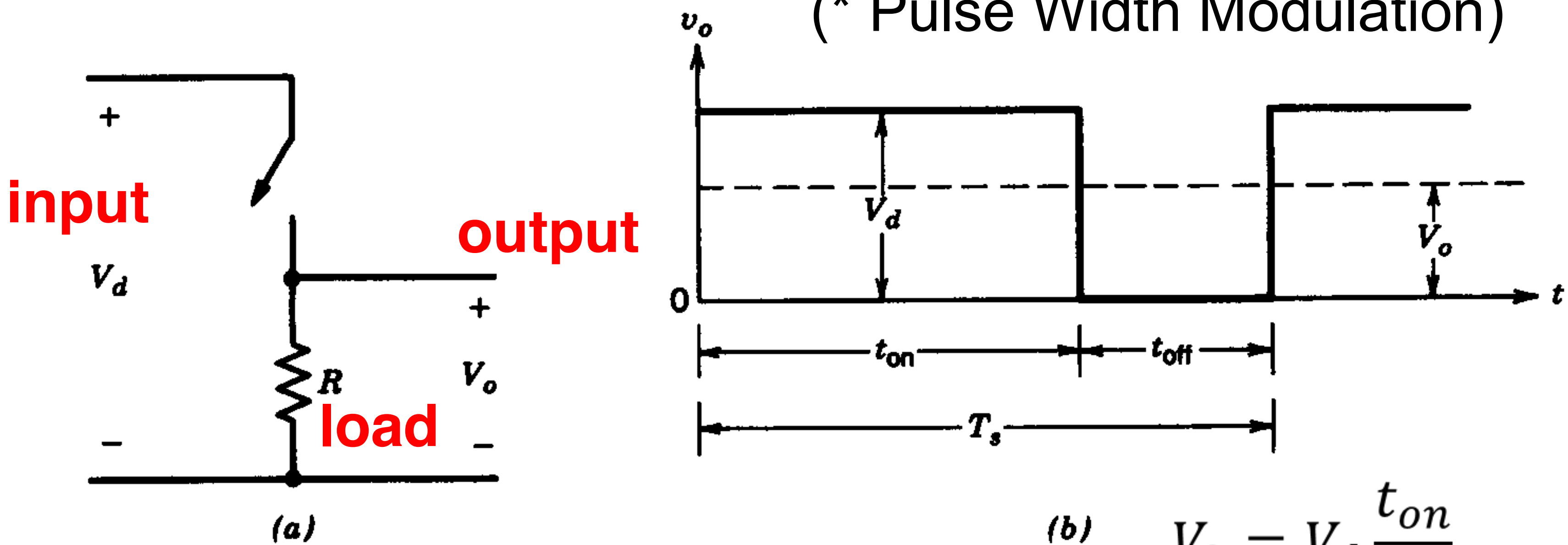


Figure 7-2 Switch-mode dc–dc conversion.

$$V_0 = V_d \frac{t_{on}}{T_s}$$

Assumptions: Switches, L, C are lossless, DC input has zero internal impedance, load is an equivalent R

This cannot work: 1. Load is inductive and can destroy switch by dissipating all stored energy, 2. output voltage must be continuous

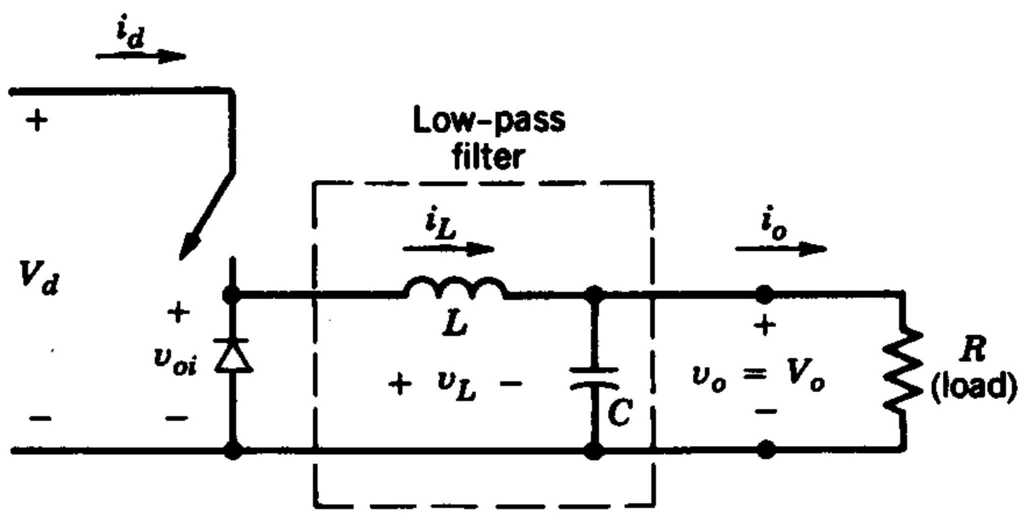
Step-down (buck) converter

DC power supplies, DC motor drives -- $V_o < V_d$

Low-pass filter keeps output voltage constant

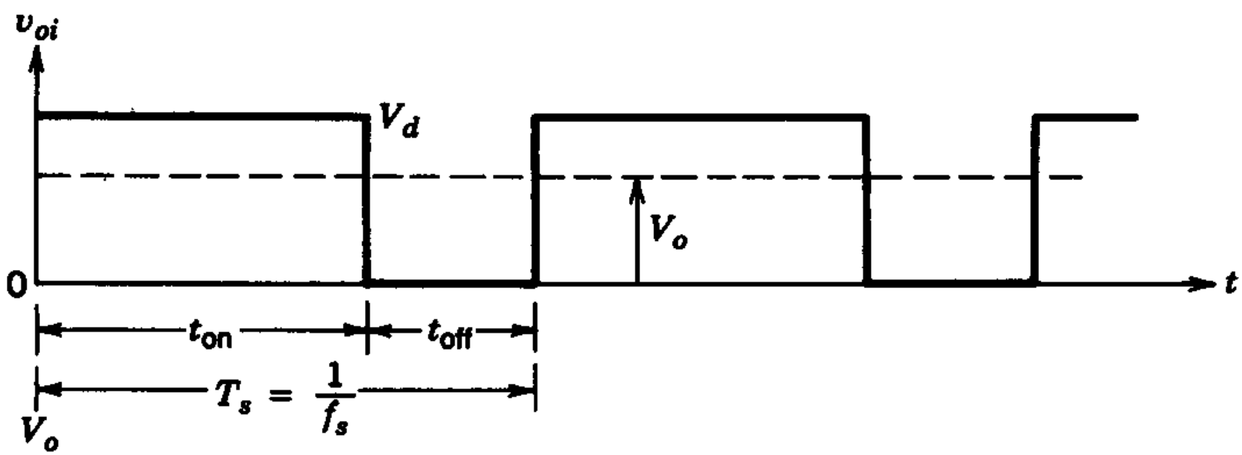
Note: 2nd order non dissipative filter

$$f_c = \frac{1}{2\pi}\frac{1}{\sqrt{LC}} \ll f_s$$



(a)

Diode avoids voltage spike on switch (when switch is off, diode provides current to L)



Continuous-conduction mode

Current in L is always > 0

- t_{on}: $\dfrac{dI}{dt} = \dfrac{V_d - V_o}{L}$
- t_{off}: $\dfrac{dI}{dt} = -\dfrac{V_o}{L}$

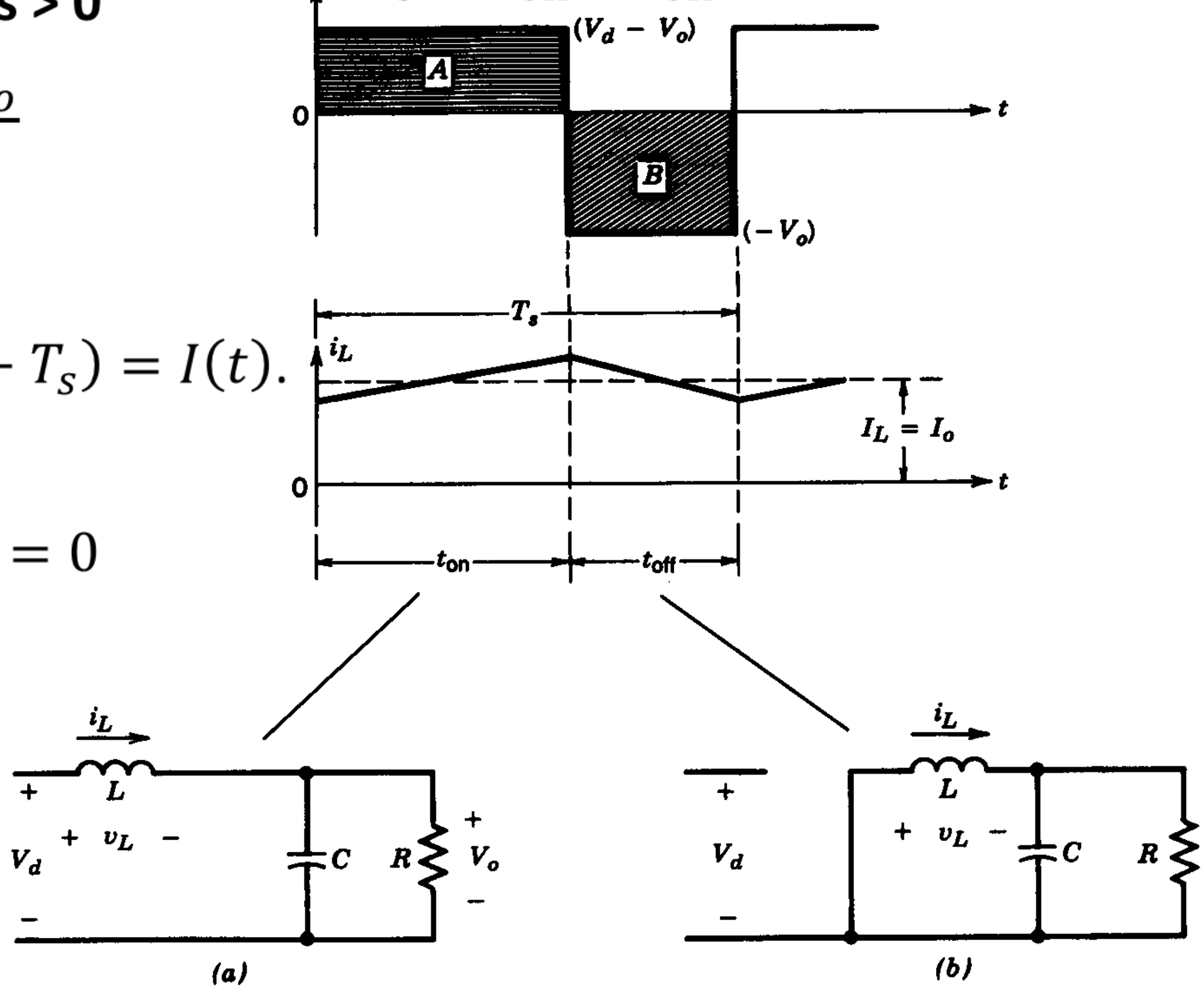
At steady state: $I(t + T_s) = I(t)$.

Therefore

$$\frac{V_d - V_o}{L} t_{\text{on}} - \frac{V_o}{L} t_{\text{off}} = 0$$

$$\frac{V_o}{V_d} = \frac{t_{on}}{T_s} = D$$

$$T_s = t_{\text{on}} + t_{\text{off}}$$



Limit of continuous conduction

If the ripple amplitude $I_{LB} \equiv \frac{I_{peak}}{2} = I_o$, the converter is at the limit of continuous conduction (i.e. $min\{I_L\} = 0$)

$$I_{LB} \equiv \frac{I_{peak}}{2} = \frac{t_{on}(V_d - V_o)}{2L} = \frac{DT_sV_d(1-D)}{2L} = I_{LBmax}4D(1-D)$$

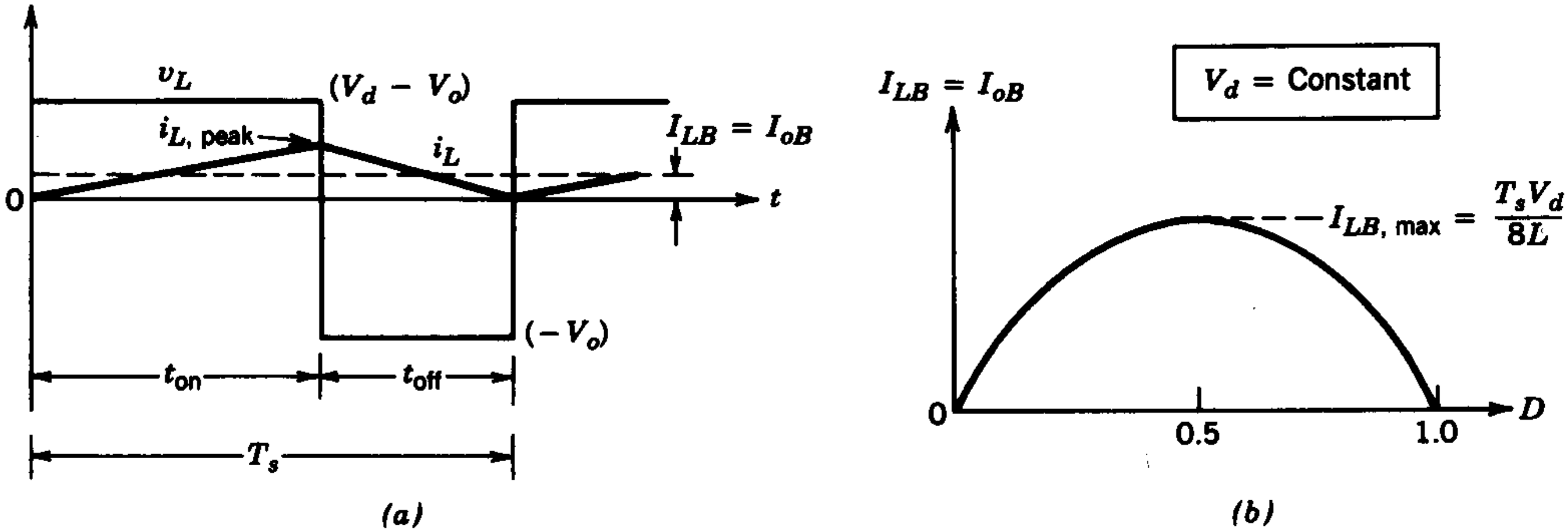


Figure 7-6 Current at the boundary of continuous–discontinuous conduction: (a) current waveform; (b) I_{LB} versus D keeping V_d constant.

Limits of continuous-discontinuous conduction (constant V_d)

Continuous

$$\frac{V_o}{V_d} = D$$

$$\frac{I_o}{I_{\mathrm{LBmax}}} > 4D(1-D)$$

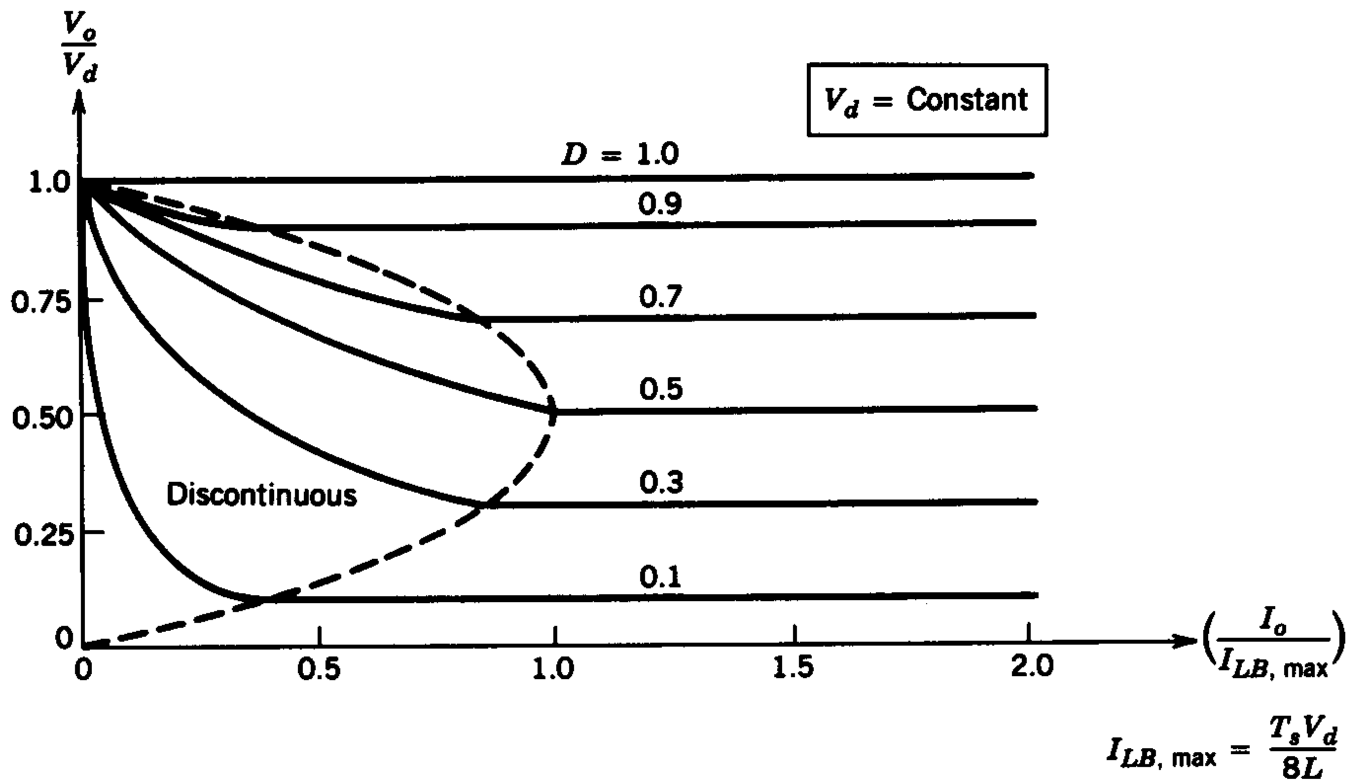


Figure 7-8 Step-down converter characteristics keeping V_d constant.

