Filtri

→ Sistemi lineari a 2 porte

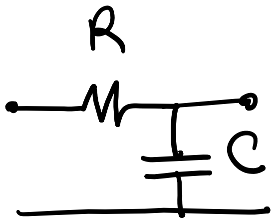
$$X(f) \rightarrow \boxed{H(f)} \rightarrow Y(f) = H(f)X(f)$$

$$H(f) = \frac{N(f)}{D(f)}$$

Filtri

- Filtri passivi : es.

  SOLO COMPONENTI LINEARI PASSIVI



$$H(f) = \frac{1}{1 + j\omega RC}$$

- Filtri attivi

  usano <u>amplificatori</u>

Classificazione dei filtri [ideale]

- Passabasso

$$|H| = \begin{cases} H_0 & \omega \leq \omega_H \\ 0 & \omega > \omega_H \end{cases}$$

- Passo alto

$$|H| = \begin{cases} 0 & \omega < \omega_L \\ H_\infty & \omega \geq \omega_L \end{cases}$$

- Passa banda

$$|H| = \begin{cases} H_{CB} & \omega_L \leq \omega \leq \omega_H \\ 0 & \text{altrimenti} \end{cases}$$

- Elimina banda

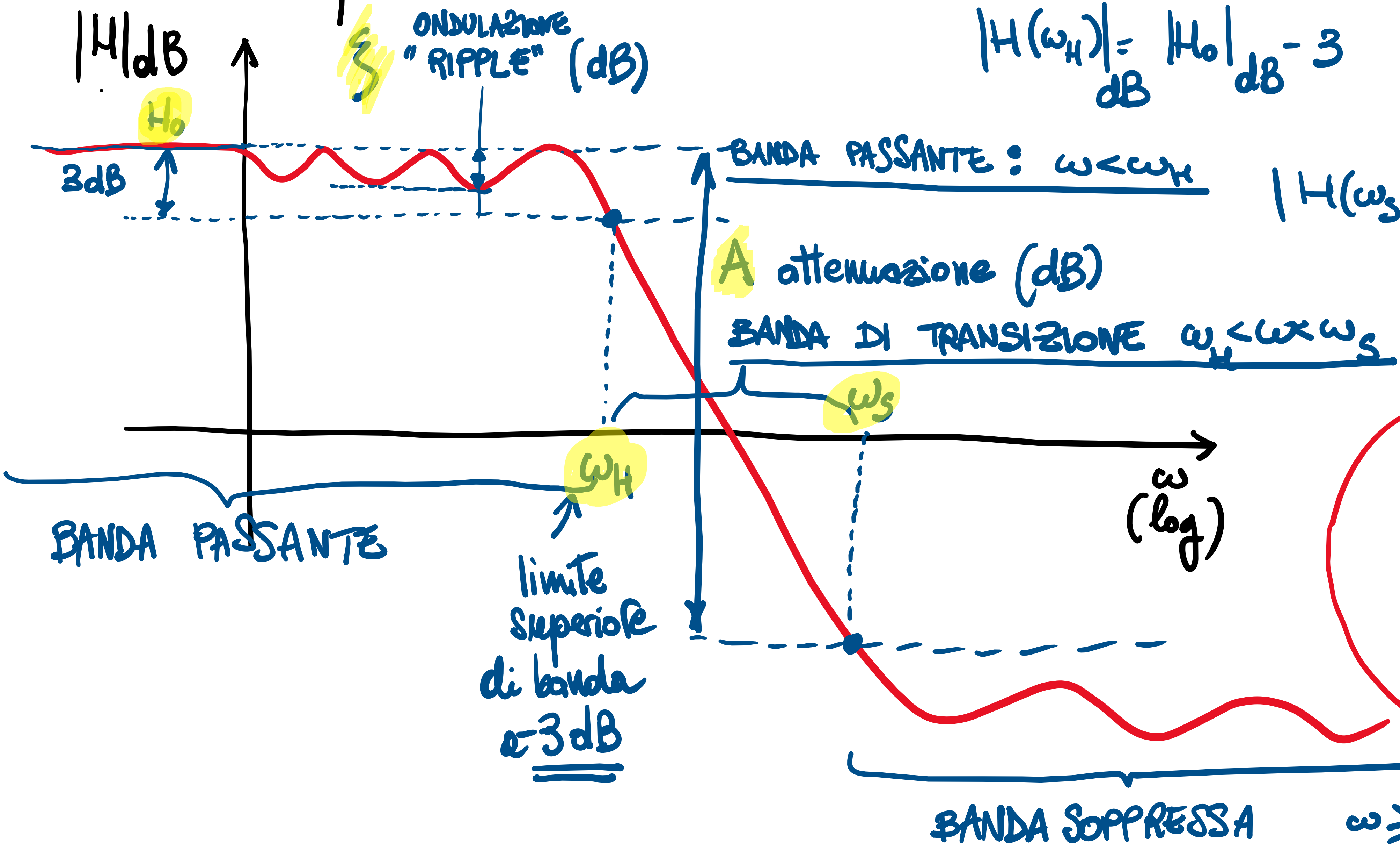
$$|H| = \begin{cases} H_0 & \omega \leq \omega_L \\ 0 & \omega_L < \omega < \omega_H \\ H_0 & \omega \geq \omega_H \end{cases}$$

Filtro passa basso reale

$$H_0 = \lim_{\omega \to 0} H(\omega)$$

$$|H(\omega_H)|_{dB} = |H_0|_{dB} - 3$$

$$|H(\omega_S)|_{dB} = |H_0|_{dB} - A_{dB}$$



IN UN FILTRO IDEALE AVREMMO

$\xi = 0\,dB$

$A = \infty\,dB$

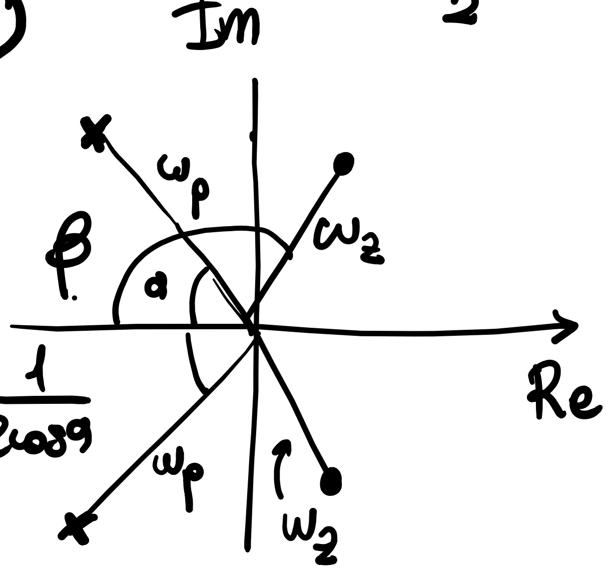
$\omega_S = \omega_H$

Filtri Biquadratici [Biquad]

$$H(s) = \frac{b_2 s^2 + b_1 s + b_0}{a_2 s^2 + a_1 s + a_0} = H_0 \frac{\left(\frac{s}{\omega_z}\right)^2 + \frac{s}{Q_z \omega_z} + 1}{\left(\frac{s}{\omega_p}\right)^2 + \frac{s}{Q_p \omega_p} + 1}$$

ω_p modulo dei poli se i poli sono c.c. (altrimenti ω_p è la media geometrica dei 2 poli)

