Modeling of nanoscale devices with carriers obeying a three-dimensional density of states

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While aggressively nanoscale field-effect transistors commonly used in CMOS technology exhibit strong quantum confinement of charge carriers in one or two dimensions, few devices have been recently proposed whose operation reminds that of vacuum tube triodes and bipolar transistors, since charge carriers are ballistically injected into a three-dimensional \( k \)-space. In this work, we derive, under the parabolic band approximation, the analytical expressions of the first three directed ballistic moments of the Boltzmann transport equation (current density, carrier density, and average kinetic energy), suitable to describe ballistic and quasi-ballistic transport in such devices. The proposed equations are applied, as an example, to describe the ballistic transport in graphene-based variable-barrier transistors.

I. INTRODUCTION

Charge transport modeling is of fundamental importance to study and predict the behavior of transistors, the basic building blocks of electronic systems. The first amplifying electron device has been the vacuum tube triode, a three-terminal device where electrons are thermionically emitted in the vacuum from a heated electrode, the cathode, while the number of electrons collected by the anode terminal per unit time is modulated by the voltage applied to the grid terminal. Charge transport occurs in the vacuum where carriers form a three-dimensional (3D) electron gas. The vacuum tube technology has dominated electronics since the beginning of the twentieth century until taken over by transistor technology, i.e., solid-state electronics, in the sixties. In bipolar transistors, transport from emitter to collector still occurs in a three-dimensional electron gas. On the other hand, in nanoscale field-effect transistors, carriers are subject to strong quantum confinement in the channel, typically along the vertical direction, but more recently along the two transversal directions, with respect to current flow, in the so-called nanowire transistors.

Recently, nanoscale solid state devices inspired to the vacuum tube triode have been presented, where charge carriers are free to move in all three directions forming a 3D electron gas. As a difference with respect to vacuum tubes or bipolar transistors, in these recent devices carrier density can be so high with respect to the density of states of the materials considered that the free electron gas becomes degenerate and therefore must be described by Fermi-Dirac statistics. Let us stress the fact that there is no contradiction between the nanometer scale, ballistic transport, and the absence of quantum confinement. Indeed, when one talks about ballistic transistors, one typically refers to transistors with nanometer size in the longitudinal direction, so that the channel length is shorter than the mean free path and the transport is ballistic or quasi-ballistic. Along the transport direction the wave function is typically non localized—when the device is on—and can typically be described by a propagating wave. Quantum confinement in transistors typically occurs in the transverse direction or directions, typically imposed by the high electric field at the silicon/oxide interface (as in conventional MOSFETs) and/or by the reduced device dimension in the transverse direction (as in heterostructures or in nanowire transistors). In this paper we will discuss transistors with nanometer scale channel lengths where lateral quantum confinement is negligible, and therefore electrons can be described by a three-dimensional density of states.

In Sec. II we present the equations describing charge transport in such devices, which are indeed very similar to those describing the original thermionic process involved in vacuum tube triodes. The main differences with respect to vacuum tube triodes are the fact that carriers move in a solid state region with a given band-structure and that the electron gas is degenerate at the emitter and therefore obeys to Fermi-Dirac statistics. From another point of view our equations can be used when studying charge transport with device simulation tools based on a three dimensional \( k \)-space, in particular Monte Carlo methods.

II. BALLISTIC DIRECTED MOMENTS FOR 3D CARRIERS

In this section we develop the equations for the first three directed moments (carrier density, current density, and average kinetic energy) of the carrier distribution in the case of ballistic transport for 3D carriers. These equations can be used to describe the ballistic transport of 3D carriers in...
homogenous semiconductor regions, e.g., in the channel of MOSFETs. In conventional nano-scale MOSFETs carriers are free to move only in two dimensions (in a plane parallel to the silicon-oxide interface) while they are confined in the third direction forming a two-dimensional (2D) electron gas. Directed moments of the carrier distribution (carrier and current density) have been extensively used as basic building blocks for 2D ballistic\textsuperscript{5,6} ad quasi-ballistic\textsuperscript{7-14} charge transport models. Two-dimensional carriers occupy discrete energy levels known as subbands. Making the assumption that only the lowest sub-band is occupied (energy levels for the considered sub-band, \( \varepsilon_j \)) and limiting the analysis to one-dimensional transport (\( x \) direction), the oppositely directed ballistic current density \( J^\pm(x) \) and carrier density \( n^\pm(x) \) can be calculated, at the top of the energy barrier in the subband \( x = x_{\text{max}} \), by the Natori equations\textsuperscript{5}