Discontinuous-conduction mode with constant V_d

Motor drives

$$I_{peak} = \frac{(V_d - V_o)DT_s}{L} = \frac{V_o \Delta_1 T_s}{L} \quad \Rightarrow \quad \frac{V_o}{V_d} = \frac{D}{(D + \Delta_1)}$$

$$I_{peak} = \frac{V_d T_s}{L} \frac{D\Delta_1}{D + \Delta_1}$$

$$I_{peak} = 8I_{LBmax} \frac{D\Delta_1}{D + \Delta_1}$$

$$I_o T_s = \frac{I_{peak}(D + \Delta_1)T_s}{2}$$

$$I_o = 4I_{LBmax} D\Delta_1$$

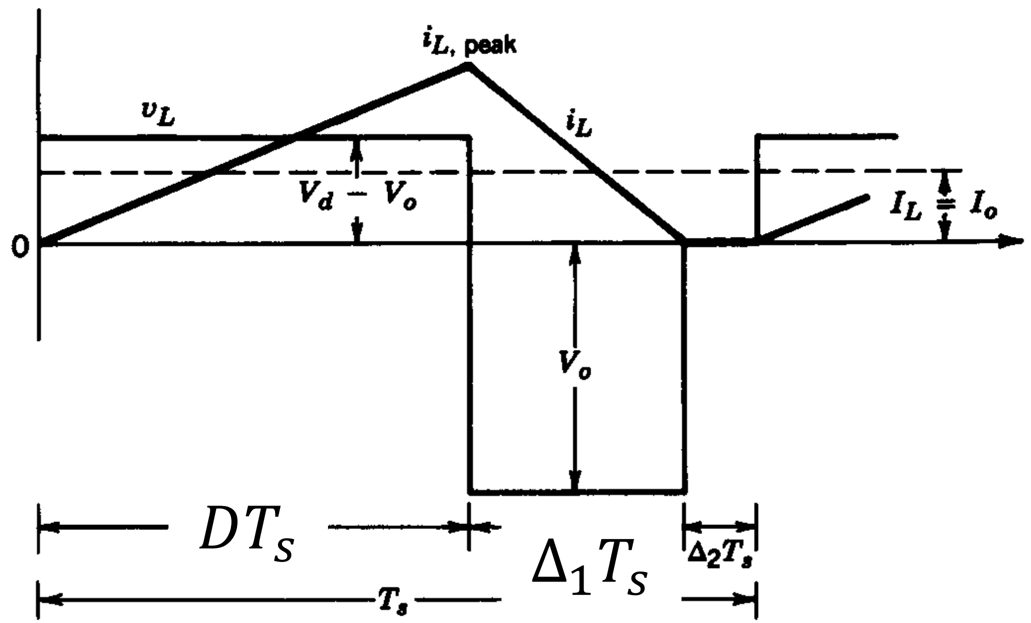


Figure 7-7 Discontinuous conduction in step-down converter.

$$\frac{V_o}{V_d} = \frac{D^2}{D^2 + I_o/(4I_{LBmax})}$$

Limits of continuous-discontinuous conduction (constant Vd)

Continuous

$$\frac{I_o}{I_{\text{LBmax}}} > 4D(1-D)$$

$$\frac{V_o}{V_d} = D$$

Discontinuous

$$\frac{I_o}{I_{\text{LBmax}}} < 4D(1-D)$$

$$\frac{V_o}{V_d} = \frac{D^2}{D^2 + \frac{I_o}{4I_{\text{LBmax}}}}$$

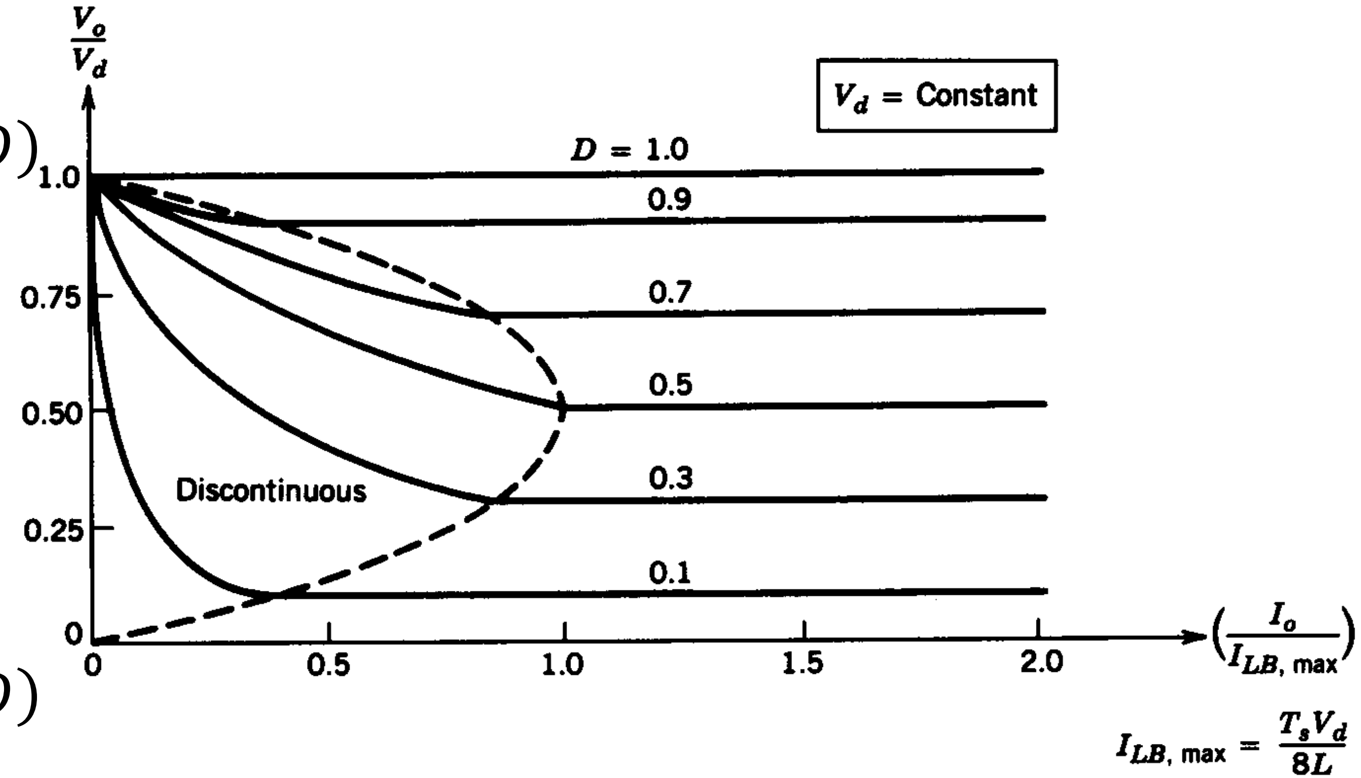


Figure 7-8 Step-down converter characteristics keeping V_d constant.

Discontinuous-conduction with constant Vo

DC voltage supply

At the limit of continuous conduction

$$I_{LB} = \frac{V_o T_s (1-D)}{2L} = I_{\mathrm{LBmax}}(1-D)$$

We can write D explicitly from:

$$I_{\mathrm{peak}} = \frac{V_o \Delta_1 T_s}{L} = 2 I_{\mathrm{LBmax}} \Delta_1$$

$$I_o = \frac{I_{\mathrm{peak}}(D+\Delta_1)}{2} = I_{\mathrm{LBmax}} \Delta_1 (D+\Delta_1)$$

$$\frac{V_d}{V_o} = \frac{D+\Delta_1}{D}$$

$$\frac{I_o}{I_{\mathrm{LBmax}}} = D^2 \frac{V_d}{V_o}\left(1-\frac{V_d}{V_o}\right) \Rightarrow D = \left[\frac{V_o}{V_d}\frac{I_o}{I_{LBmax}}\left(1-\frac{V_d}{V_o}\right)^{-1}\right]^{\frac{1}{2}}$$

Discontinuous-conduction with constant Vo

DC voltage supply

Continuous: $I_o > I_{LB}$

$$D > 1 - \frac{I_o}{I_{LBmax}}$$

$$D = \frac{V_o}{V_d}$$

Discontinuous: $I_o < I_{LB}$

$$D < 1 - \frac{I_o}{I_{LBmax}}$$

$$D = \left[\frac{V_o}{V_d}\frac{I_o}{I_{LBmax}}\left(1 - \frac{V_d}{V_o}\right)^{-1}\right]^{\frac{1}{2}}$$

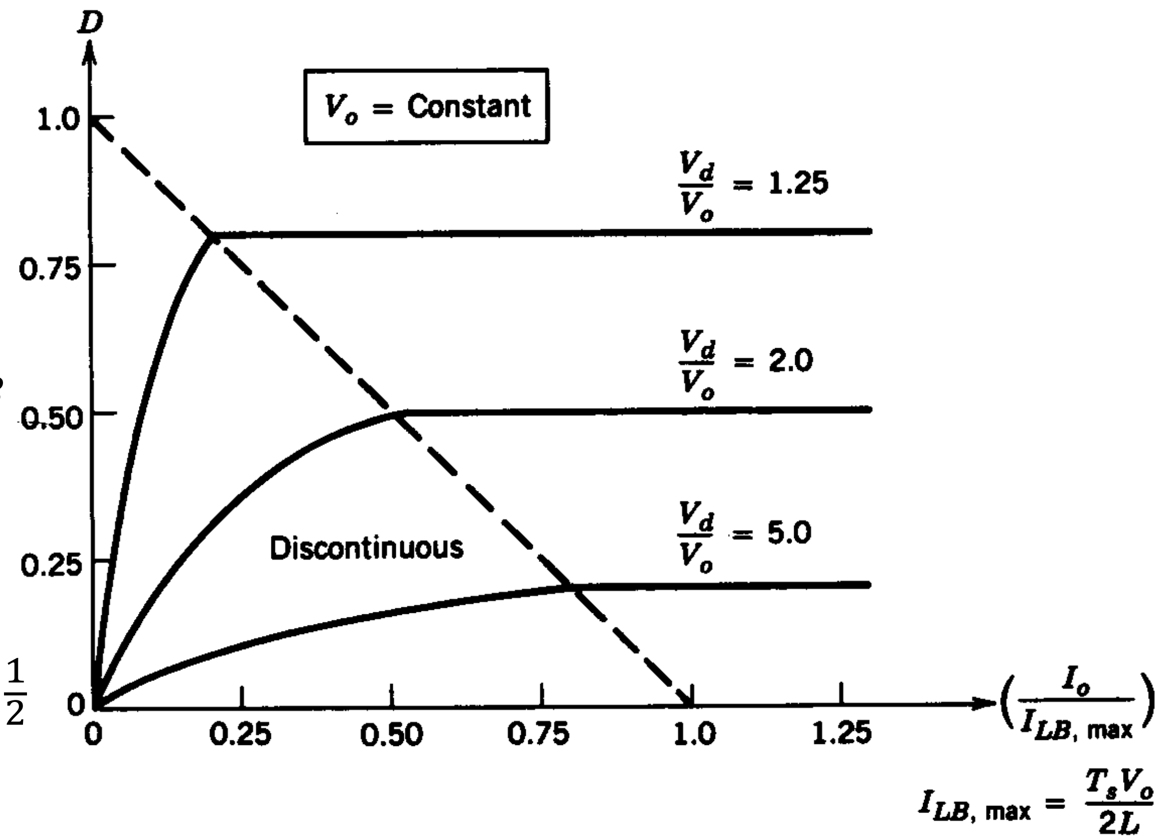


Figure 7-9 Step-down converter characteristics keeping V_o constant.

Output voltage ripple

First order calculation:

The average iL flows in the load, and the ripple component in C.

Additional charge:

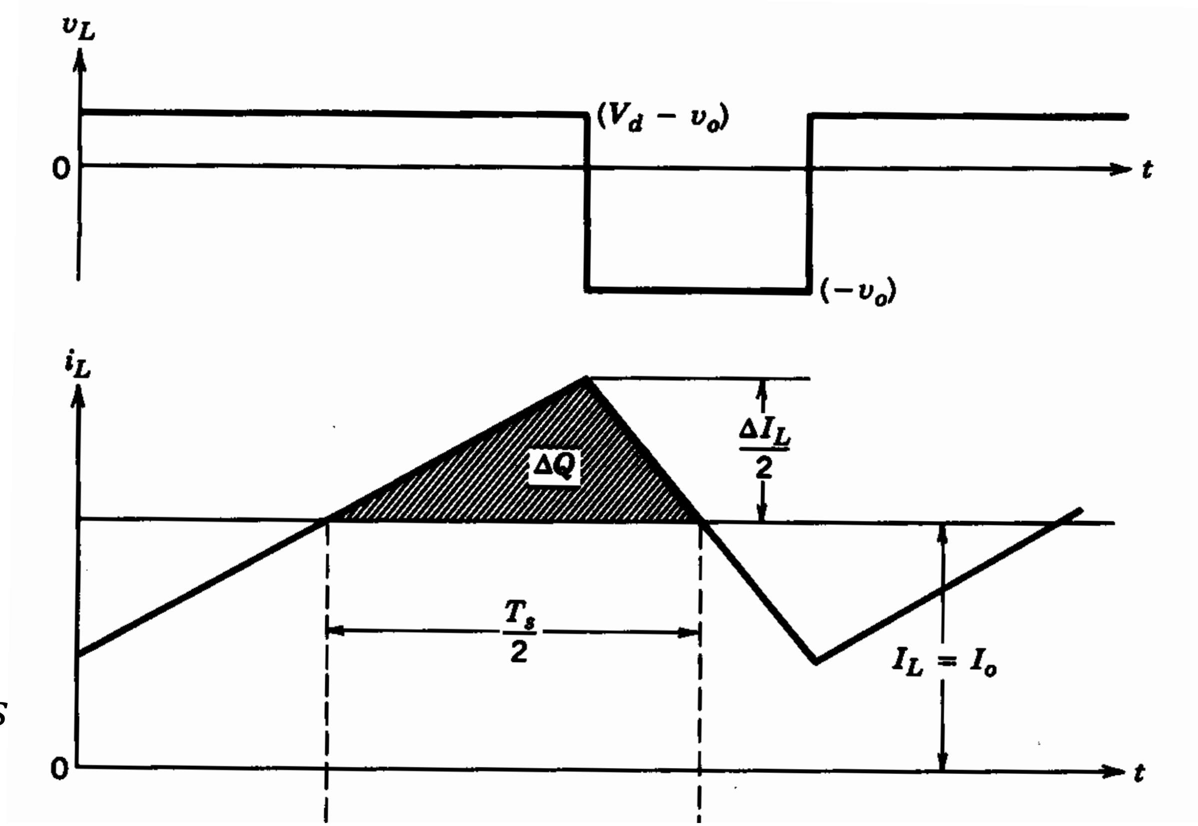
$$\Delta Q = \frac{1}{2}\frac{\Delta I_L}{2}\frac{T_s}{2}$$