ES



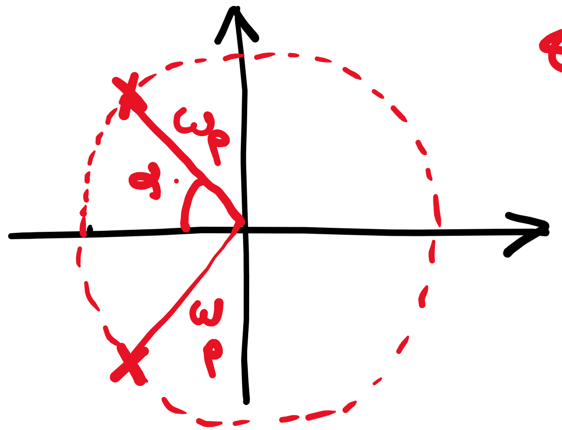
$$Q_p = \frac{1}{2\cos\alpha}$$

$$Q_z = \frac{1}{2\cos\beta}$$

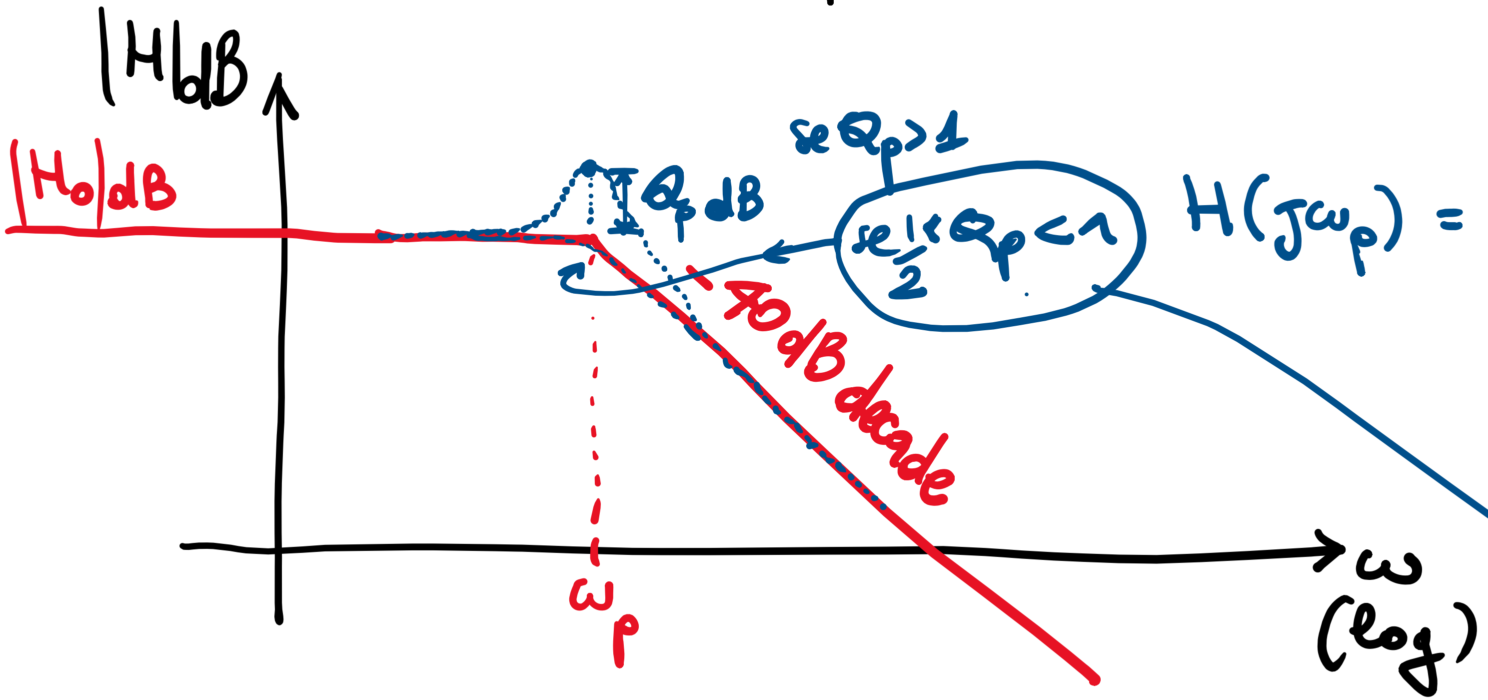
Filtro Biquad Passa basso

i) 2 poli, nessuno zero $\rightarrow$ $b_2=0$, $b_1=0$

$$H = \frac{H_0}{\left(\frac{s}{\omega_p}\right)^2 + \frac{s}{Q_p\omega_p} + 1}$$



$$Q_p = \frac{1}{2\cos\alpha}$$



$$H(j\omega_p) = \frac{H_0}{\cancel{-1} + \frac{j}{Q_p} + \cancel{1}} = -jH_0Q_p$$

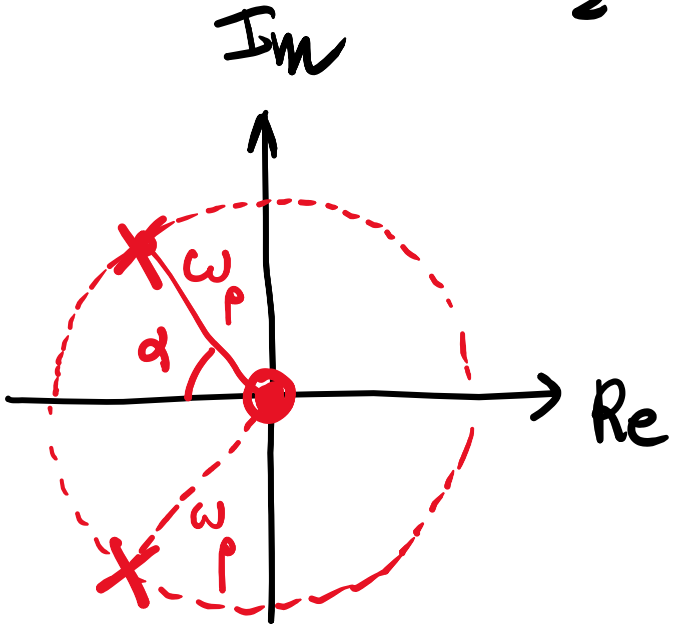
$$\underline{\underline{|H(j\omega_p)|_{dB} = |H_0|_{dB} + |Q_p|_{dB}}}$$

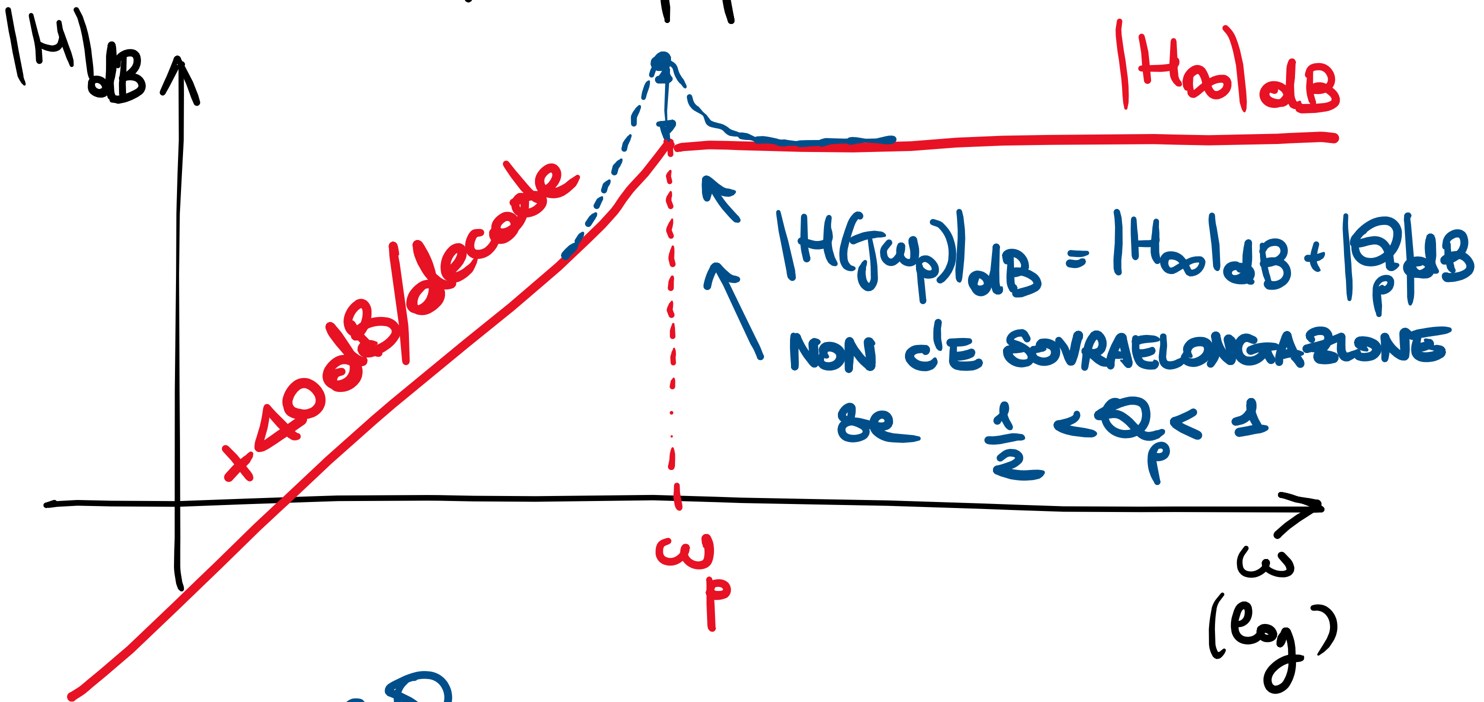
cerchiamo di avere $\underline{\underline{\frac{1}{2} < Q < 1}}$