\[
J^+(x_{\text{max}}) = \frac{N_{2D}}{2} \frac{2kT}{\hbar^2} \frac{\varepsilon_{FS} - \varepsilon_j(x_{\text{max}})}{\hbar^2} \frac{\hbar^2}{m_j^*} \frac{1}{kT} N_{2D} = kT g_{2D},
\]

\[
n^+(x_{\text{max}}) = \frac{N_{2D}}{2} \frac{3}{2} \left( \frac{\varepsilon_{FS} - \varepsilon_j(x_{\text{max}})}{kT} \right),
\]

where \( q \) is the electronic charge, \( k \) the Boltzmann constant, \( T \) the absolute temperature, \( g_{2D} \) the two dimensional density of states for the considered sub-band, \( m^*_j \) the effective conduction mass for the considered sub-band, \( \varepsilon_j \) the Fermi-Dirac integral of order \( j \), and \( E_{FS} \) the source quasi Fermi level. When quantum confinement in the transversal direction (with respect to transport) is negligible, carriers are better described by a 3D gas. In order to evaluate the upper performance limit of such devices, we can assume fully ballistic transport and use a model similar to that proposed by Natori for 2D carriers (Eq. (1)). In order to calculate the ballistic moments for 3D carriers in a homogeneous semiconductor, we make two approximations similar to the one in the Natori model: one-dimensional analysis (transport along the direction \( x \)) and parabolic bands with effective mass approximation in the channel.

![Energy band diagram](image)

**FIG. 1.** Energy band diagram along the transport direction \( x \) in a device with two injecting contacts (source and drain). In the case of ballistic transport, carriers injected by contacts can only be backscattered by the energy barrier between the contact and the channel. At the top of the barrier (\( x = x_{\text{max}} \)), positive directed carriers are all injected by the source, while negative directed carriers are all injected by the drain.

### A. Carrier density

In this subsection we develop the equation for the carrier density of directed carriers in a homogeneous semiconductor. Because we are dealing with ballistic transport, the Fermi-Dirac distribution is still a solution of the Boltzmann transport equation (BTE) so that the final result (Eq. (6)) coincides with the well known expression for the equilibrium case. However, we proceed in the calculation of the carrier density because the same procedure is later reused for the calculation of current density and average kinetic energy.

To start, let us consider a carrier with wave vector \( \mathbf{k} = (k_x, k_y, k_z) \) and total energy \( E_T \) equal to

\[
E_T = E_C + E = E_C + \frac{\hbar^2 k_x^2}{2m_x} + \frac{\hbar^2 k_y^2}{2m_y} + \frac{\hbar^2 k_z^2}{2m_z},
\]

where \( E_C \) is the bottom energy of the conduction band (indicated above as \( E_j \) in Fig. 1), \( E \) is the kinetic energy, \( m^*_j \) is the effective mass in the channel along direction \( j \), and \( \hbar \) is the reduced Plank constant. Because scattering is neglected, we know that at the top of the barrier carriers with \( k_x > 0 \) are injected from the source and carriers with \( k_x < 0 \) are injected from the drain. The concentration of carriers, at \( x = x_{\text{max}} \), in the positive directed flux is

\[
n^+ = \sum_{\text{valley}} \frac{2}{(2\pi)^3} \int_0^\infty dk_z \int_{-\infty}^{+\infty} dk_y \int_0^{+\infty} dk_x f(E_T, E_{FS}).
\]