Current ripple:

$$\Delta I_L = (V_o/L)(1-D)T_s$$

Voltage ripple:

$$\Delta V_o = \frac{\Delta Q}{C} = \frac{V_o}{8LC}T_s^2(1-D)$$



$$f_c = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$$

$$\frac{\Delta V_o}{V_o} = \frac{\pi^2}{2}(1-D)\frac{f_c^2}{f_s^2}$$

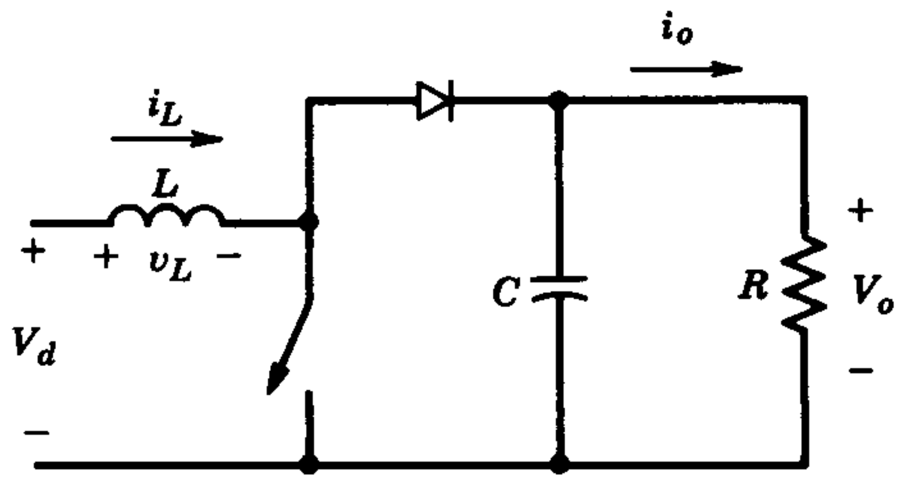
Step-up (boost) converter

- DC power supplies
- Regenerative breaking of DC motors

Output voltage always larger than the input

Switch on → diode off, output isolated, L accumulates energy from input

Switch off → diode on, load receives energy from input and from L



Continuous-conduction mode

Periodic conditions:

$$\frac{t_{\text{on}}V_d}{L} + \frac{t_{\text{off}}(V_d - V_o)}{L} = 0$$

if $t_{\text{on}} = DT_s$ and

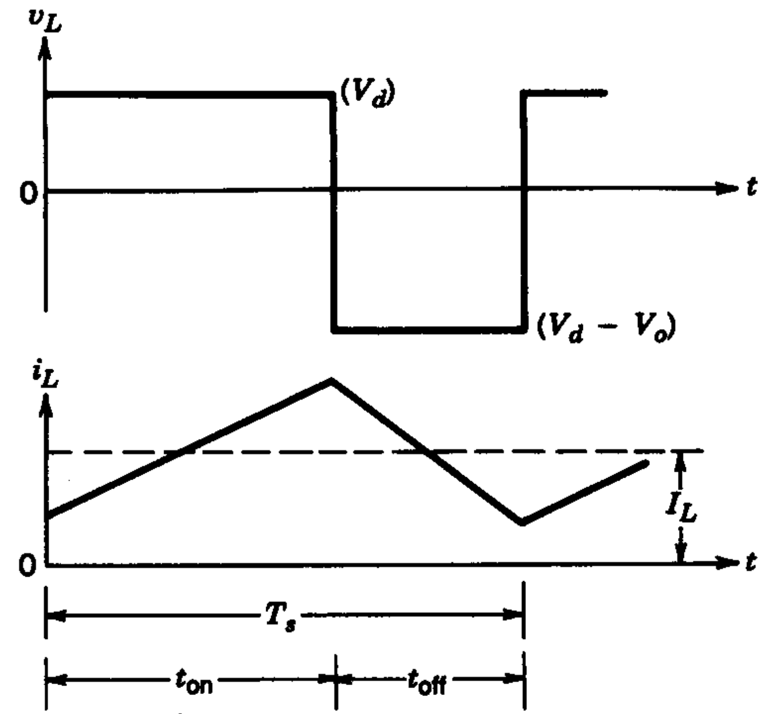
$$t_{\text{off}} = (1 - D)T_s$$

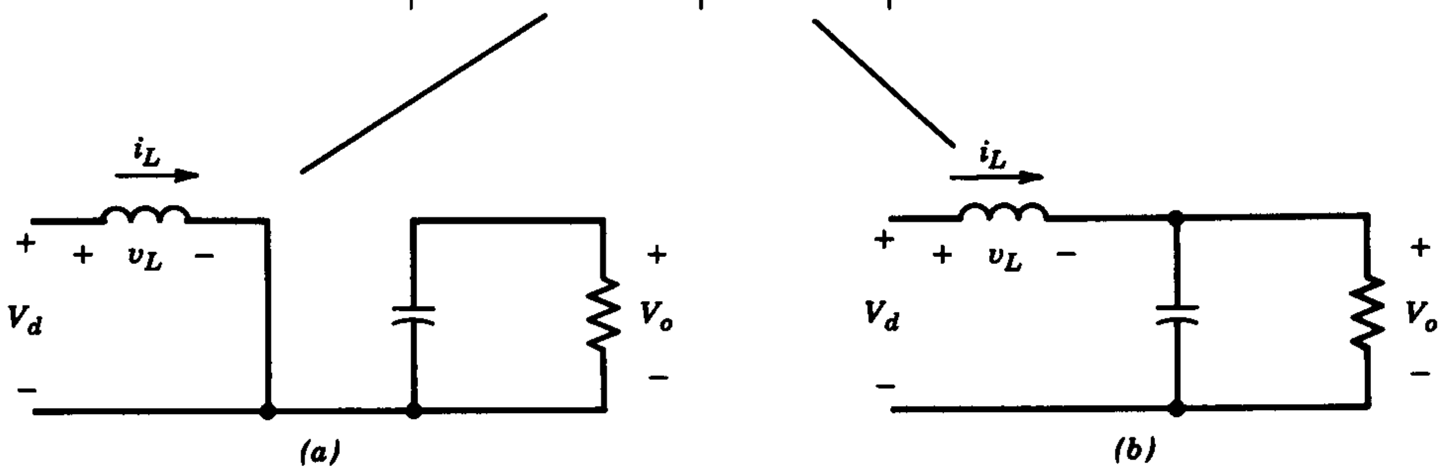
$$T_sV_d + T_s(1 - D)V_o = 0$$

$$\frac{V_o}{V_d} = \frac{1}{1 - D}$$

No losses:

$$V_oI_o = V_dI_d$$





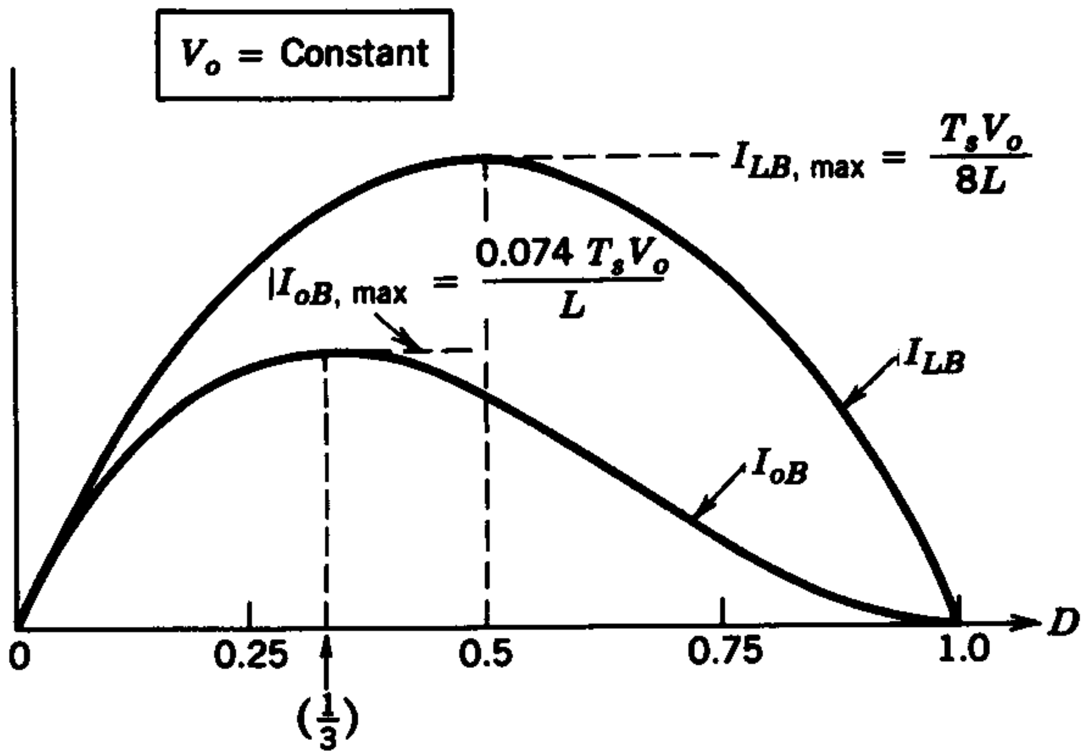
Continuous-discontinuous boundary

Average current in L = ripple :

$$I_{LB} = \frac{V_d t_{on}}{2L} = \frac{V_o(1-D)T_s D}{2L}$$

Average output current at the limit:

$$I_{oB} = I_{LB}(1-D) = \frac{V_o T_s (1-D)^2 D}{2L}$$



I_{LB} is max if D=0.5 → $I_{LBmax} = \frac{V_o T_s}{8L}$,

I_{oB} is max if D=1/3 → $I_{oBmax} = \frac{2V_o T_s}{27L}$ → $I_{oB} = \frac{27}{4}(1-D)^2 D I_{oBmax}$

Discontinuous conduction mode (constant V_o)

Periodic conditions:

$$\frac{DT_sV_d}{L} + \frac{\Delta_1 T_s(V_d - V_o)}{L} = 0$$

$$\frac{V_o}{V_d} = 1 + \frac{D}{\Delta_1} = \frac{I_d}{I_o}$$

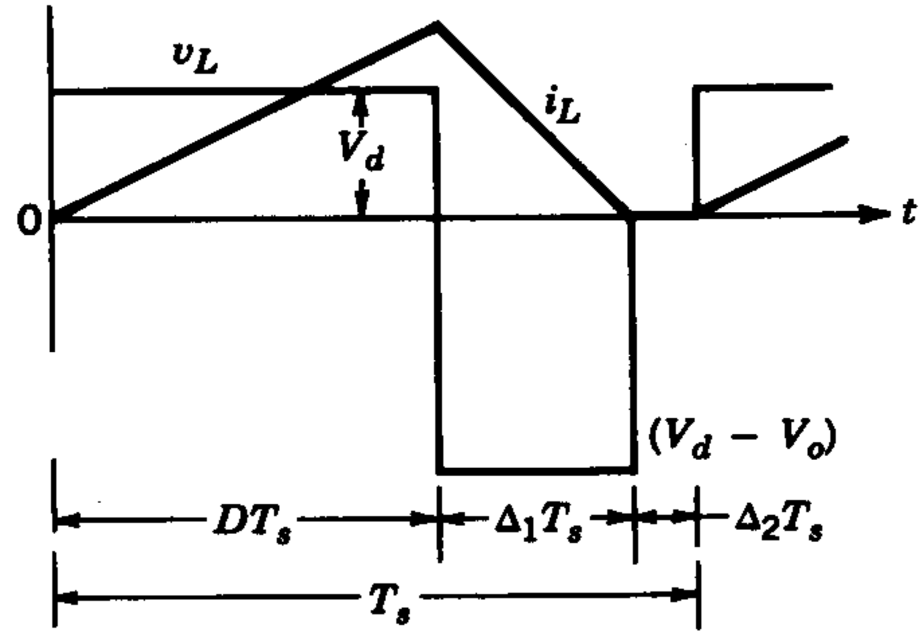
Average current in L

$$I_d T_s = \frac{DT_sV_d}{L}\frac{(D+\Delta_1)T_s}{2}$$

Average output current

$$I_o = I_d\frac{\Delta_1}{D+\Delta_1} = \frac{T_sV_d}{2L}D\Delta_1$$

$$= \frac{27}{4}I_{oBmax}\frac{V_d}{V_o}D^2\frac{V_d}{V_o - V_d}$$



$$D = \left[\frac{4}{27}\frac{V_o}{V_d}\left(\frac{V_o}{V_d} - 1\right)\frac{I_o}{I_{oBmax}}\right]^{\frac{1}{2}}$$

Continuous-discontinuous mode (constant V_o)

Continuous mode:

$$I_o > I_{oB}$$

$$= I_{oBmax}\frac{27(1-D)^2D}{4}$$

$$D = 1 - \frac{V_d}{V_o}$$

Discontinuous mode:

$$I_o < I_{oB}$$

$$D = \left[\frac{4}{27}\frac{V_o}{V_d}\left(\frac{V_o}{V_d}-1\right)\frac{I_o}{I_{oBmax}}\right]^{\frac{1}{2}}$$

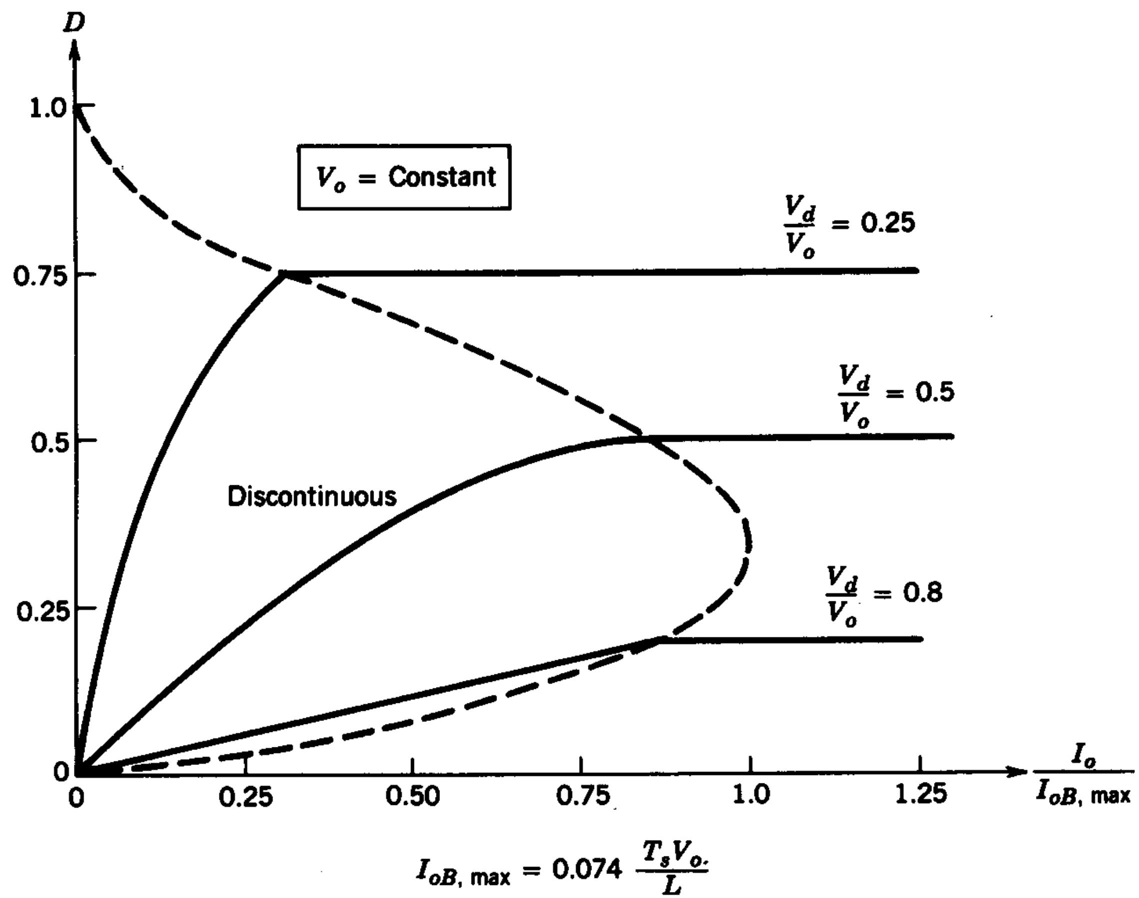


Figure 7-15 Step-up converter characteristics keeping V_o constant.

Losses and ripple

Losses: inductor, capacitor, switch, diode

Ripple: first order assumption: when the switch is on the C is discharged through the load

$$\Delta V_o = \frac{\Delta Q}{C} = \frac{I_o D T_s}{C} = \frac{V_o D T_s}{RC}$$

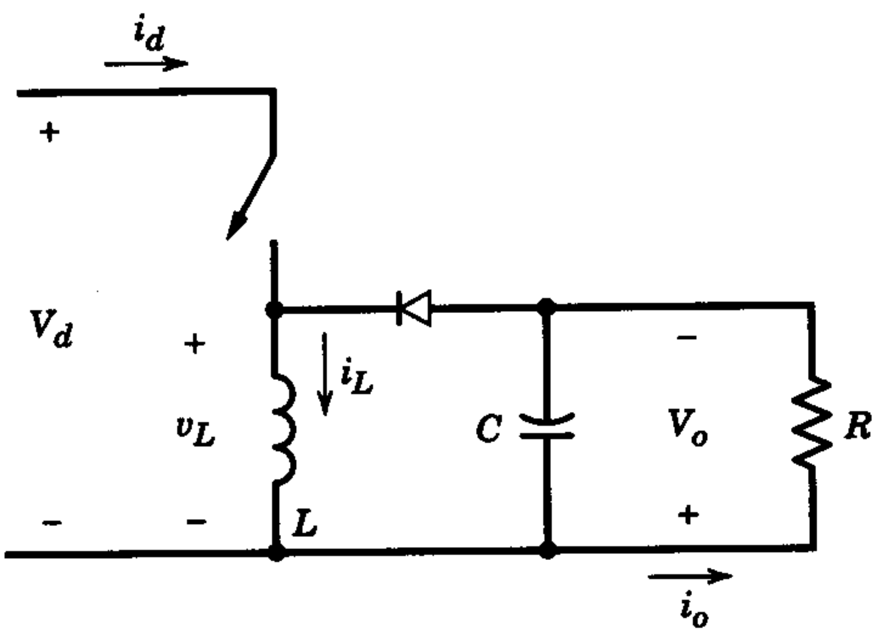
$$\frac{\Delta V_o}{V_o} = D\,\frac{T_s}{\tau}$$

Buck-boost converter

Negative DC power supply

Switch on: inductance accumulates energy, diode off, C supplies the load

Switch off: diode on, inductance transfers energy to the capacitance and to the load