Filtro Biquadratico Passa alto

· 2) 2 poli, 2 zeri nell'origine $\Rightarrow$ $b_1 = 0,\ b_2 = 0$

$$H = \frac{b_2 s^2}{a_2 s^2 + a_1 s + a_0} = \frac{\left(\frac{s}{\omega_p}\right)^2}{\left(\frac{s}{\omega_p}\right)^2 + \frac{s}{Q_p \omega_p} + 1} H_\infty \qquad \left[b_2 = \frac{H_\infty}{\omega_p^2} \right]$$





$$Q_p = \frac{1}{2\cos\varphi} \rightarrow Q_p < 1$$

$\hookrightarrow \cos\varphi > \frac{1}{2}$ $\quad \underline{\varphi < 60°}$ $\quad Q_p$:

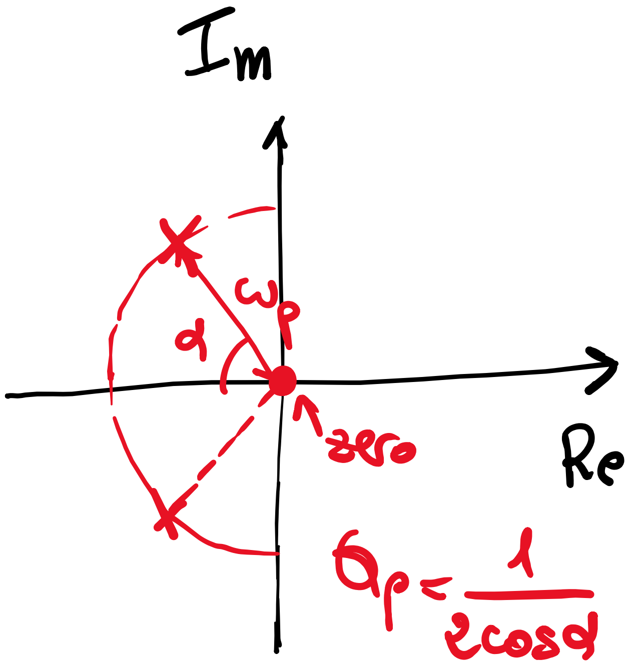
Filtro biquadratico passa banda

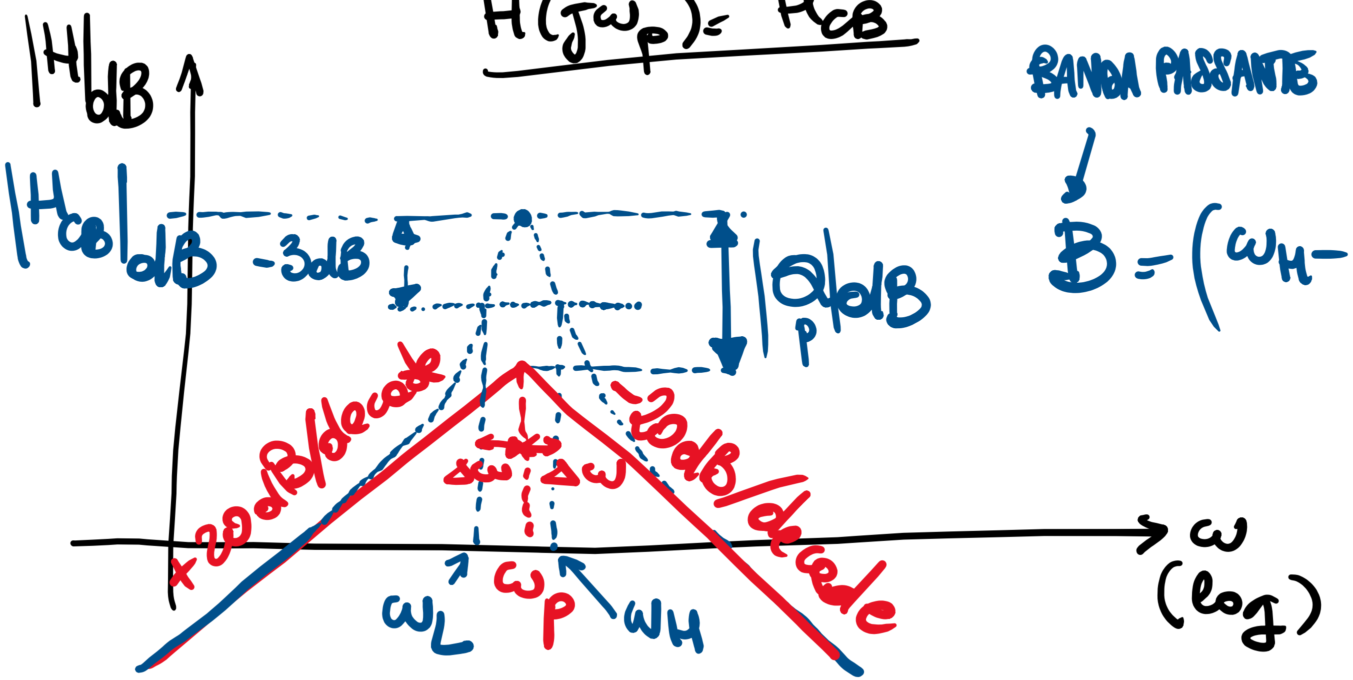
·) 2 poli complessi coniugati + 1 zero nell'origine $\Rightarrow$ $b_2=0, b_0=0$

$$H(s)=\frac{b_1 s}{a_2 s^2+a_1 s+a_0} = \frac{\frac{s}{Q_p\omega_p}}{\left(\frac{s}{\omega_p}\right)^2+\frac{s}{Q_p\omega_p}+1} H_{CB}$$

↳ $b_1=\frac{H_{CB}}{Q_p\omega_p}$

$$H(j\omega_p)=H_{CB}$$





BANDA PASSANTE

$$B=(\omega_H-\omega_L)$$

IPOTESI $\quad Q_p \gg 1$

$$H(j\omega) = \frac{\frac{j\omega}{Q_p\omega_p}}{-\frac{\omega^2}{\omega_p^2} + \frac{j\omega}{Q_p\omega_p} + 1} H_{CB} = \frac{\frac{j}{Q_p}\left(1+\frac{\Delta\omega}{\omega_p}\right) \cdot H_{CB}}{\left[1 - \frac{(\omega_p^2 + 2\omega_p\Delta\omega + \Delta\omega^2)}{\omega_p^2}\right] + \frac{j}{Q_p}\left(1+\frac{\Delta\omega}{\omega_p}\right)}$$

scriviamo $\omega = \omega_p + \Delta\omega$

→ in corrispondenza dei limiti di banda a −3dB $\quad |H(j\omega_H)| = \frac{|H_{CB}|}{\sqrt{2}}$

$$\frac{1}{\sqrt{2}} = \left| \frac{\frac{j}{Q_p}\left(1+\cancel{\frac{\Delta\omega}{\omega_p}}\right)}{\left[-2\frac{\Delta\omega}{\omega_p} - \cancel{\frac{(\Delta\omega)^2}{(\omega_p)^2}}\right] + \frac{j}{Q_p}\left[1+\cancel{\frac{\Delta\omega}{\omega_p}}\right]} \right| = \left| \frac{\frac{j}{Q_p}}{-\frac{2\Delta\omega}{\omega_p} + \frac{j}{Q_p}} \right| = \left| \frac{1}{j\frac{2Q_p\Delta\omega}{\omega_p} + 1} \right|$$