Now, changing the integration variables from \((k_x, k_y, k_z)\) to \((\rho, \phi, \theta)\) in ellipsoidal coordinates

\[
k_x = \rho \sin \phi \cos \theta; \quad k_y = \rho \sin \phi \sin \theta; \quad k_z = \rho \cos \phi,
\]

\[
\rho \in [0, \infty], \theta \in [-\pi/2, +\pi/2], \phi \in [0, \pi],
\]

the positive directed carrier density at \( x_{\text{max}} \) can be calculated as

\[
n^+ = \sum_{\text{valley}} \frac{2}{(2\pi)^3} \int_0^{\pi/2} \frac{\sin \phi d\phi}{\sqrt{m_xm_ym_z}} \int_{-\pi/2}^{\pi/2} d\theta \int_0^{\infty} \sqrt{m_xm_ym_z} \rho^2 d\rho
\]

\[
\times \left[ 1 + \exp \left( \frac{E_C(x_{\text{max}}) + \frac{\hbar^2 \rho^2}{2kT} - E_{FS}}{kT} \right) \right]^{-1},
\]

from which one obtains

\[
n^+ = \frac{N_{3D}}{2} \frac{3}{2} \left( \frac{\varepsilon_{FS} - E_C(x_{\text{max}})}{kT} \right),
\]

with

\[
N_{3D} = \sum_{\text{valley}} \frac{2}{(2\pi)^3} \int_0^{\infty} \frac{\rho^2 d\rho}{\sqrt{m_xm_ym_z}} \frac{kT}{\hbar^2}, \quad \eta_x = \frac{E_{FS} - E_C(x_{\text{max}})}{kT},
\]

where \( N_{3D} \) is the three dimensional effective density of states and \( m_{XYZ} = (m_xm_ym_z)^{1/3} \). Silicon has six equivalent anisotropic valley minima and the band curvature along a direction...
where $g$ is the number of valleys (6 for silicon) and $m_{\text{DOS}} = g^{3/2}m_{\text{XYZ}}$ is the density-of-states effective mass which is $1.08m_0$ for silicon.

In the case of MOSFETs, similar results can be obtained for carriers injected from the drain, so that the total charge density in the case of ballistic transport at $x_{\text{max}}$ is

$$n = n^+ + n^- = N_{3D} \frac{1}{2} \Theta_{3/2}(\eta_S) + N_{3D} \frac{1}{2} \Theta_{1/2}(\eta_D),$$

$$\eta_D = \frac{E_{\text{FD}} - E_c(x_{\text{max}})}{kT},$$

where $E_{\text{FD}}$ is the drain quasi Fermi level. Particular cases are: (i) in saturation ($V_{DS} \gg kT/q$) carriers injected by the drain are reflected back by the channel-drain energy barrier so that $n \approx n^+$; (ii) in equilibrium ($V_{DS} = 0$) $\eta_S = \eta_D$ so that $n = 2n^+ = N_{3D} \Theta_{3/2}(\eta_S)$.

### B. Current density

The result for the calculation of the positive directed current density in a homogeneous semiconductor is obtained following a similar procedure

$$J^+ = \sum_{\text{valley}} J^+_v$$

$$= q \sum_{\text{valley}} \int d \mathbf{k} \int d \mathbf{y} \int_0 \mathbf{k}_X f(E_T, E_F) v_X(E),$$

where $J^+_v$ is the current density contribution due to a single valley and

$$v_X = \frac{\partial E}{\partial k_X} = \frac{\hbar}{m_X} k_X,$$

is the carrier velocity along the transport direction $x$.

The value of $v_X$ depends on the valley considered and the expression inside the integral of Eq. (10) is not symmetrical with respect to $x, y,$ and $z,$ like in the case of carrier concentration (3), so that the sum over the valleys cannot be replaced by a simple multiplication constant $g$. By changing the integration variables like in Eq. (4), the positive-directed current density due to a single valley is

$$J^+_v = q \left(\frac{2 \pi m_{\text{XYZ}} kT}{h^2}\right)^{3/2} \sqrt{\frac{2kT}{\pi m_X}} \Theta_1(\eta_S).$$

Changing the valley, the band curvature, and the effective mass along $x$ change, so that $J^+_v$ depends on the particular valley. The total positive directed current density can be obtained summing over all valleys

$$J^+ = \sum_{\text{valley}} J^+_v = q \left(\frac{2 \pi m_{\text{XYZ}} kT}{h^2}\right)^{3/2} \sqrt{\frac{2kT}{\pi m_X}} \Theta_1(\eta_S) \sum_{\text{valley}} \frac{1}{m_X}.$$