Periodic conditions in continuous conduction mode:

$$\frac{DT_sV_d}{L} - \frac{V_o(1-D)T_s}{L} = 0$$

$$\frac{V_o}{V_d} = \frac{D}{1-D} = \frac{I_d}{I_o}$$

$$I_L = I_o + I_d = \frac{I_o}{1-D}$$

Continuous-discontinuous boundary

Current in L at the boundary

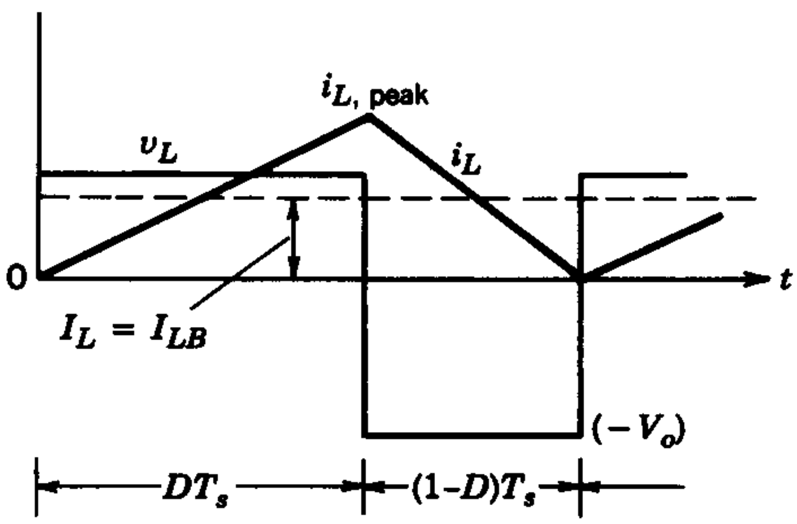
$$I_{LB} = \frac{DT_sV_d}{2L}$$

$$I_{LB} = I_{LBmax}(1 - D)$$

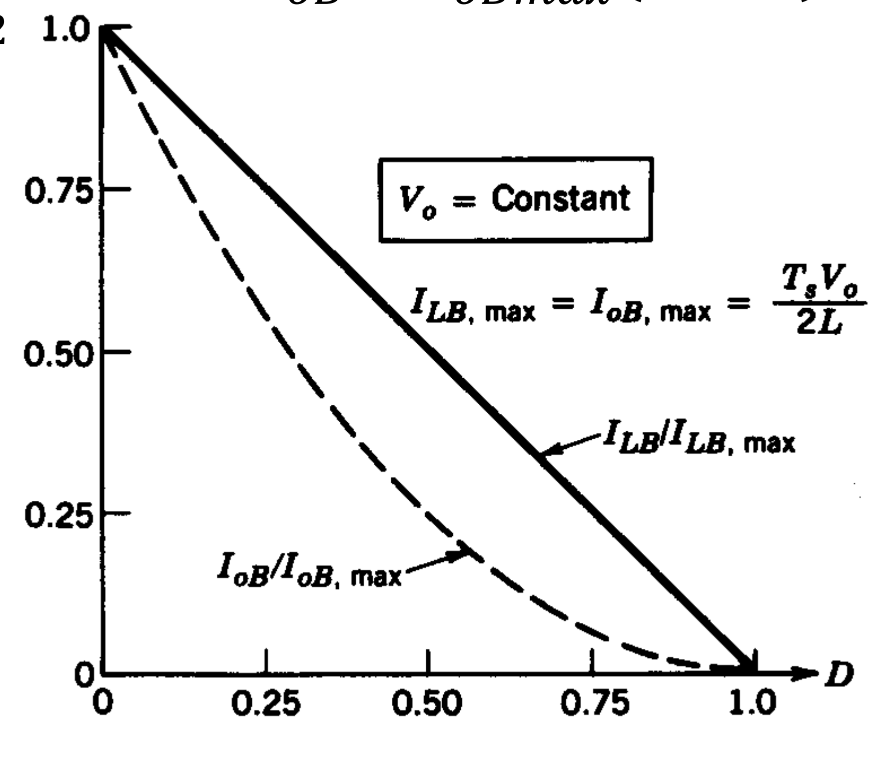
Output current at the boundary:

$$I_{oB} = I_{oBmax}(1 - D)^2$$

$$I_{oB} = I_{LB}(1 - D) = \frac{T_sV_o}{2L}(1 - D)^2$$



(a)



(b)

Discontinuous conduction

Periodic conditions:

$$\frac{DV_dT_s}{L} - \frac{V_o\Delta_1 T_s}{L} = 0$$

$$\frac{V_o}{V_d} = \frac{D}{\Delta_1} = \frac{I_d}{I_o}$$

Average current in L:

$$I_LT_s = \frac{V_dDT_s}{L}\frac{(D+\Delta_1)T_s}{2}$$

Therefore:

$$I_L = I_o\left(1 + \frac{D}{\Delta_1}\right) = \frac{V_dT_s}{2L}D(D+\Delta_1)$$

$$\frac{I_o}{I_{oBmax}} = D\Delta_1\frac{V_d}{V_o} = D^2\left(\frac{V_d}{V_o}\right)^2 \rightarrow D = \frac{V_o}{V_d}\sqrt{\frac{I_o}{I_{oBmax}}}$$

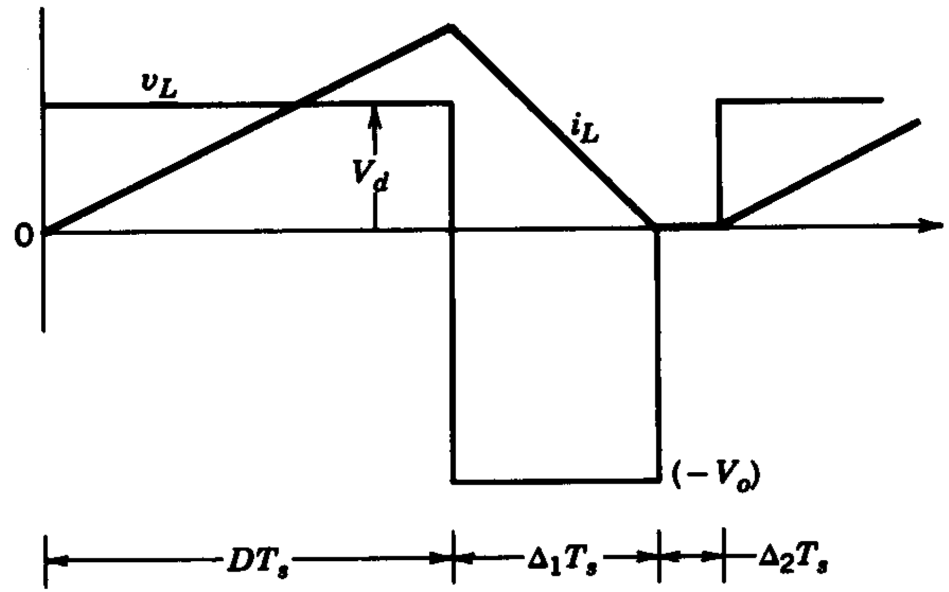


Figure 7-21 Buck–boost converter waveforms in a discontinuous-conduction mode.

Continuous-discontinuous mode

Continuous operation

$$I_o > I_{oB} = I_{oBmax}(1-D)^2$$

$$D = \frac{V_o}{V_d - V_o}$$

Discontinuous operation

$$I_o > I_{oB}$$

$$D = \frac{V_o}{V_d}\sqrt{\frac{I_o}{I_{oBmax}}}$$

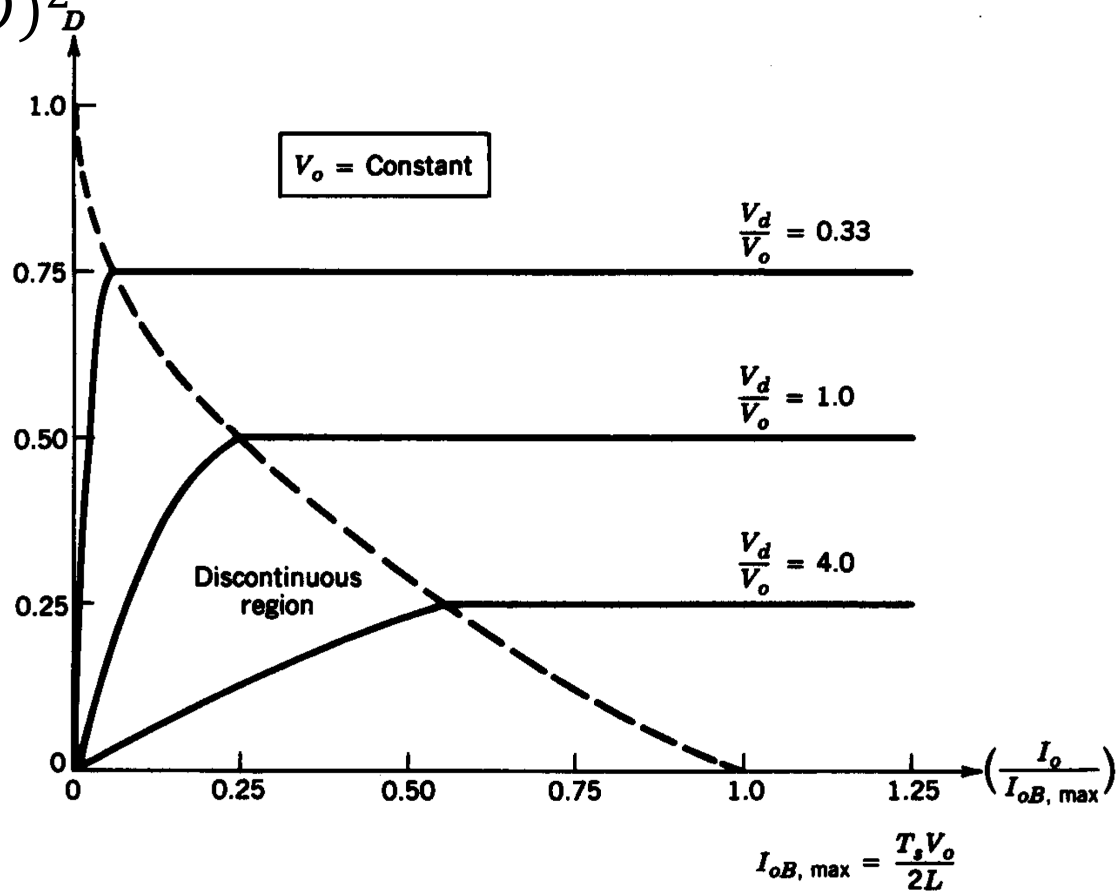


Figure 7-22 Buck–boost converter characteristics keeping V_o constant.

Output voltage ripple

When the switch is ON, C is discharged through the load

$$\Delta V_o = \frac{\Delta Q}{C} = \frac{DT_sV_o}{RC} \rightarrow \frac{\Delta V_o}{V_o} = D\frac{T_s}{\tau}$$

Cuk DC-DC converter

Negative DC power supply

DC analysis: $V_{C1} = V_d + V_o$ note: ($V_{C1} > V_d$)

Assumption: Large C1 (Voltage almost constant)

Switch OFF: C1 is charged through L1 and the input, Diode ON, L2 supplies energy to R (currents in L1 and L2 decrease)

Switch ON: L1 receives energy, Diode OFF, C supplies current to R, C1 gives energy to L2 (currents in L1 and L2 increase)

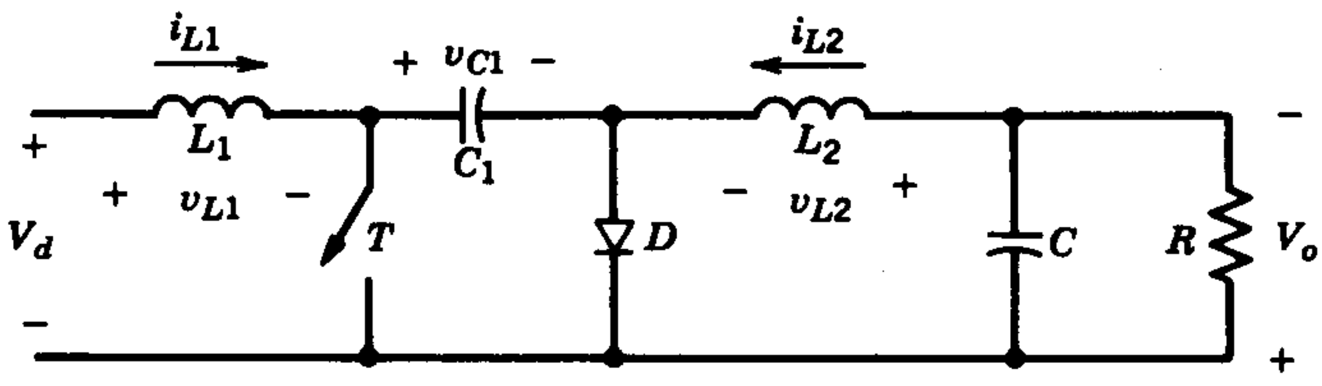


Figure 7-25 Cúk converter.

Cuk

Periodic conditions in L1

$$V_d D T_s + (1 - D) T_s (V_d - V_{C1}) = 0$$

$$V_{C1} = \frac{V_d}{1 - D}$$

Periodic conditions in L2

$$(V_{C1} - V_o) D T_s - \mathrm{V_o}(1 - \mathrm{D}) T_s = 0$$

$$V_{C1} = \frac{V_o}{D}$$

Therefore

$$\frac{V_o}{V_d} = \frac{D}{1 - D}$$

Pro: currents

in L1 and L2 ripple free

Con: C1 must be large

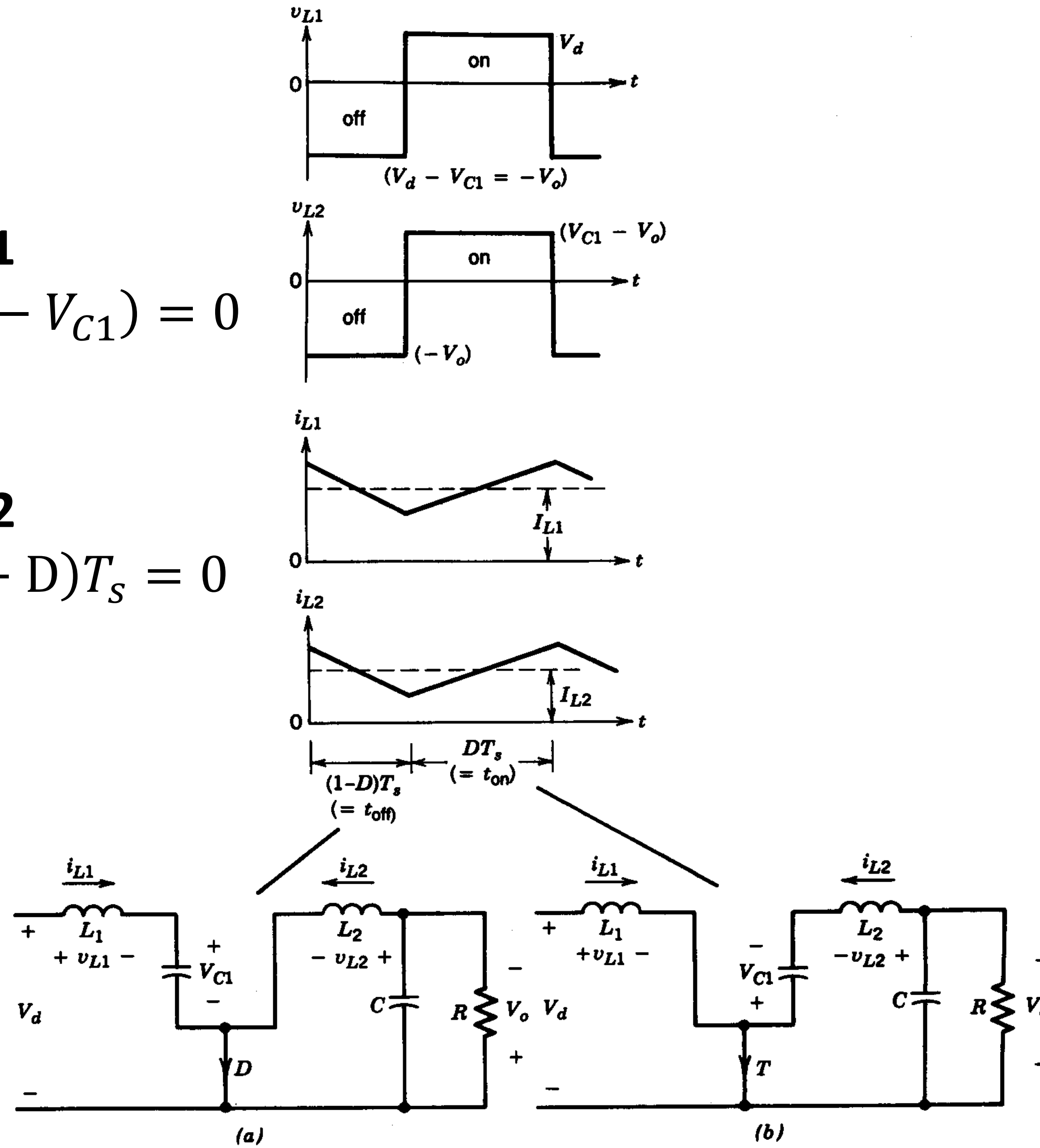


Figure 7-26 Cúk converter waveforms: (a) switch off; (b) switch on.

Full bridge DC-DC converter

Applications:

- DC motor drives
- DC to AC conversion in UPS
- DC to AC conversion in transformer isolated power supply

Fixed V_d.