$\underline{\underline{\text{Hp}}} \quad \underline{\underline{\Delta\omega \ll \omega_p}}$ → approx al 1° ordine in $\frac{\Delta\omega}{\omega_p} \ll 1$

$$\boxed{\Delta\omega = \pm\frac{\omega_p}{2Q_p}}$$

$$\omega_H = \omega_p + \Delta\omega \qquad \Delta\omega = \frac{\omega_p}{2Q_p}$$

$$\omega_L = \omega_p - \Delta\omega$$

$$B = \omega_H - \omega_L = 2\Delta\omega = \frac{\omega_p}{Q_p}$$

$$\boxed{B = \frac{\omega_p}{Q_p}}$$

VALE SE $Q_p \gg 1$

FILTRO PASSA-BANDA SELETTIVO

(vuol dire $B \ll \omega_p$)

Filtro Biquadratico Elimina banda

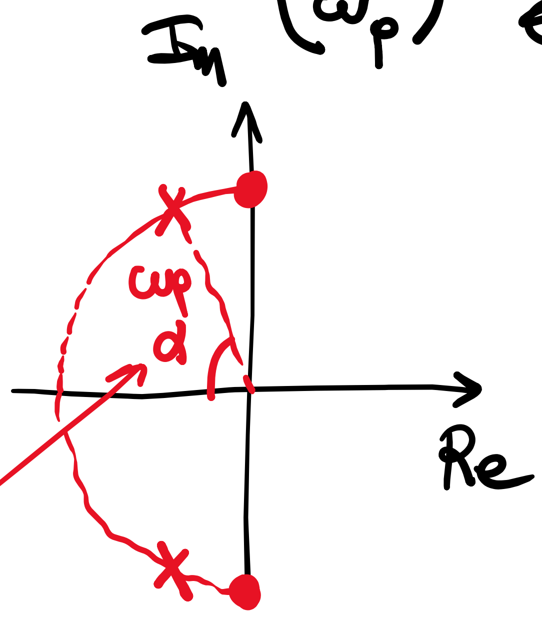
i) 2 poli complessi coniugati, 2 zeri immaginari puri alla stessa pulsazione dei poli $\Rightarrow \underline{b_1 = 0}$

$$H = \frac{b_2 s^2 + b_0}{a_2 s^2 + a_1 s + a_0} = \frac{\left(\frac{s}{\omega_p}\right)^2 + 1}{\left(\frac{s}{\omega_p}\right)^2 + \frac{s}{Q_p \omega_p} + 1} \cdot H_0$$

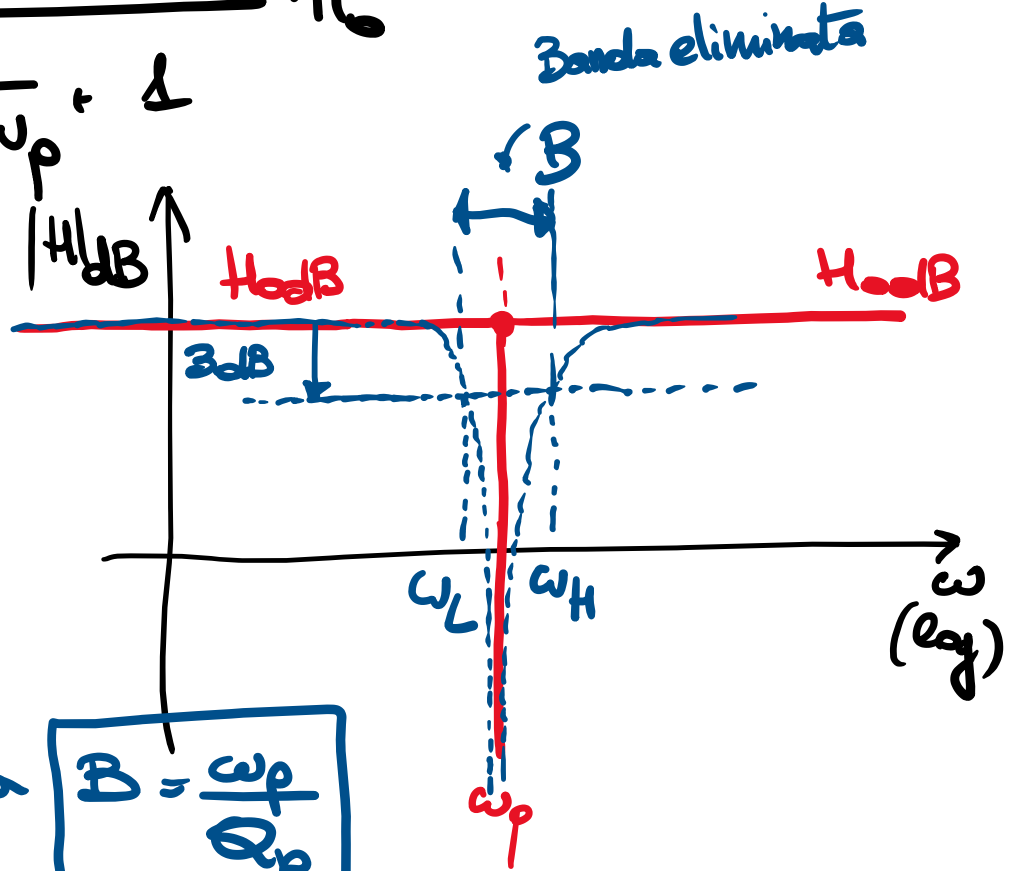
$H(0) = H_0$

$H(j\omega_p) = 0$

$\lim_{\omega \to \infty} H(j\omega) = H_0$

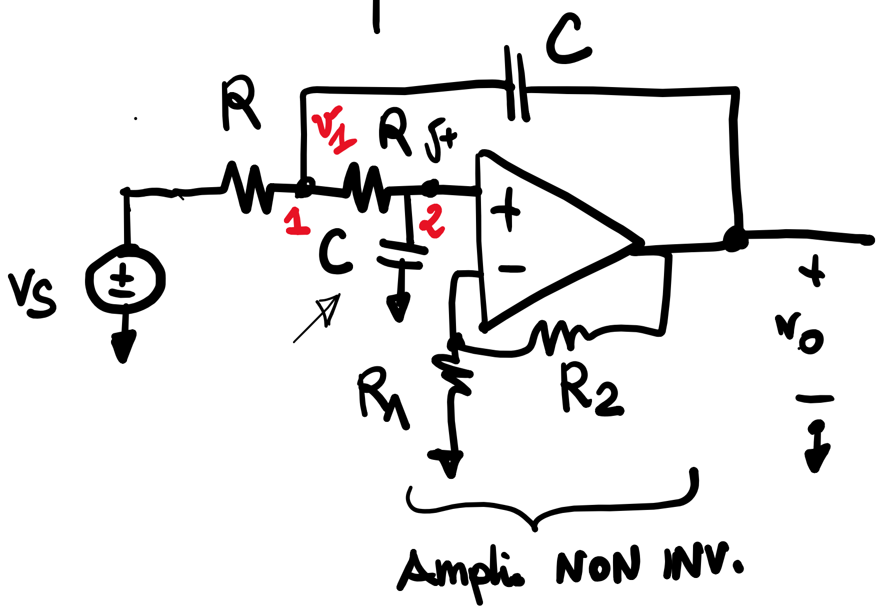


$$Q = \frac{1}{2\cos\alpha}$$



se $Q_p \gg 1$ allora $\boxed{B = \frac{\omega_p}{Q_p}}$

Filtro passabasso di Sallen-Key



$$A = 1 + \frac{R_2}{R_1} \qquad v^+ = \frac{v_o}{A}$$

Eq. ai nodi

1) $$v_1\left[\frac{2}{R} + Cs\right] - v_s\frac{1}{R} - v_o Cs + \overset{V_o/A}{v^+}\frac{1}{R} = 0$$