Control polarity and amplitude of Vo

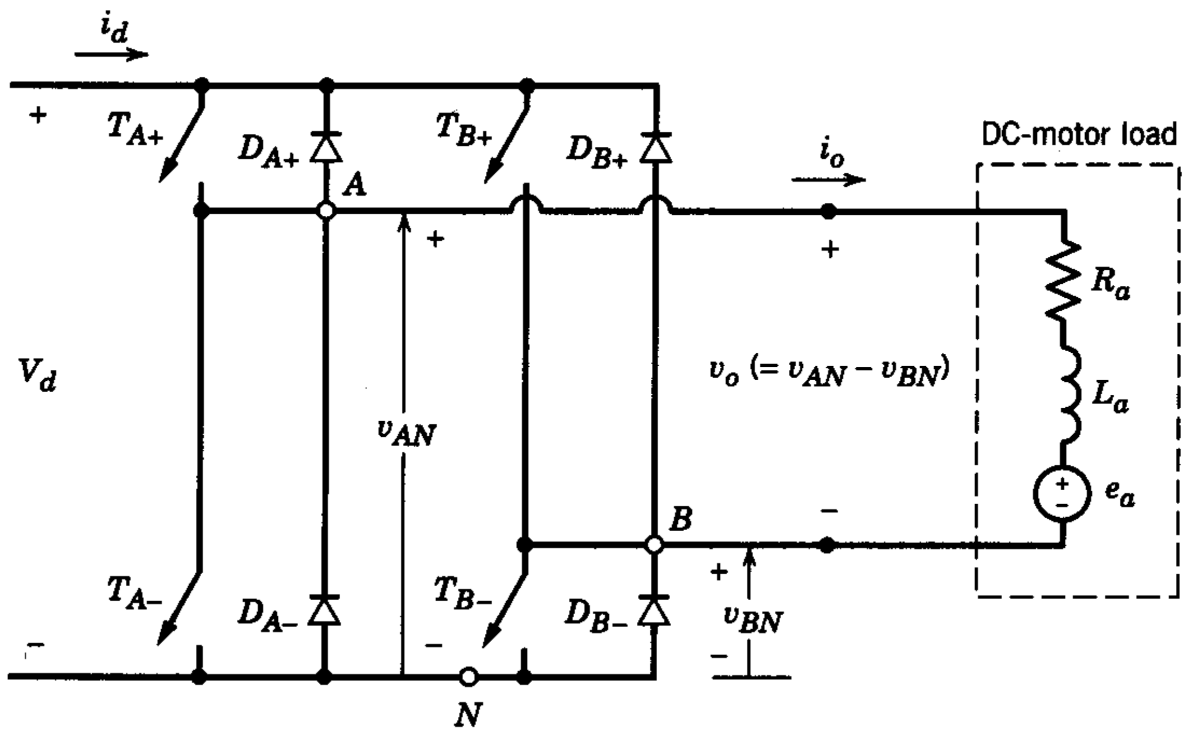


Figure 7-27 Full-bridge dc–dc converter.

Two legs: A and B. **Only one switch** in each leg is ON at any time

Full bridge DC-DC converter

When switch TA+ is on:

$i_o > 0$: i_o through TA+

$i_o < 0$: i_o through DA+

$V_{AN} = V_d \text{dutycycle}(TA^+)$

When switch TB+ is on:

$i_o < 0$: i_o through TB+

$i_o > 0$: i_o through DB+

$V_{BN} = V_d \text{dutycycle}(TB^+)$

$$V_o = V_{AN} - V_{BN}$$

Four quadrant operation on V_o, I_o

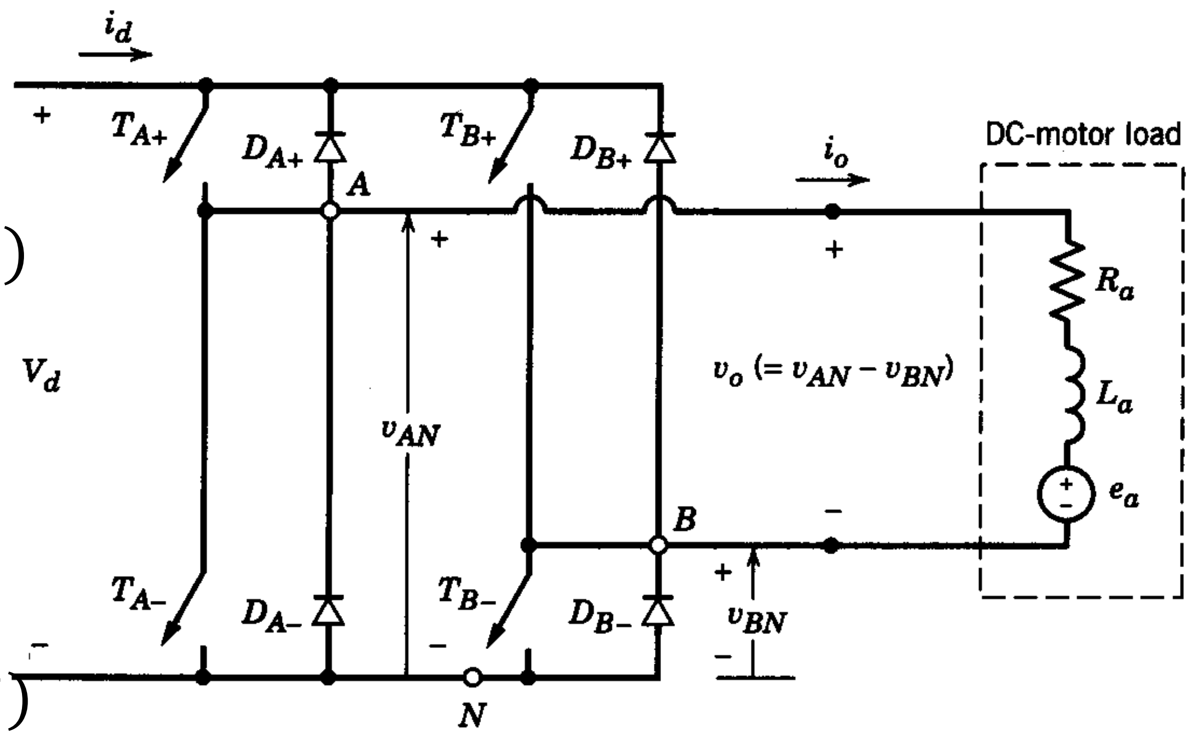


Figure 7-27 Full-bridge dc–dc converter.

PWM with bipolar voltage switching

When $v_{control} > v_{tri}$,

TA+ and TB- are **ON**

Duty cycle

$$D_1 = \frac{1}{2} + \frac{v_{control}}{\widehat{V_{tri}}}\frac{1}{2}$$

When $v_{control} < v_{tri}$,

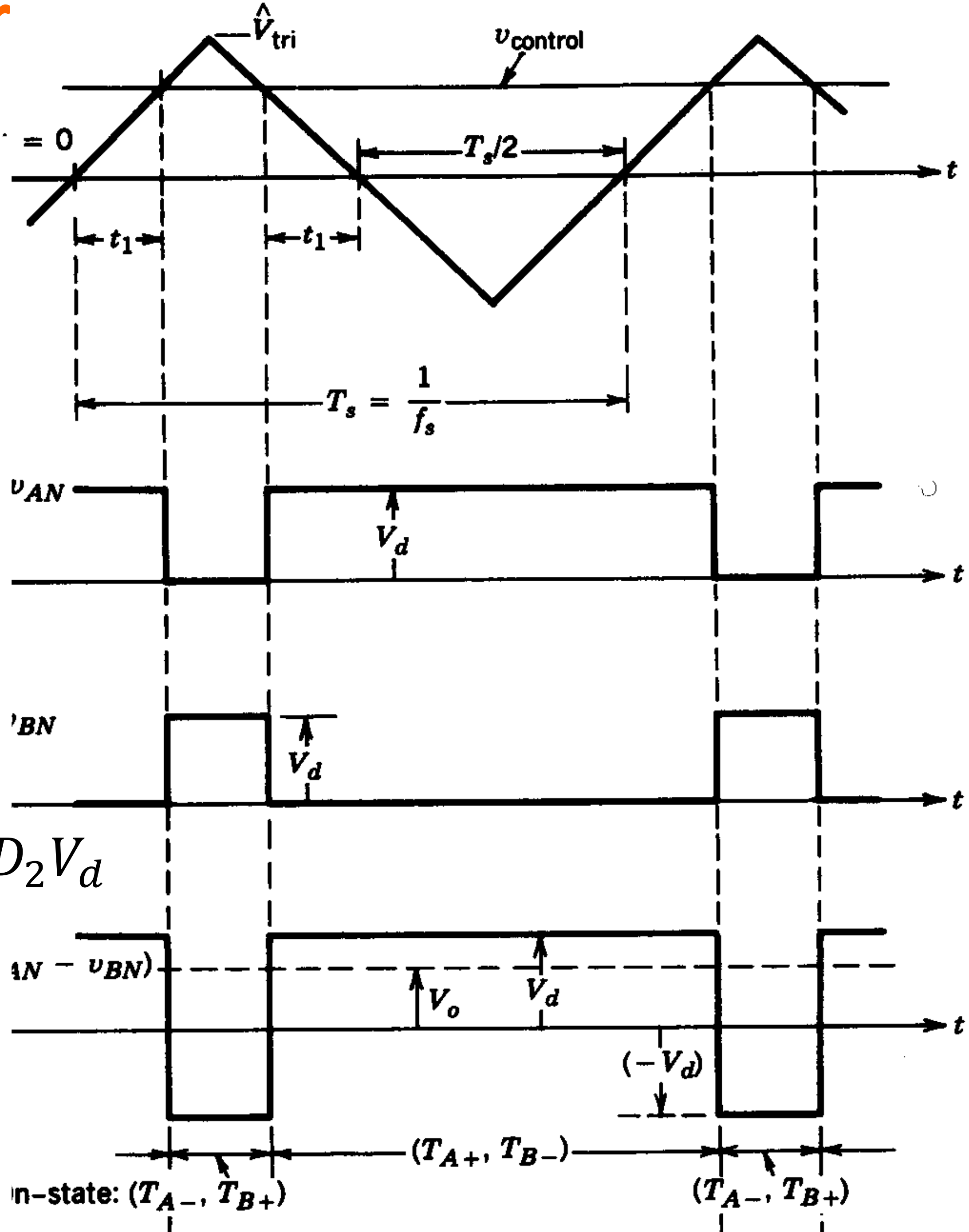
TA- and TB+ are **ON**

$$D_2 = 1 - D_1$$

$$V_o = V_{AN} - V_{BN} = D_1 V_d - D_2 V_d$$

$$= (2D_1 - 1)V_d$$

$$= \frac{V_d}{\widehat{V_{tri}}} v_{control}$$



PWM with unipolar voltage switching

When $v_{control} > v_{tri}$,

TA+ and TB- are **ON**

Duty cycle

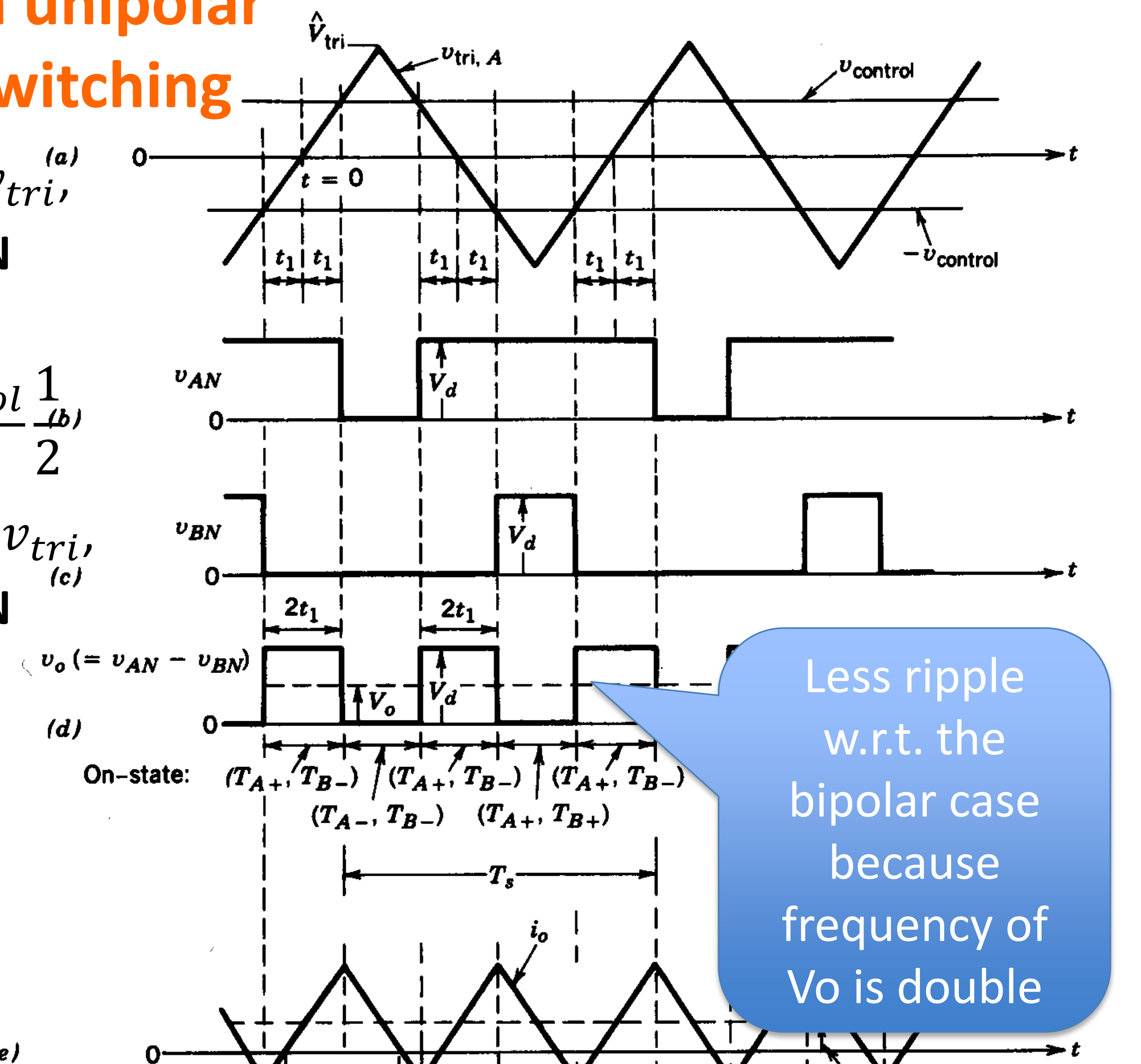
$$D_1 = \frac{1}{2} + \frac{v_{control}}{\widehat{V_{tri}}}\frac{1}{2}$$

When $-v_{control} < v_{tri}$,

TA- and TB+ are **ON**

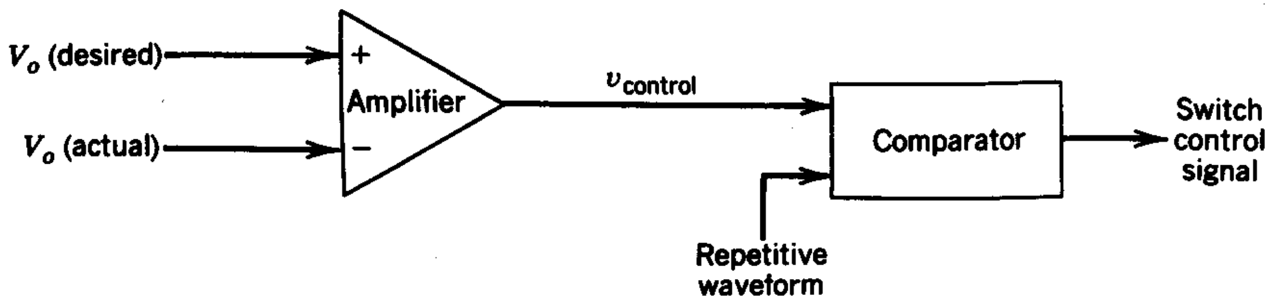
$$D_2 = 1 - D_1$$

$$V_o = V_{AN} - V_{BN}$$
$$= D_1 V_d - D_2 V_d$$
$$= (2D_1 - 1)V_d$$
$$= \frac{V_d}{\widehat{V_{tri}}} v_{control}$$

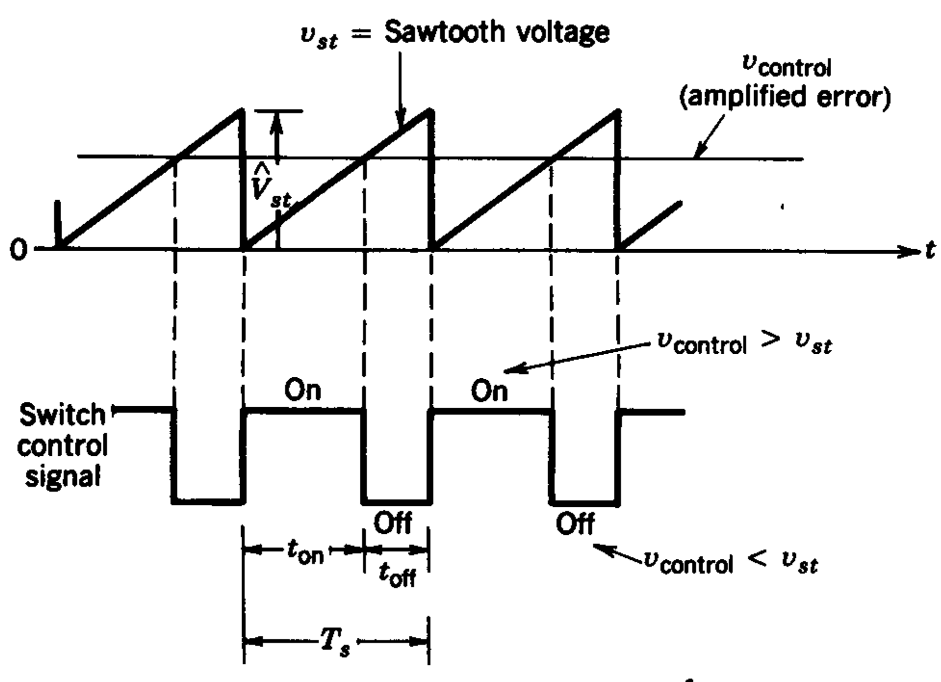


Less ripple w.r.t. the bipolar case because frequency of Vo is double

PWM signal generation



(a)