2) $$\underset{\frac{v_o}{A}}{v^+}\left[\frac{1}{R} + Cs\right] - v_1\frac{1}{R} = 0 \rightarrow v_1 = \frac{v_o}{A}\left(1 + RCs\right)$$

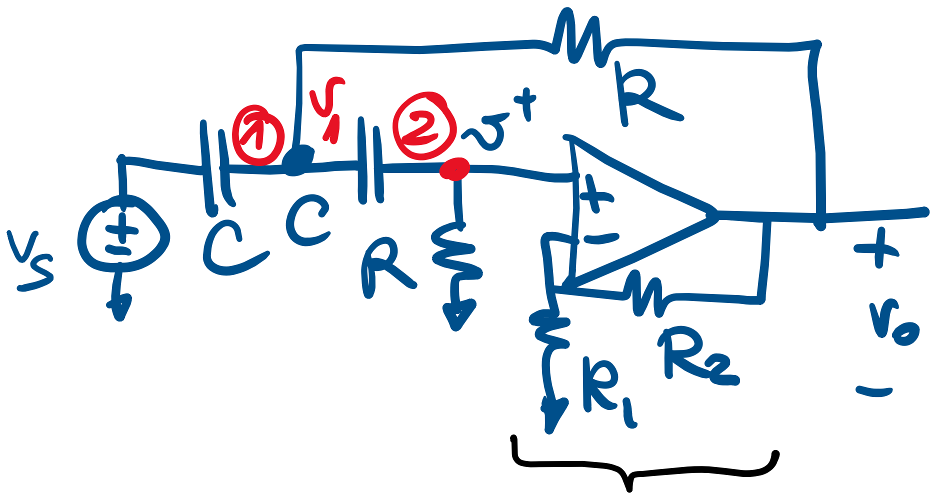
1) $$\frac{v_o}{A}\left[1 + RCs\right]\left[2 + RCs\right] - v_s - v_o RCs + \frac{v_o}{A} = 0$$

$$v_o\left[2 + RCs + 2RCs + R^2C^2s^2 - ARCs + 1\right] = v_s \Rightarrow \frac{v_o}{v_s} = \frac{\overset{\text{nessuno}}{1}}{\underbrace{R^2C^2s^2}_{\left(\frac{1}{\omega_p}\right)^2} + \underbrace{(3-A)RCs}_{\frac{1}{Q_p\omega_p}} + 1}$$

2 poli

$$\boxed{\omega_p = \frac{1}{RC} \qquad Q_p = \frac{1}{3-A}}$$

Filtro Passa alto Sallen-Key



$v_o = A v^+ \rightarrow v^+ = \frac{v_o}{A}$

$A = \left(1+\frac{R_2}{R_1}\right)$

eq. ai nodi

$$① \quad v_1\left[2Cs+\frac{1}{R}\right] - v_s Cs - \overset{\frac{v_o}{A}}{v^+} Cs - \frac{v_o}{R} = 0$$

$$② \quad v^+\left[Cs+\frac{1}{R}\right] - v_1 Cs = 0 \Rightarrow v_1 = \underset{\frac{v_o}{A}}{v^+}\left(\frac{RCs+1}{RCs}\right)$$

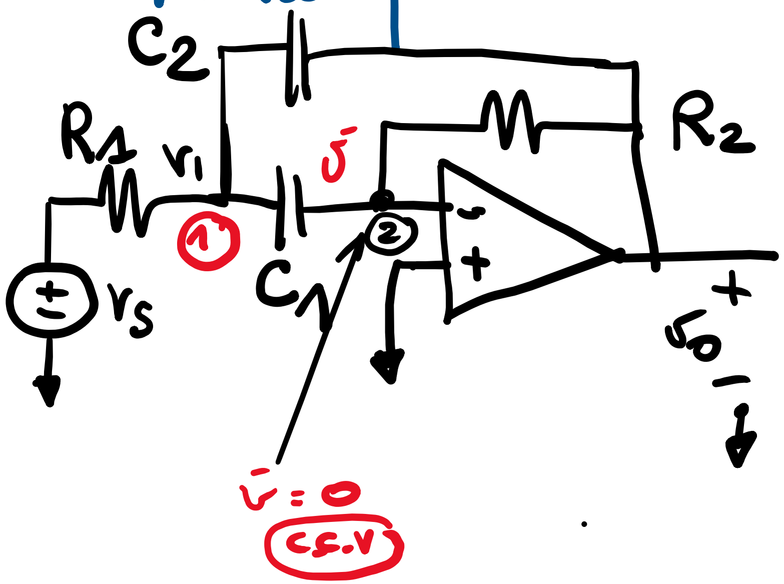
$$① \quad \frac{v_o}{A}\left[\frac{RCs+1}{RCs}\right]\left[2RCs+1\right] - v_s RCs - \frac{v_o}{A}RCs - v_o = 0$$

$$v_o\left[(RCs+1)(2RCs+1) - R^2C^2s^2 - ARCs\right] = v_s R^2C^2s^2 \Rightarrow \frac{v_o}{v_s} = \frac{R^2C^2s^2}{R^2C^2s^2+(3-A)RCs+1}$$

$$\frac{\frac{s^2}{\omega_p^2}}{\frac{s^2}{\omega_p^2} + \frac{s}{Q_p\omega_p} + 1}$$

$$\boxed{\omega_p = \frac{1}{RC} \qquad Q_p = \frac{1}{3-A}}$$

Filtro passabanda di Delyannis



eq. al nodo ①

$$v_1 \cdot \left[\frac{1}{R_1} + C_1 s + C_2 s\right] - \frac{v_s}{R_1} - v_o C_2 s = 0$$

② $$\underset{=0}{v^-}\left[\frac{1}{R_2} + C_1 s\right] - v_1 C_1 s - \frac{v_o}{R_2} = 0 \Rightarrow v_1 = \frac{-v_o}{R_2 C_1 s}$$

① $$\frac{-v_o}{R_2 C_1 s}\left[\frac{1}{R_1} + (C_1 + C_2)s\right] - \frac{v_s}{R_1} - v_o C_2 s = 0$$

$$v_o\left\{\left[1 + R_1(C_1+C_2)s\right] + R_1R_2C_1C_2 s^2\right\} = v_s R_2 C_1 s$$

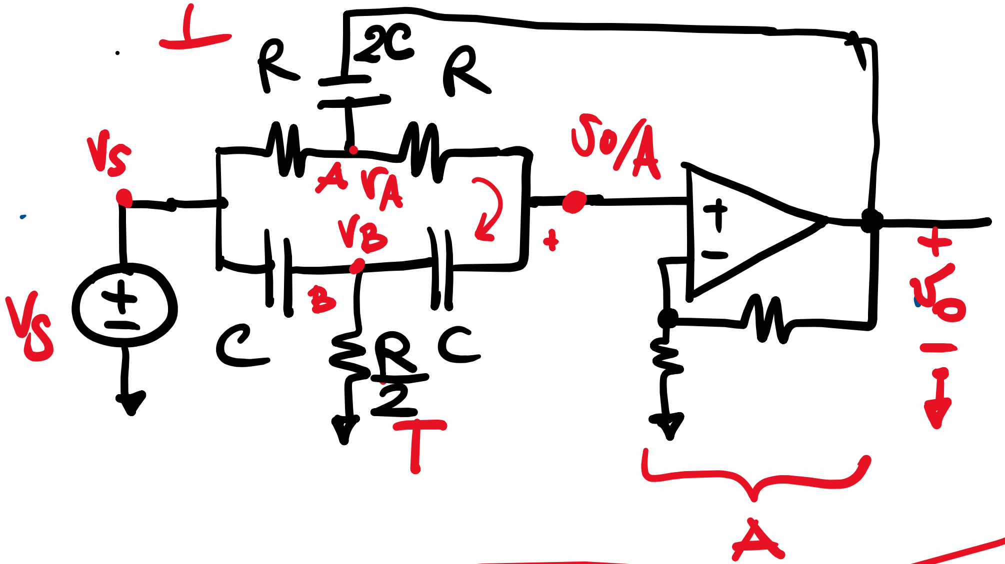
$$\frac{v_o}{v_s} = \frac{R_2 C_1 s}{\underbrace{(R_1R_2C_2C_1)}_{\frac{1}{\omega_p^2}} s^2 + \underbrace{R_1(C_1+C_2)}_{\frac{1}{Q_p\omega_p}} s + 1}$$

$$\omega_p = \frac{1}{\sqrt{R_1R_2C_1C_2}}$$

$$Q_p = \frac{1}{\omega_p R_1(C_1+C_2)} = \frac{\sqrt{R_1R_2C_1C_2}}{R_1(C_1+C_2)} = \sqrt{\frac{R_2}{R_1}} \cdot \frac{\sqrt{C_1C_2}}{(C_1+C_2)}$$