(switching frequency $f_s = \frac{1}{T_s}$)

Buck-boost

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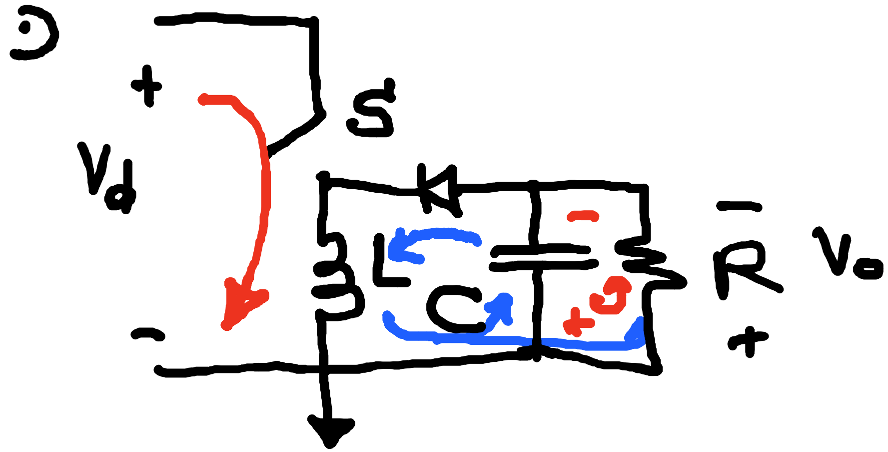
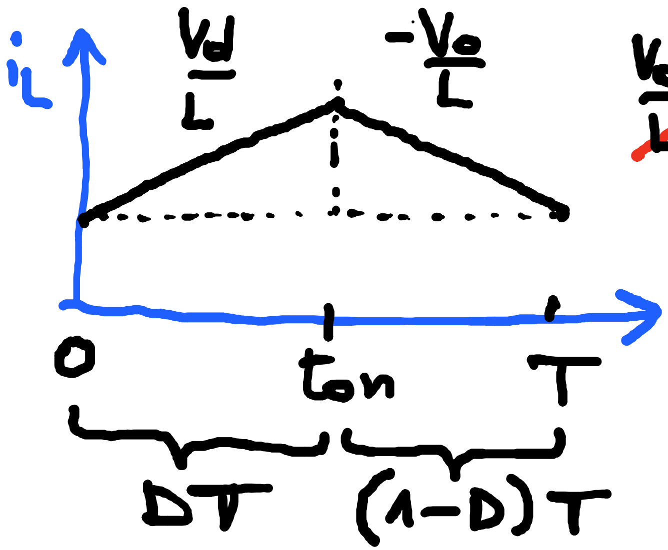




$$\frac{V_d}{\cancel{L}} D\cancel{T} - \frac{V_o}{\cancel{L}}(1-D)\cancel{T} = 0$$

$$\boxed{V_o = V_d \frac{D}{1-D}}$$

S ON   Diodo OFF

$$\frac{di_L}{dt} = \frac{V_d}{L}$$

La C sostiene V_o

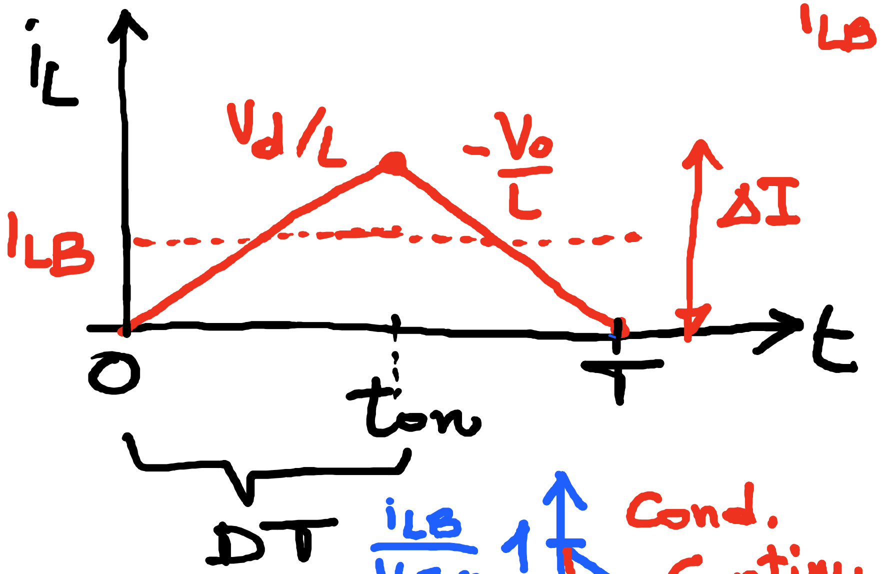
S OFF   Diodo ON

La L carica C e sostiene V_o

$$\frac{di_L}{dt} = -\frac{V_o}{L}$$

Limite TRA conduzione continua e discontinua

Wednesday, 10 May 2017 09:01

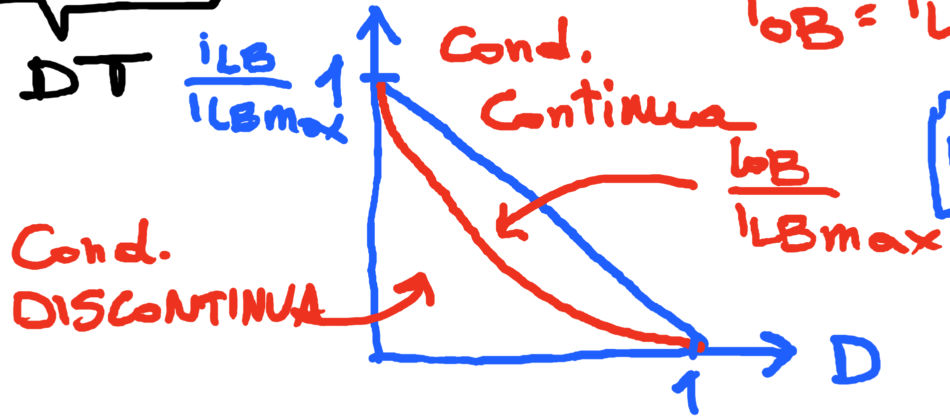


$$i_{LB} = \frac{\Delta I}{2} = \frac{V_d}{L} DT \frac{1}{2} = \frac{V_o}{L}(1-D)T\frac{1}{2}$$

$$i_{LB} = i_{LBmax}(1-D) \qquad \left[i_{LBmax} \triangleq \frac{V_o T}{2L} \right]$$

$$\langle i_o \rangle = \langle i_L \rangle (1-D)$$

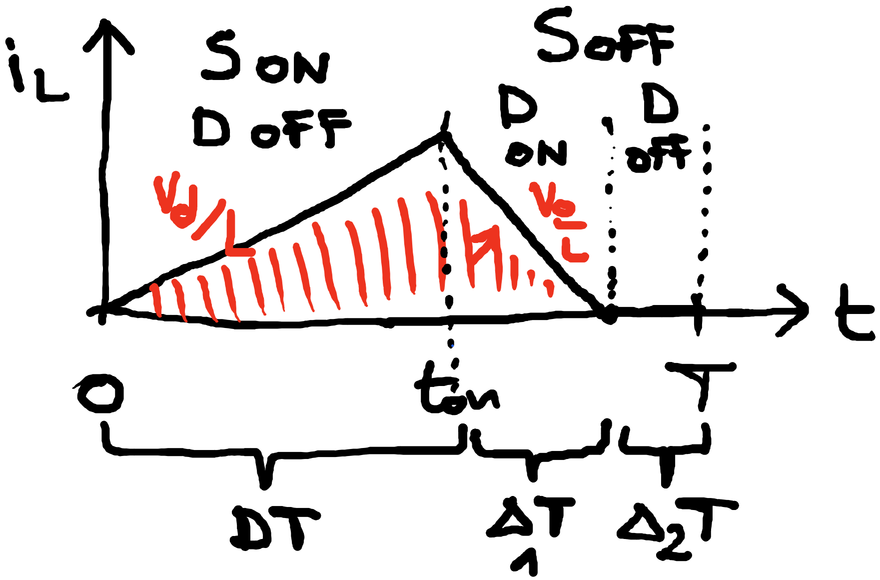
$$i_{oB} = i_{LB}(1-D) = i_{LBmax}(1-D)^2$$



$\boxed{V_o \text{ costante}}$

Conduzione discontinua

Wednesday, 10 May 2017 09:09



$$\langle I_L \rangle = \frac{V_d}{L} DT (D+\Delta_1)\cancel{T}\,\frac{1}{2}\,\frac{1}{\cancel{T}}$$

$$\langle i_o \rangle = \frac{\Delta_1}{\Delta_1 + D}\langle i_L \rangle =$$

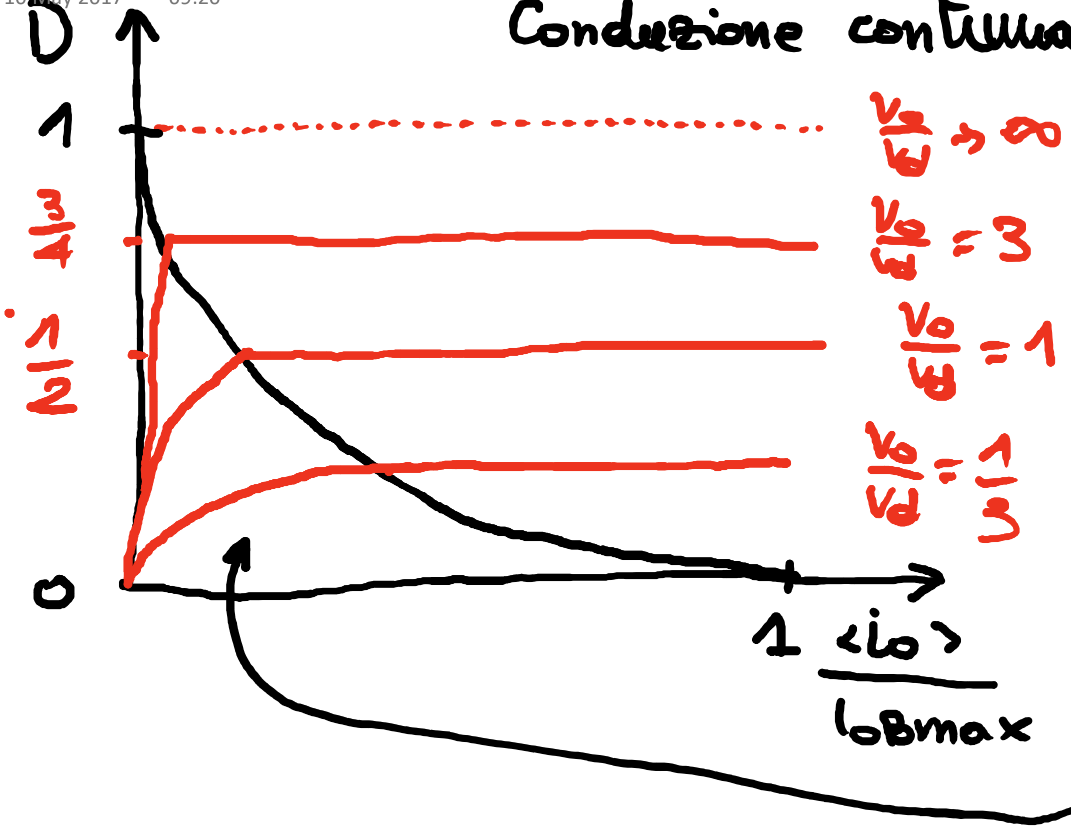
$$= \frac{\Delta_1}{\cancel{\Delta_1 + D}}\,\frac{V_d}{L}\,\frac{DT}{2}\cancel{(D+\Delta_1)} = \frac{V_d}{L}\,\frac{DT}{2}\Delta_1$$

$$\frac{V_d}{L} DT = \frac{V_o}{L}\Delta_1 T \rightarrow V_d D = V_o \Delta_1 \rightarrow \Delta_1 = \frac{V_d D}{V_o} \Rightarrow \langle i_o \rangle = \frac{V_d}{L} D^2 \left(\frac{V_d}{V_o}\right)\frac{T}{2} =$$

$$i_{LBmax} = i_{oBmax} = \frac{V_o T}{2L} \qquad \langle i_o \rangle = i_{oBmax}\left(\frac{V_d}{V_o}\right)^2 D^2 \rightarrow D \propto \sqrt{\langle i_o \rangle}$$

Conduzione continua

$$\frac{V_o}{V_d} = \frac{D}{1-D}$$



Conduzione discontinua

$$D = \left(\frac{V_o}{V_d}\right)\sqrt{\frac{\langle i_o \rangle}{i_{oBmax}}}$$

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Ripple :

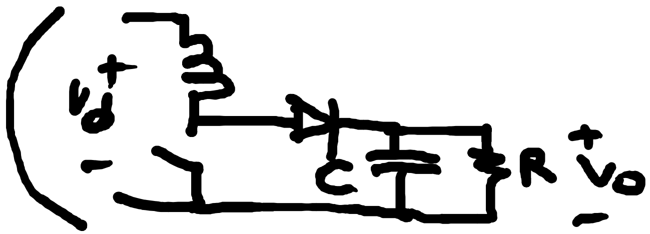
Durante $t_{on} = DT$ la C si scarica su R

$$\Delta V = \frac{\Delta Q}{C} = \langle I_o \rangle \frac{DT}{C} = \frac{V_o}{R}\frac{DT}{C}$$

$$\boxed{\Delta V \propto \frac{1}{C f_s}}$$

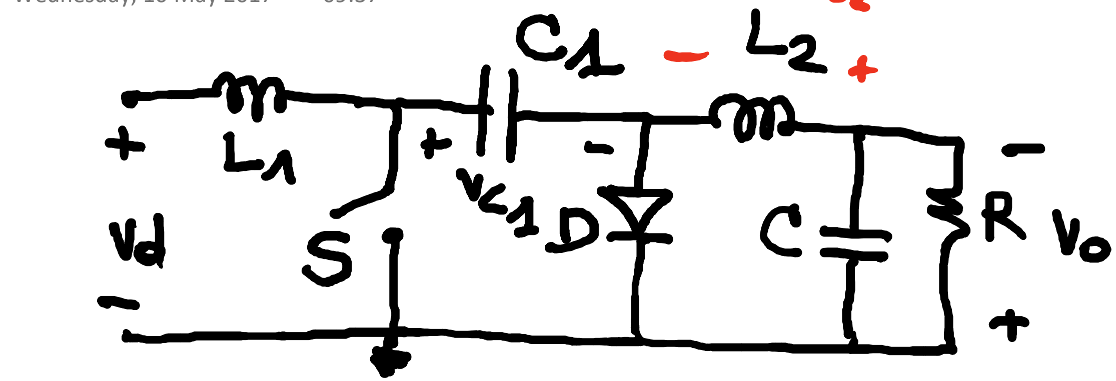
Buck-Boost

Boost

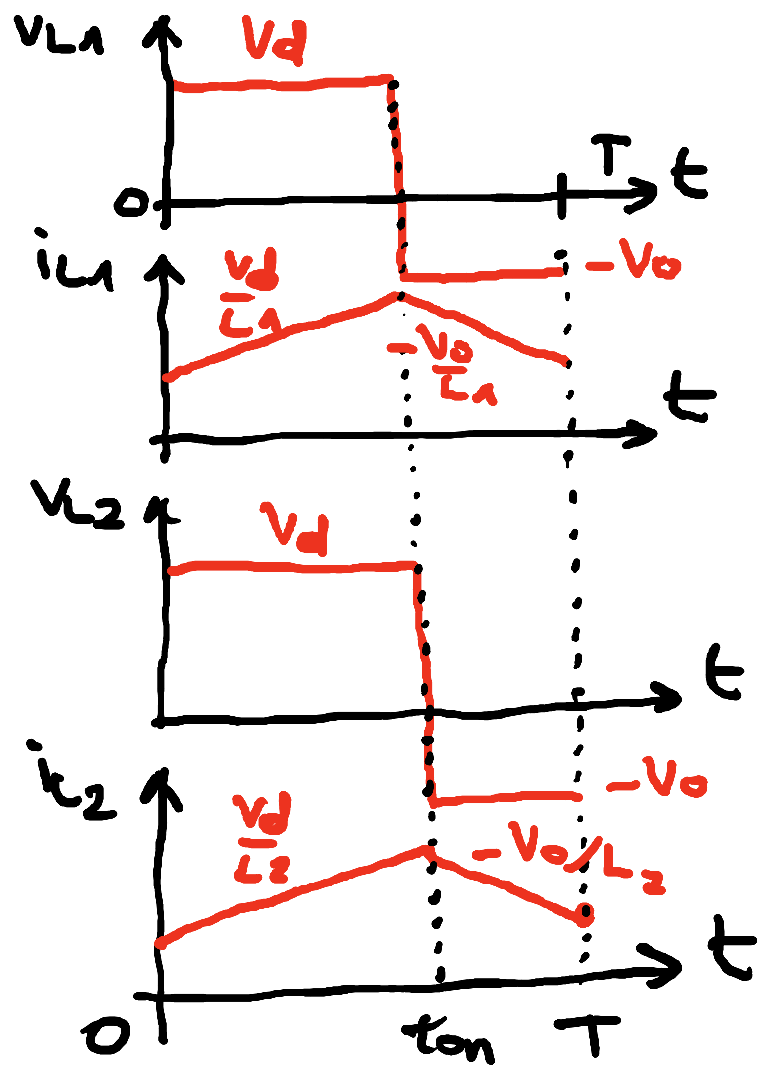


Ćuk Converter

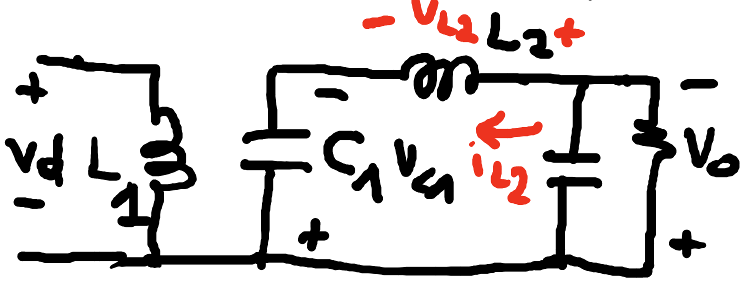
Wednesday, 10 May 2017 09:37



$V_{c1} = V_d + V_o$

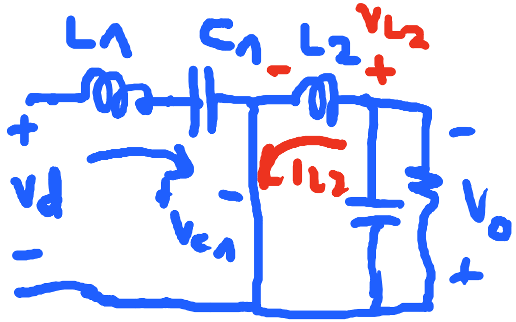


Ton (D off)