Q_p è sempre > 0

Filtro eliminabanda a doppio T



Eq ai nodi

(A) $v_A\left[\frac{2}{R}+2Cs\right]-\frac{v_s}{R}-\frac{v_o}{A}\frac{1}{R}-2Csv_o=0$

(B) $v_B\left[\frac{2}{R}+2Cs\right]-v_sCs-\frac{v_o}{A}Cs=0$

(+) $\frac{v_o}{A}\left[\frac{1}{R}+Cs\right]-v_A\frac{1}{R}-v_BCs=0$

(+) $\frac{v_o}{A}\left[1+RCs\right]-v_A-v_BRCs=0$

$$\frac{v_o}{A}\left[1+RCs\right]\left[\frac{2}{R}+2Cs\right]-v_A\left[\frac{2}{R}+2Cs\right]-v_B\left[\frac{2}{R}+2Cs\right]RCs=0$$

$$v_o\left[1+RCs\right]\left[2+2RCs\right]-v_sA-v_o-2ARCsv_o-v_sAR^2C^2s^2-v_oR^2C^2s^2=0$$

$$v_o\left[(1+RCs)(2+2RCs)-1-2ARCs-R^2C^2s^2\right]=v_s\left(A+AR^2C^2s^2\right)$$

$$\frac{v_o}{v_s}=\frac{\overbrace{A}^{H_0}\left(1+\overbrace{R^2C^2}^{\frac{1}{\omega_p^2}}s^2\right)}{\underbrace{R^2C^2}_{\left(\frac{1}{\omega_p}\right)^2}s^2+\underbrace{(4-2A)}_{\frac{1}{Q_p}}\underbrace{RC}_{\frac{1}{\omega_p}}s+1}=\frac{H_0\left(1+\frac{s^2}{\omega_p^2}\right)}{\frac{s^2}{\omega_p^2}+\frac{s}{Q_p\omega_p}+1}$$

$$\omega_p=\frac{1}{RC}$$

$$Q_p=\frac{1}{4-2A}$$

$$H_0=A$$

$$(A<2)$$

Filtri di ordine superiore al secondo

Processo di progettazione di un filtro

- SPECIFICHE (insieme dei requisito)

↓

- TIPO DI FILTRO [Categoria di polinomio per il denominatore]

  Es. polinomio di Butterworth ←

  polinomio di Chebyshev

  ...

↓

- ORDINE DEL POLINOMIO DEL DENOMINATORE

↓

- POLI E ZERI DEL FILTRO

↓

- CIRCUITO

Filtri di Butterworth

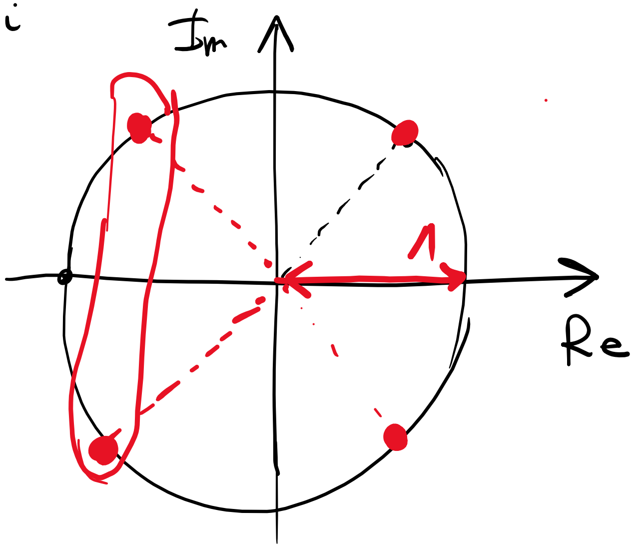
(il denominatore è un polinomio di Butterworth)

POLINOMI DI BUTTERWORTH

- se il grado n è PARI → le radici del Polinomio sono le radici 2n-esime a parte reale negativa di -1
- se il grado n è DISPARI → le radici del polinomio sono le radici 2n-esime a parte reale negativa 1

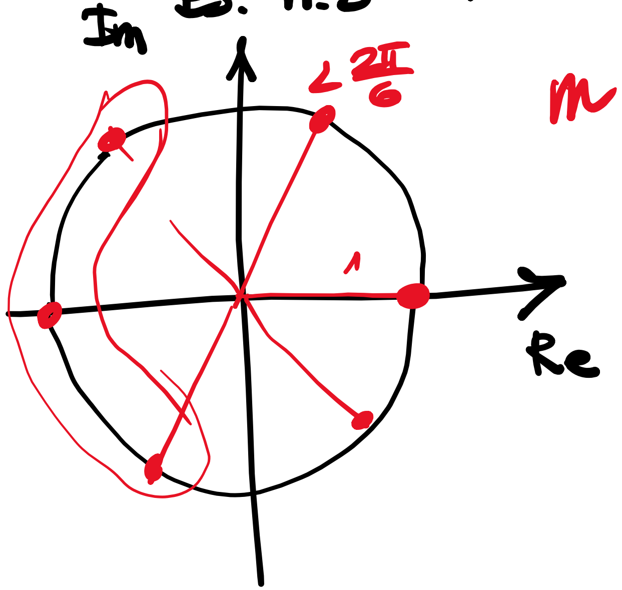
ES $n=2$ → radici quarte di -1 $\quad e^{j\frac{\pi}{4}} + i\, e^{j\frac{\pi}{2}} \; (i=0,3)$

n pari



Es. $n=3$ → radici seste di 1

$m\, e^{j\frac{\pi}{3}} \; (m=0 \ldots 5)$



Polinomi di Butterworth di ordine n

$B^n(s)$:

$s \in \mathbb{C}$

n pari $\quad |B^n(s)|^2 = 1 + s^{2n} \quad$ [è zero se $s^{2n} = -1$]

n dispari $\quad |B^n(s)|^2 = 1 - s^{2n} \quad$ [è zero se $s^{2n} = 1$]

se sostituisco $s = j\omega$ $\quad$ n pari $\quad 1 + j^{2n}\omega^{2n} = 1 + \omega^{2n}$

$s =$ $\quad$ n dispari $\quad 1 - j^{2n}\omega^{2n} = 1 + \omega^{2n}$

$$\boxed{|B^n(j\omega)|^2 = 1 + \omega^{2n}}$$

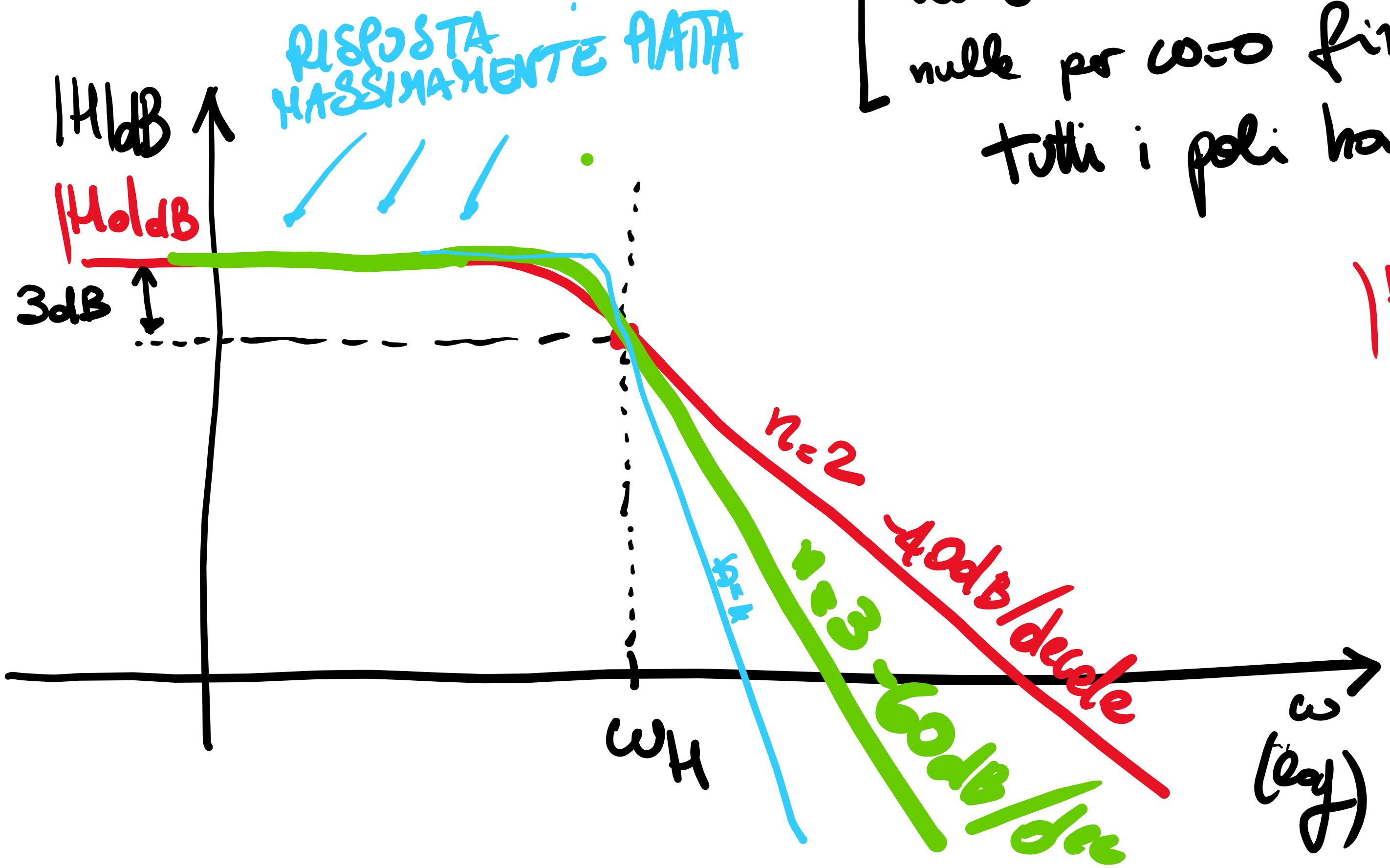
Proprietà più importante del polinomio di B_

Filtro Passabasso di Butterworth di ordine n

$$H(s) = \frac{H_0}{B^n\left(\frac{s}{\omega_H}\right)} \rightarrow |H(j\omega)|^2 = \frac{H_0^2}{\left|B^n\left(j\frac{\omega}{\omega_H}\right)\right|^2} = \frac{H_0^2}{1+\left(\frac{\omega}{\omega_H}\right)^{2n}}$$

[tutte le derivate rispetto a ω di $|H|^2$ sono nulle per $\omega=0$ fino all'ordine $2n-1$]

tutti i poli hanno modulo $\underline{\underline{\omega_H}}$



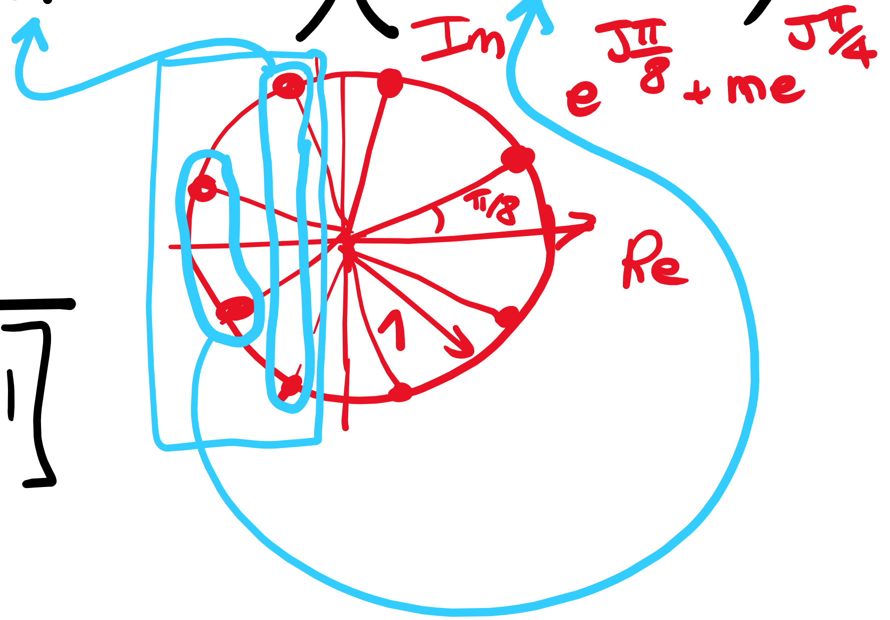
$$|H(j\omega_H)| = \frac{H_0}{\sqrt{2}}$$

$$|H(j\omega_H)|_{dB} = H_0 - 3dB$$

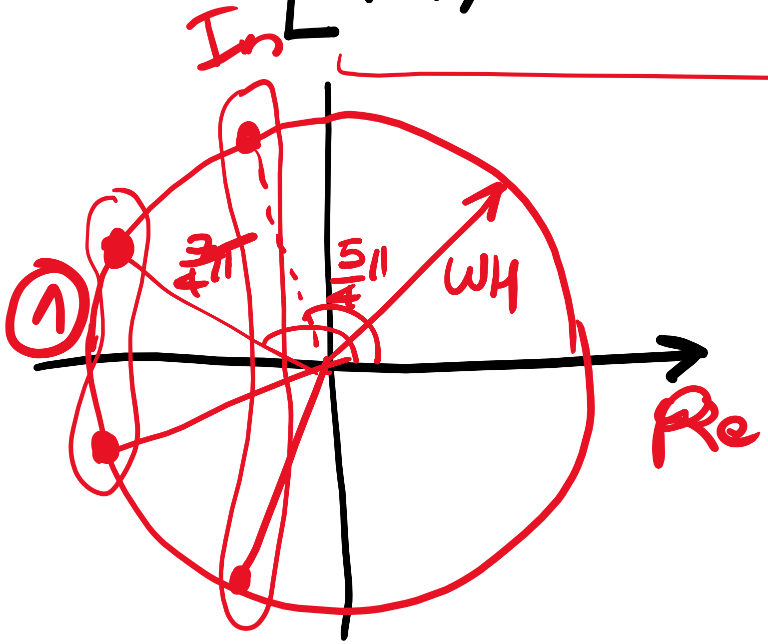
ω_H è il limite superiore di banda a -3 dB (per qualunque n)