$V_{L2} = V_{c1} - V_o = V_d$

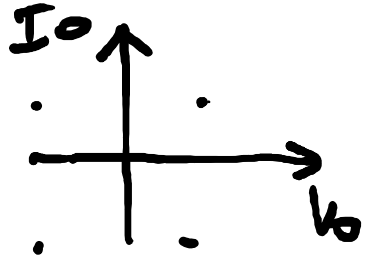
TOFF (D ON)

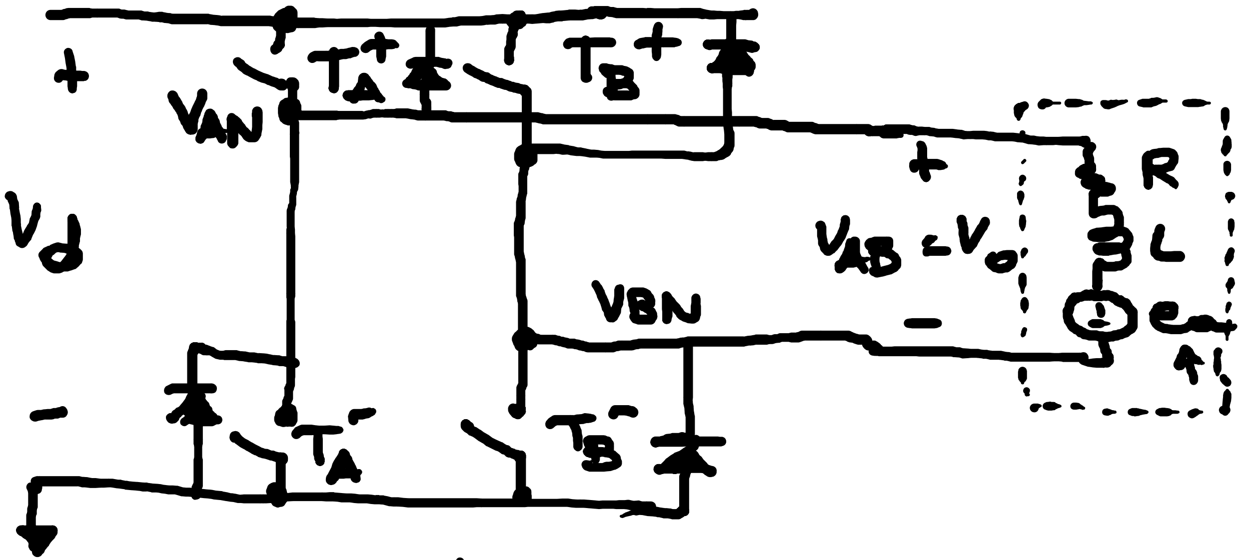


$V_{L1} = V_d - V_{c1} = -V_o$

Full Bridge DCDC converter
(a 4 quadranti)

Wednesday, 10 May 2017 09:53





A B

D_A D_B

$$\langle V_{AN} \rangle = D_A V_d$$

$$\langle V_{BN} \rangle = D_B V_d$$

$$V_{AB} = V_{AN} - V_{BN}$$

$$\langle V_{AB} \rangle = \langle V_{AN} \rangle - \langle V_{BN} \rangle$$

$$\langle V_{AB} \rangle = V_d (D_A - D_B)$$

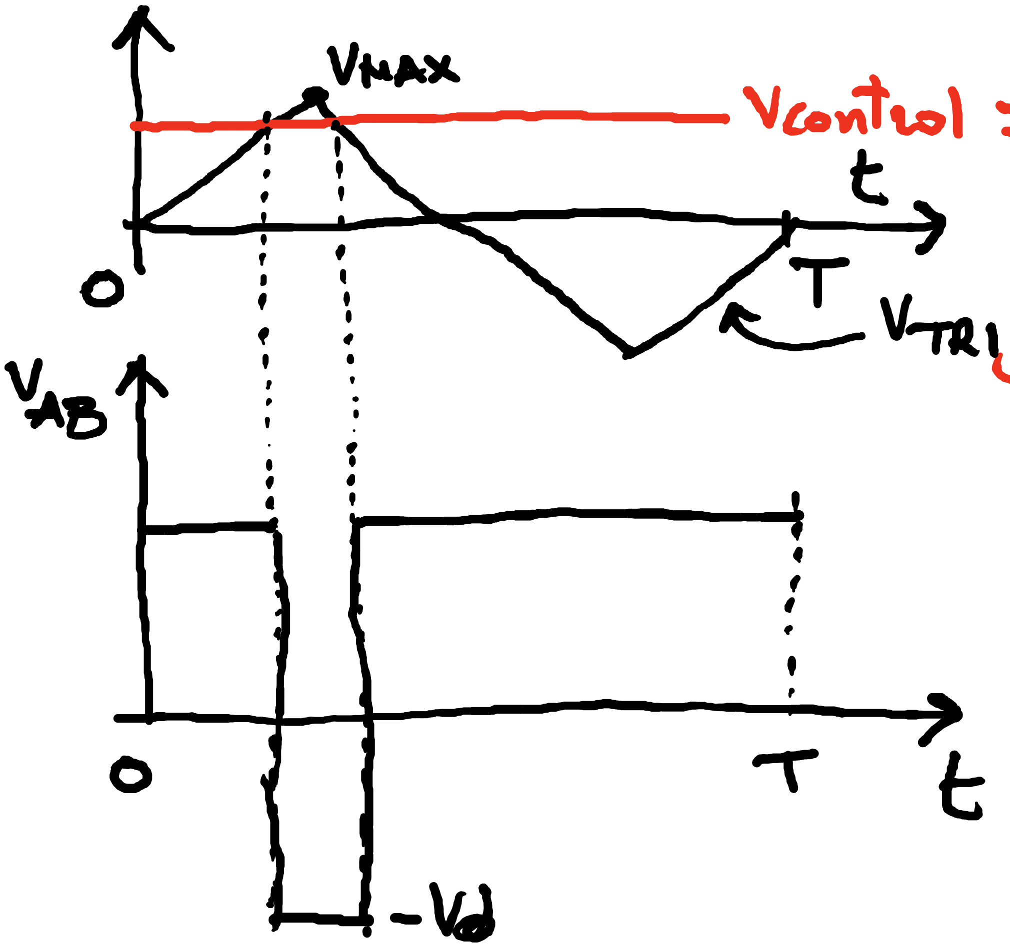
CONTROLLO

PWM → UNIPOLARE
→ BIPOLARE

Pulse Width Modulation

PWM Bipolare

Wednesday, 10 May 2017 10:07



$\left\{ \begin{array}{l} T_A^- \text{ ON} \\ T_B^+ \text{ ON} \end{array} \right. \Rightarrow V_{AB} = -V_d$

$\left\{ \begin{array}{l} T_A^+ \text{ ON} \\ T_B^- \text{ ON} \end{array} \right. \Rightarrow V_{AB} = +V_d$

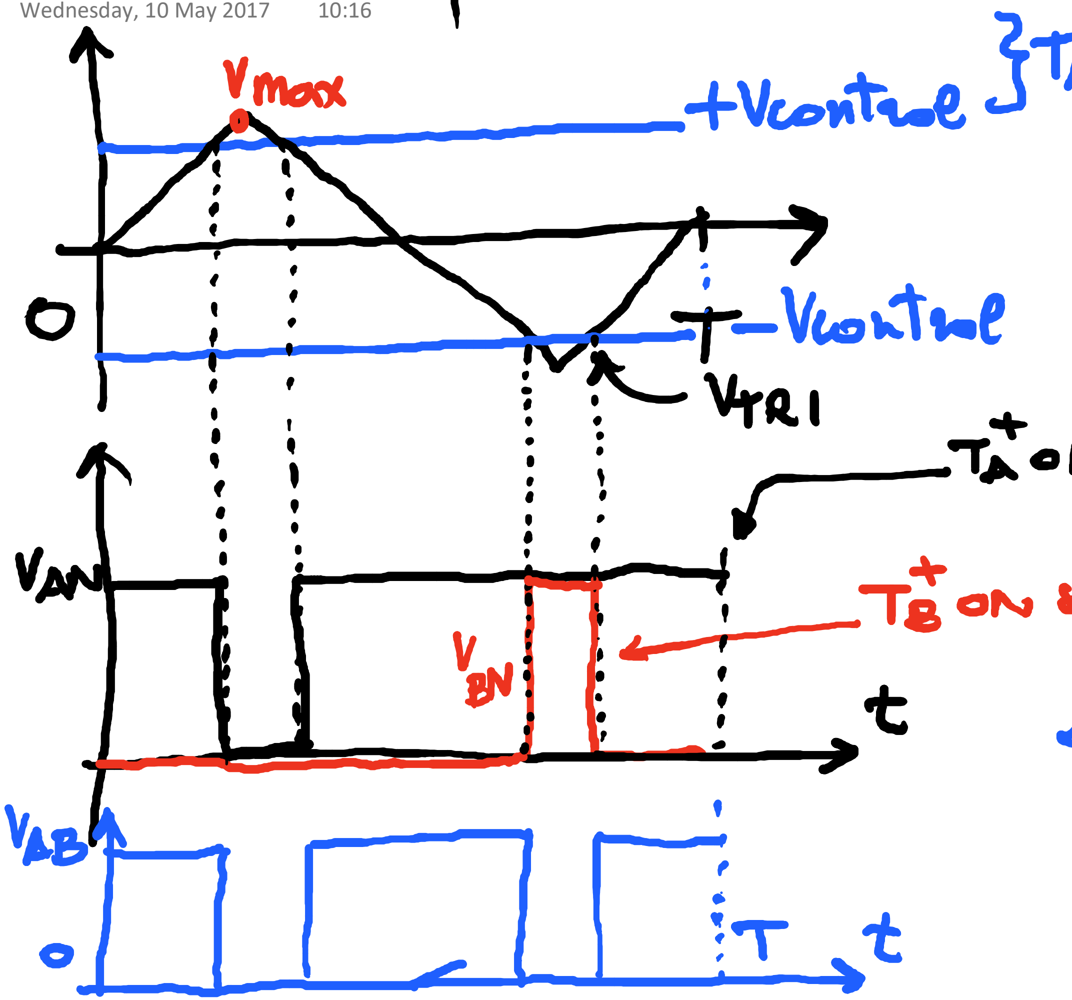
$$D_A = \frac{1}{2} + \frac{1}{2} \frac{V_{control}}{V_{max}}$$

$$D_B = \frac{1}{2} - \frac{1}{2} \frac{V_{control}}{V_{max}}$$

$$\langle V_{AB} \rangle \; (D_A - D_B) V_d = \left(\frac{V_{control}}{V_{max}} \right) \cdot V_d$$

PWM Unipolare

Wednesday, 10 May 2017 10:16



T_A^+ ON

T_B^+ ON

$D_A = \frac{1}{2} + \frac{1}{2}\frac{V_{control}}{V_{MAX}}$

$D_B = \frac{1}{2} - \frac{1}{2}\frac{V_{control}}{V_{MAX}}$

T_A^+ ON se $V_{control} > V_{TRI}$

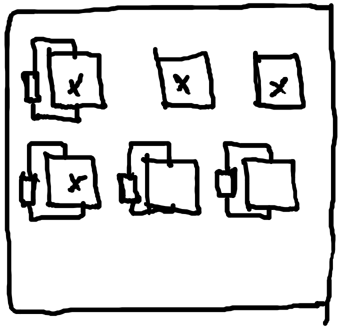
T_B^+ ON se $-V_{control} > V_{TRI}$

$\langle V_{AB} \rangle = (D_A - D_B) V_d = \left(\frac{V_{control}}{V_{MAX}}\right) V_d$

Convertitori DC DC inductorless

Tuesday, 16 May 2017 08:56

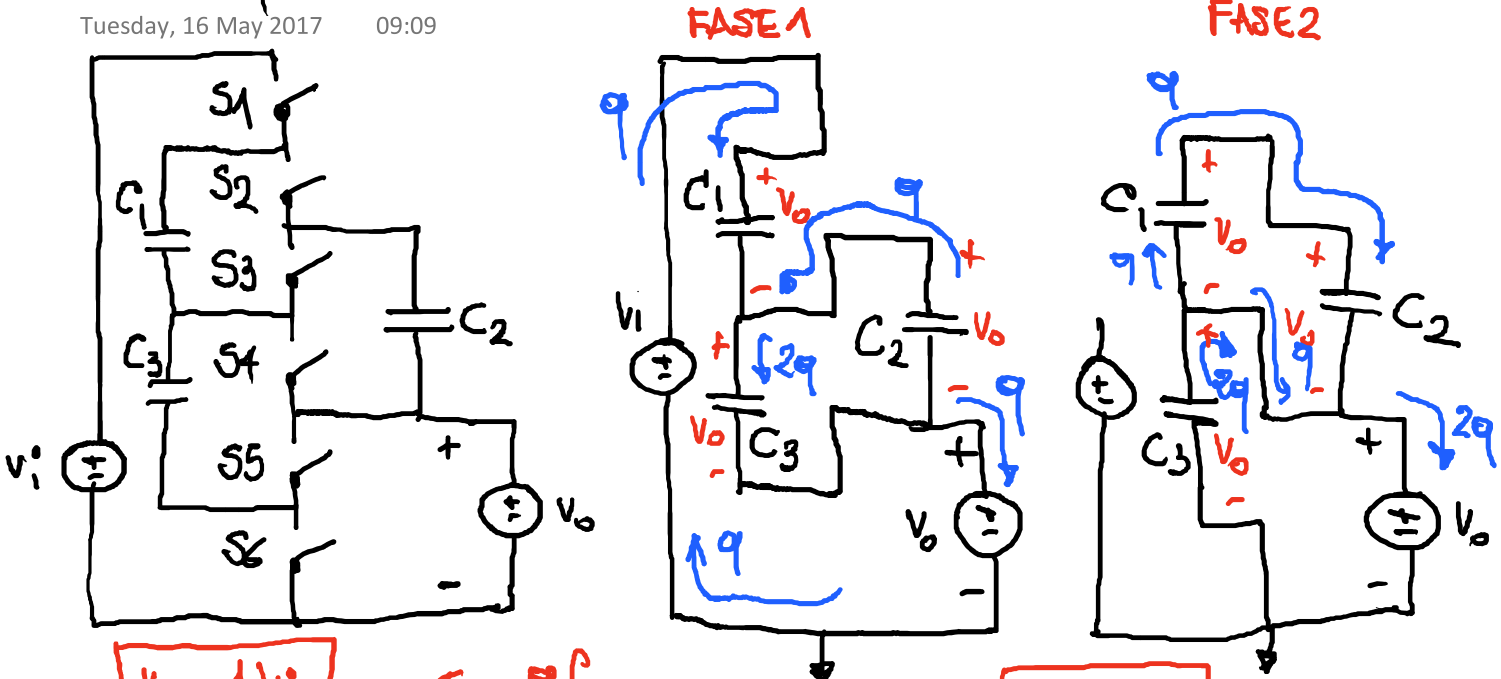
↳ Switched Cap vs inductor-based



- ↑ RIDOTTA AREA
- ↑ RIDOTTA ALTEZZA
- ↑ Possibilità di integrazione completa (per piccole potenze) e fornitura di power supply distribuita
- ↓ RAPPORTO DI CONVERSIONE IMPOSTO DALLA TOPOLOGIA
- ↓ EFFICIENZA DI CONVERSIONE RIDOTTA SE IL RAPPORTO DI CONVERSIONE NON È FRAZIONARIO

Esempio: LADDER 3:1

Tuesday, 16 May 2017 09:09



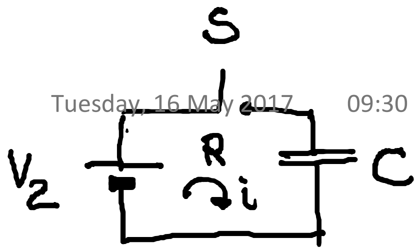
$\boxed{V_o = \frac{1}{3} V_i}$ $I_i = q f_s$ $I_o = 3 q f_s \Rightarrow \boxed{I_o = 3 I_i}$

Limiti di funzionamento

Tuesday, 16 May 2017 09:26

- SSL Slow Switching Limit [bassa f_s]
  - → durante ciascuna fase le capacità raggiungolo lo stato finale di carica
  - → si può trascurare la potenza dissipata nella resistenza dei switch
- FSL Fast Switching Limit [alta f_s]
  - → la tensione sulle capacità si può considerare costante durante ciascuna fase.

Tuesday, 16 May 2017 09:30



$t=0$ chiudo S

$$V_c(0) = V_1$$

$$i(t) = \frac{V_2 - V_1}{R} e^{-\frac{t}{RC}}$$

ENERGIA EROGATA DA V_2

$$\int_0^\infty V_2\, i(t)\, dt = V_2 \left(\frac{V_2 - V_1}{R}\right) RC = V_2 (V_2 - V_1) C$$

ENERGIA DISSIPATA NEL SWITCH

$$\int_0^\infty R\, i^2(t)\, dt = \int_0^\infty \left(\frac{V_2 - V_1}{R}\right)^2 e^{-\frac{2t}{RC}}\, dt = \frac{(V_2 - V_1)^2}{\cancel{R}} \frac{\cancel{R}C}{2} = \frac{C}{2} (V_2 - V_1)^2$$

ENERGIA ACCUMULATA DA C $\nearrow$ $= \frac{1}{2} C (V_2^2 - V_1^2)$

SSL

Tuesday, 16 May 2017 09:38

→ vettore moltiplicatore di carica

$$\vec{a}^{(1)} = \left\{ a_{out}^{(1)},\ a_{c_1}^{(1)},\ a_{c_2}^{(1)},\ a_{c_3}^{(1)},\ a_{in}^{(1)} \right\}$$

FASE 1

carica che passa durante la fase

NORMALIZZATA rispetto alle carice di uscita in un periodo

ES. LADDER

$$\vec{a}^{(1)} = \left\{ \frac{1}{3},\ \frac{1}{3},\ -\frac{1}{3},\ \frac{2}{3},\ \frac{1}{3} \right\}$$

$$\vec{a}^{(2)} = \left\{ \frac{2}{3},\ -\frac{1}{3},\ \frac{1}{3},\ -\frac{2}{3},\ 0 \right\}$$

Teorema di Tellegen

Tuesday, 16 May 2017 09:46

$$\sum_i v_i i_i = 0$$

$$V_{out} i_{out} + \sum_{i \in Cop} v_{c_i} i_{c_i} - V_{in} i_{in} = 0$$

$$\left[i_{out} = T/2 = a_{out}\, i_{out} \right]$$

$\boxed{\text{Fase 1}}$

$$V_{out}\, a_{out}^{(1)} + \sum_{i \in Cop} V_{c_i}^{(1)} a_{c_i}^{(1)} - V_{in} a_{in}^{(1)} = 0$$

$\boxed{\text{Fase 2}}$

$$V_{out}\, a_{out}^{(2)} + \sum_{i \in Cop} V_{c_i}^{(2)} a_{c_i}^{(2)} - V_{in} a_{in}^{(2)} = 0$$

$\boxed{\text{Sommo}}$

$$V_{out} + \sum_{i \in Comp} \left[V_{c_i}^{(1)} - V_{c_i}^{(2)} \right] a_{c_i} - V_{in} \underbrace{\left[a_{in}^{(1)} + a_{in}^{(2)} \right]}_{= V_{out}^{o}} = 0$$

N.B.

$$a_{out}^{(1)} + a_{out}^{(2)} = 1$$

$$a_{c_i}^{(1)} = -a_{c_i}^{(2)} = a_{c_i}$$

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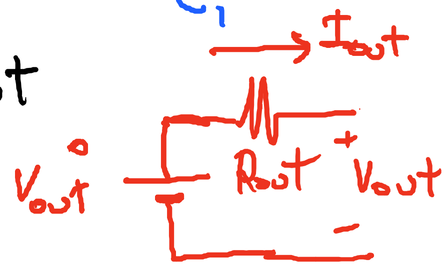
$$\Delta V_{out} = V_{out}^{0} - V_{out} = \sum_{i \in C_{op}} \underbrace{\left[v_{c_i}^{(1)} - v_{c_i}^{(2)} \right]}_{\frac{a_{c_i} q_{out}}{C_i}} a_{c_i} = \sum_{i \in C_{op}} \frac{a_{c_i}^2}{C_i} q_{out}$$

perdita di tensione rispetto al caso a vuoto

$$\Delta V_{out} = \underbrace{\left[\sum_{i \in C_{op}} \frac{a_{c_i}^2}{C_i f_s} \right]}_{R_{out}^{SSL}} \cdot I_{out}$$



$$R_{out}^{SSL} I_{out}^2 \leftarrow$$ Potenza dissipata internamente

FSL

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R_i RESISTENZA SERIE DEL SWITCH i

$\vec{a}$ vettore di moltiplicatore di carica per i switch

ES.

$$\begin{array}{cccccccc} & & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 \\ \vec{a}_s^{(1)} & = & \Big\{ \frac{1}{3} & 0 & \frac{1}{3} & 0 & -\frac{2}{3} & 0 \Big\} \\ \vec{a}_s^{(2)} & = & \Big\{ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & -\frac{2}{3} \Big\} \end{array}$$

switch S_i

$$i_{s,i}^{(1)} = 2 a_{s,i}^{(1)} q_{out} f_s \quad ; \quad i_{s,i}^{(2)} = 2 a_{s,i}^{(2)} q_{out} f_s$$

$$P_{FSL} = \sum_{i \in switch} R_i \, i_{s,i}^2 = \sum_{i \in switch} 4 R_i \, q_{out}^2 f_s^2 \, a_{s,i}^2 = R_{out}^{FSL} I_{out}^2$$

LA FASE NON NULLA

$I_{out} = q_{out} f_s$

$$R_{out}^{FSL} = \sum_{i \in switch} 4 R_i \left[a_{s,i}\right]^2$$

Nel caso intermedio tra i due limiti

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$$R_{out} = \sqrt{{R_{out}^{FSL}}^2 + {R_{out}^{SSL}}^2}$$

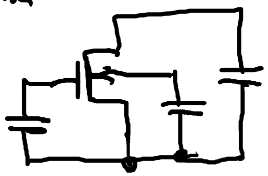
Altre perdite

Tuesday, 16 May 2017 10:28

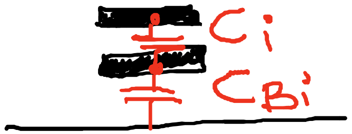
- Switching Loss

$$P_{SW} = f_S \sum_{i \in Switch} \left[C_{G_i} V_{G_i}^2 + C_{D_i} V_{D_i}^2 + C_{B_i} V_{B_i}^2 \right] \qquad \propto f_S, A_S$$

A_S ← AREA COMPLESSIVA OCCUPATA DAI SWITCH



- Bottom-plate parasitic capacitance



$$P_{CAP} = f_S \sum_{i \in Cap} C_{B_i} V_{B_i}^2 \qquad \propto f_S, A_C$$

A_C ← AREA COMPLESSIVA OCCUPATA DALLE CAPACITÀ

- RESISTENZA SERIE EQUIVALENTE

$$P_{ESR} = R_{ESR} I_{out}^2$$

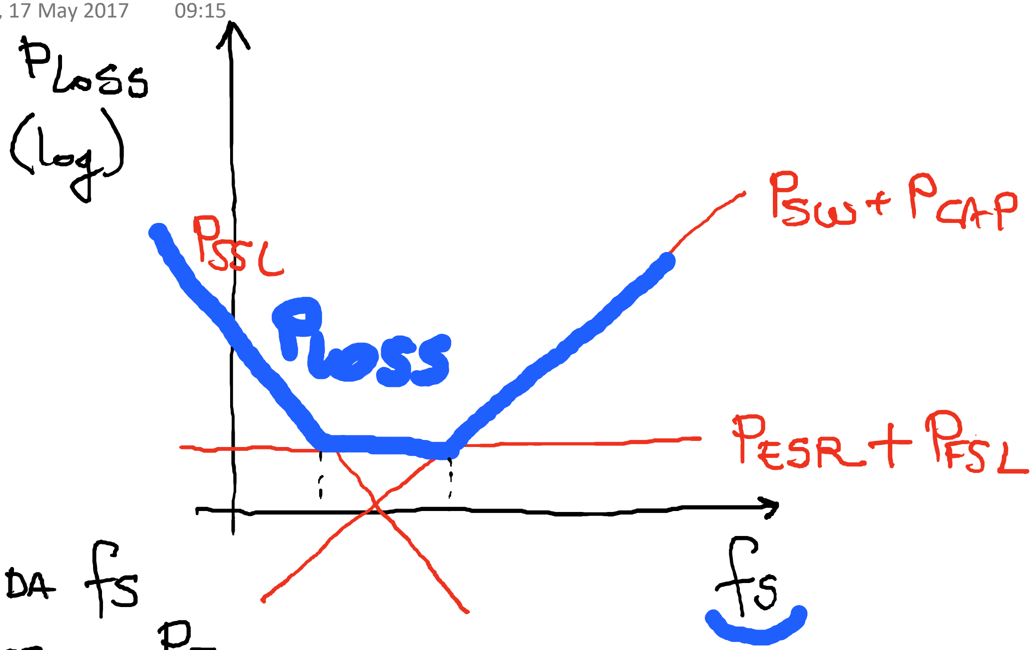
$$P_{LOSS} = R_{out} I_{out}^2 + P_{SW} + P_{CAP} + P_{ESR}$$

$$P_{out} = V_{out} I_{out}$$

$$\eta = \frac{P_{out}}{P_{out} + P_{LOSS}} \rightarrow \underline{\underline{50\% - 90\%}}$$

EFFICIENZA DI CONVERSIONE

Wednesday, 17 May 2017 09:15



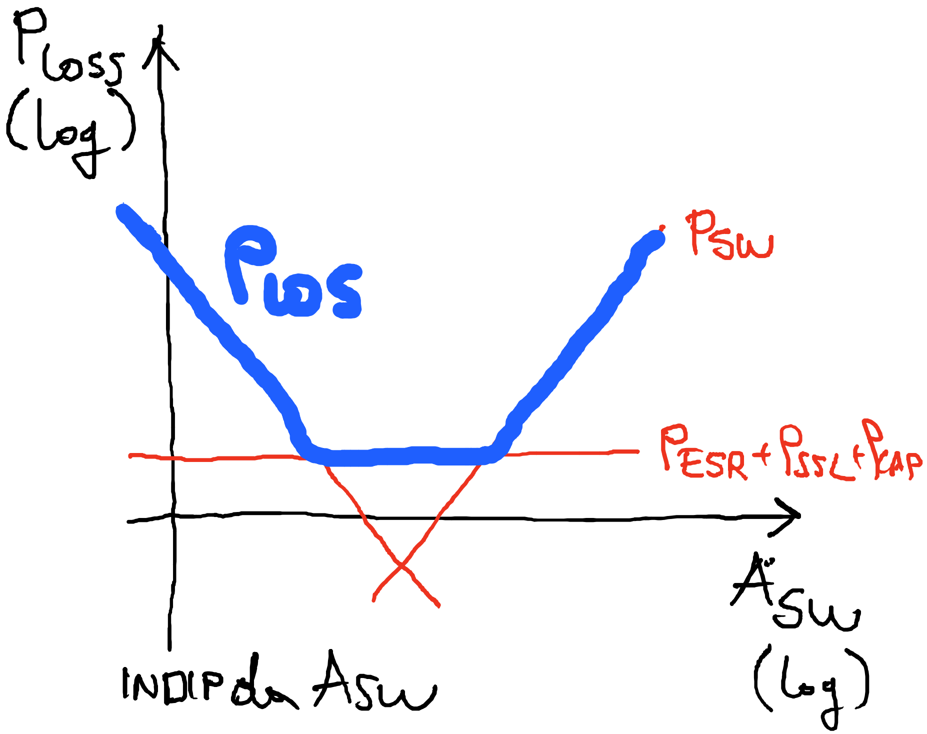
INDIP. DA f_S

P_{ESR} , P_{FSL}

$R_{out} I_{out}^2 = \sqrt{R_{out}^{FSL\,2} + R_{out}^{SSL\,2}} \cdot I_{out}^2 = \sqrt{P_{FSL}^2 + P_{SSL}^2}$

$\propto f_S$ P_{SW}, P_{CAP}

$\propto 1/f_S$ P_{SSL}

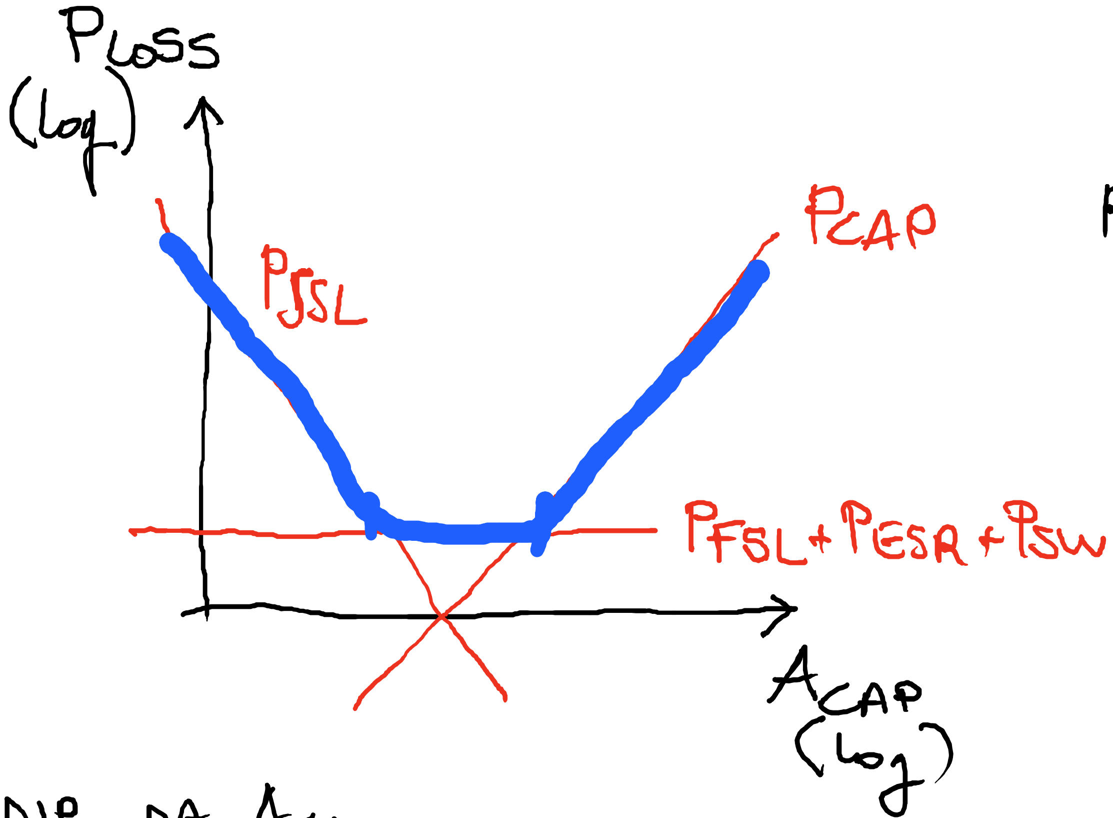


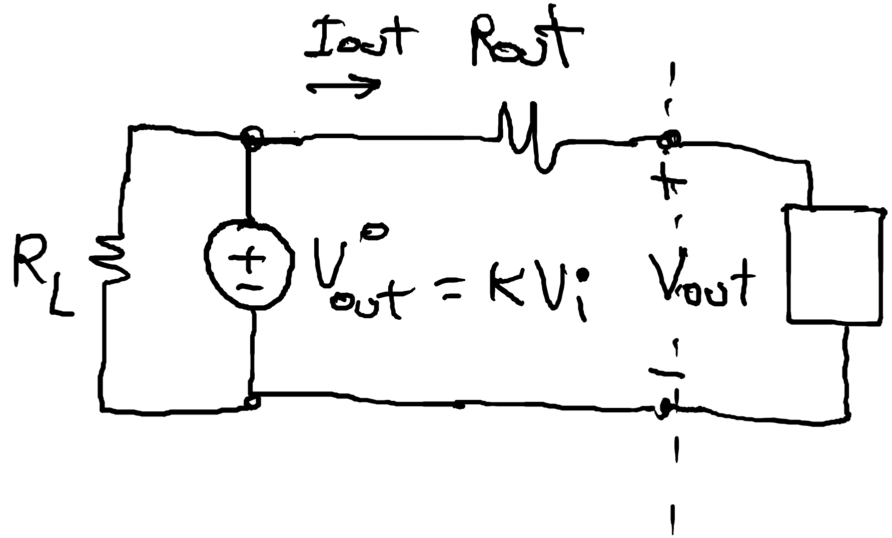
INDIP da A_{SW}

P_{ESR}, P_{SSL}, P_{CAP}

$\propto A_{SW}$ P_{SW}

$\propto \frac{1}{A_{SW}}$ P_{FSL}





INDIP DA A_{CAP}

P_{FSL}, P_{ESR}, P_{SW}

$\propto A_{CAP} \rightarrow P_{CAP}$

$\propto 1/A_{CAP} \rightarrow P_{SSL}$