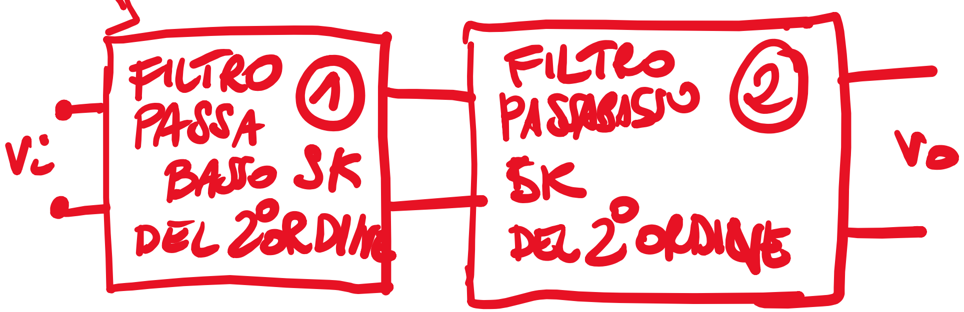
Es filtro di Butterworth del 4 ordine

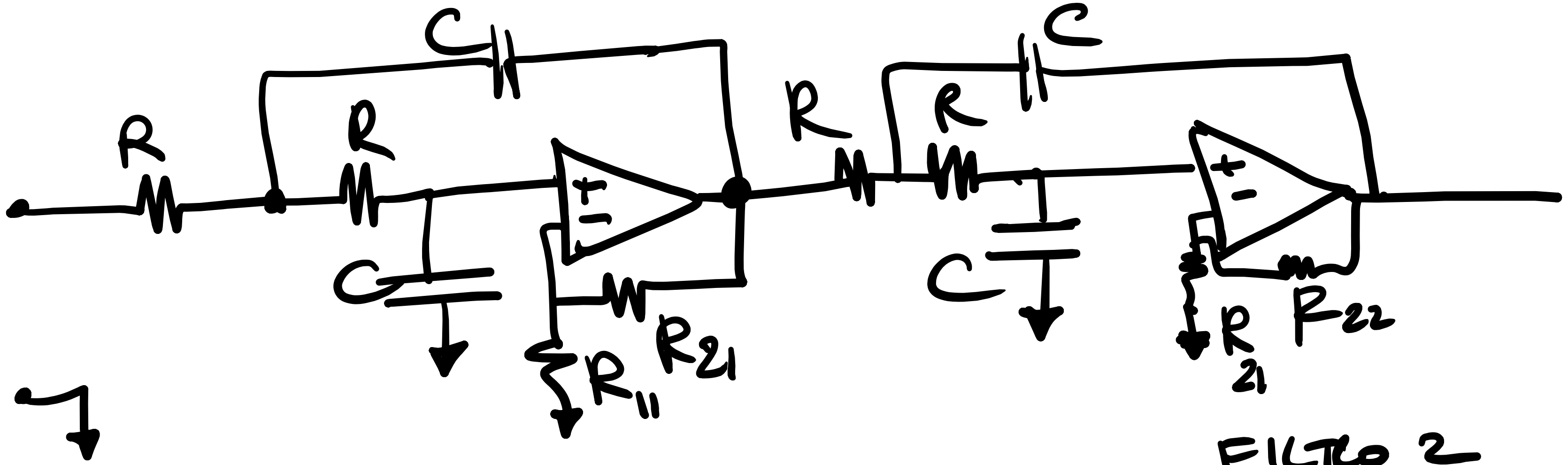
$$H(s) = \frac{H_0}{B^4\left(\frac{s}{\omega_H}\right)} \qquad B^4(x) = \left(x^2 + 0.7654\,x + 1\right)\left(x^2 + 1.8478\,x + 1\right)$$



$$H(s) = \frac{H_0}{\left[\left(\frac{s}{\omega_H}\right)^2 + 0.7654\left(\frac{s}{\omega_H}\right) + 1\right]\left[\left(\frac{s}{\omega_H}\right)^2 + 1.8478\left(\frac{s}{\omega_H}\right) + 1\right]}$$







FILTRO 1

$$\omega_{p_1} = \omega_H = \frac{1}{RC} \Rightarrow RC = \frac{1}{\omega_H}$$

$$Q_{p_1} = \frac{1}{0.7654} = \frac{1}{3-A_1}$$

$$A_1 = 3 - 0.7654 = 2.3346$$

$$= 1 + \frac{R_{21}}{R_{11}} \rightarrow R_{11} = 10k\Omega$$

$$R_{21} = 13{,}346\,k\Omega$$

FILTRO 2

$$\omega_{p_2} = \omega_H = \frac{1}{RC}$$

$$Q_{p_2} = \frac{1}{1.8478} = \frac{1}{3-A_2}$$

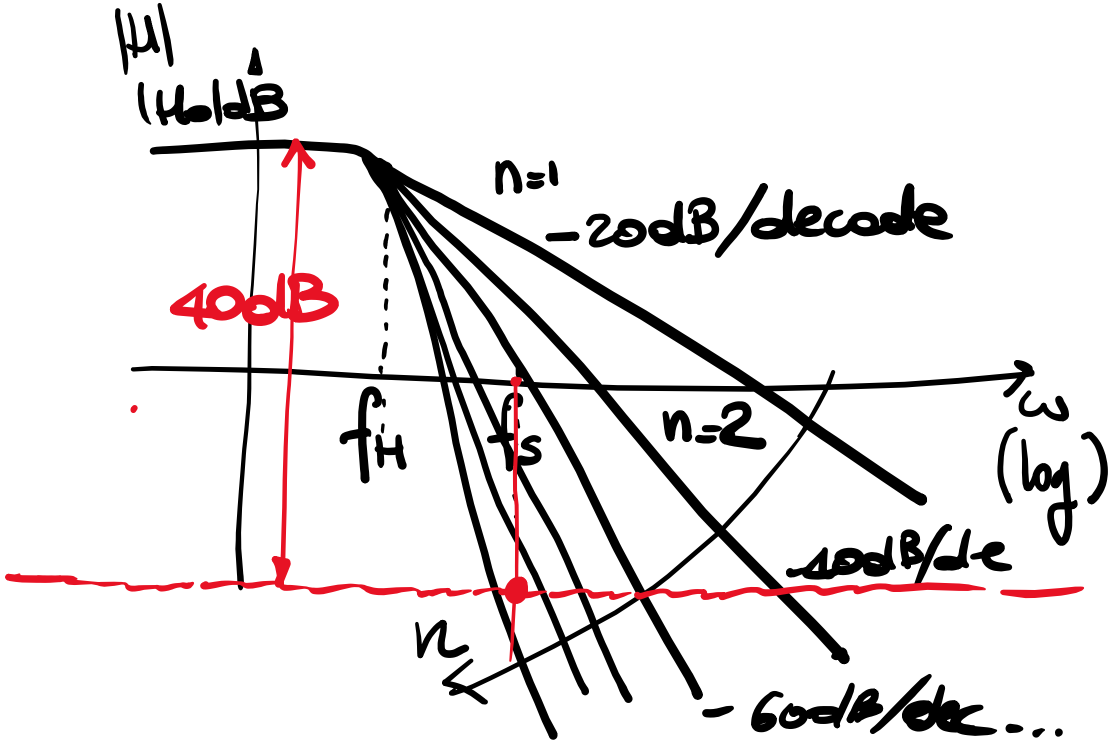
$$A_2 = 3 - 1.8478 = 1.1522$$

$$= 1 + \frac{R_{22}}{R_{21}} \rightarrow R_{21} = 10k\Omega$$

$$R_{22} = 1{,}522\,k\Omega$$

ES. SPECIFICHE

- Filtro passabasso con lim. sup. di banda a $-3dB$ $\quad f_H = 20KHz$
- RIPPLE NULLO
- limite della banda bloccata a $f_s = 40 KHz$
- attenuazione della banda bloccata > 100 (40 dB)



Filtro di Butterworth

$$|H(\omega)|^2 = \frac{|H_0|^2}{1+\left(\frac{\omega}{\omega_H}\right)^{2n}}$$

$$|H(\omega_s)|^2 = \frac{\cancel{|H_0|^2}}{1+\left(\frac{\omega_s}{\omega_H}\right)^{2n}} < \frac{\cancel{|H_0|^2}}{100^2}$$

$$1+\left(\frac{\omega s}{\omega H}\right)^{2n} > 10000$$

$$\downarrow$$

$$1+2^{2n} > 10000$$

$$2^{2n} > 9999$$

$$2n > \log_2(9999) \rightarrow n > \frac{1}{2}\log_2(9999)$$

$$n > 6.67$$

$$\boxed{n=7}$$