In the case of silicon, assuming that the 6 band minima are populated with the same probability, $m_X = m_l$ with probability $4/6$ and $m_X = m_l$ with probability $2/6$, so that the sum over effective masses can be replaced by

$$\sum_{\text{valley}} \frac{1}{m_X} = \frac{4}{\sqrt{m_0}} + \frac{2}{\sqrt{m_i}} = \frac{6}{\sqrt{m_c}}.$$

where $m_c$ is a three dimensional effective conduction mass, which results equal to 0.283$m_0$. Let us notice that this value is slightly different from 0.26$m_0$ which is typically assumed for mobility-related problems (in this case the effective mass is calculated as a weighted average of the inverse of the directional effective masses and not of the inverse of their square root). Taking into account this effective mass, the total positive current density and the average carrier velocity of the positive directed flux are

$$J^+ = q \left(\frac{2 \pi m_{\text{DOS}} kT}{h^2}\right)^{3/2} \sqrt{\frac{2kT}{\pi m_c}} \Theta_1(\eta_S),$$

$$v^+ = \frac{J^+}{q n^+} = \sqrt{\frac{2kT}{\pi m_c}} \Theta_{3/2}(\eta_S),$$

where the term in the square root is the non degenerate one-dimensional average carrier velocity and the term $\Theta_{3/2}(\eta_S)/\Theta_{1/2}(\eta_S)$ is the velocity degeneration factor.

In the case of MOSFETs similar results can be obtained for carriers injected from the drain, and the total negative current density and the average carrier velocity of the negative directed flux at $x_{\text{max}}$ are

$$J^- = q \left(\frac{2 \pi m_{\text{DOS}} kT}{h^2}\right)^{3/2} \sqrt{\frac{2kT}{\pi m_c}} \Theta_1(\eta_D),$$

$$v^- = \frac{J^-}{q n^-} = \sqrt{\frac{2kT}{\pi m_c}} \Theta_{3/2}(\eta_D).$$

The total current density in the case of ballistic transport is therefore

$$J = J^+ - J^- = q \left(\frac{2 \pi m_{\text{DOS}} kT}{h^2}\right)^{3/2} \sqrt{\frac{2kT}{\pi m_c}} [\Theta_1(\eta_S) - \Theta_1(\eta_D)].$$

It is worth noticing that (i) in saturation ($V_{DS} \gg kT/q$) carriers injected by the drain are reflected back by the channel-drain energy barrier so that $J^- \approx 0$ and $J \approx J^+$; (ii) in equilibrium ($V_{DS} = 0$) $\eta_S = \eta_D$ so that $J = 0$. 

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C. Average carrier kinetic energy

By changing the integration variables like in Eq. (4), the average kinetic energy of the positive directed carriers in a homogeneous semiconductor can be calculated as

$$E^+ = \frac{1}{\pi} \int_{-\infty}^{+\infty} dk_y \int_{-\infty}^{+\infty} dk_x E_x (E_T, E_{FS})$$

$$= \frac{3}{2} kT \frac{3 \eta_S}{\pi}$$

where $3 \eta_S$ is the Fermi-Dirac integral of order 3/2. The term $3/2kT$ is the well known thermodynamic energy of a 3D electron gas, whereas the term in the fraction is the energy degeneration factor.

In the case of MOSFETs similar results can be obtained for carriers injected from the drain

$$E^- = \frac{3}{2} kT \frac{3 \eta_D}{\pi}$$

The reported equations in Subsections II A and II B appear very similar to the corresponding 2D case developed by Natori (Eq. (1)). What changes is that the 2D effective density of states ($N_{2D}$) is substituted with the 3D effective density of states ($N_{3D}$) and the order of Fermi-Dirac integrals is increased by 1/2. They can be useful for device modeling and parameter extraction.

III. APPLICATION TO GRAPHENE VARIABLE-BARRIER TRANSISTORS

In this section we make an example of application of the ballistic current equation (15) developed in Sec. II considering n-type graphene variable-barrier transistors. These are three-terminal devices based on a hybrid interface formed by a layer of 2D graphene acting as source electrode and by a layer of n$^+$ doped silicon, acting as both a channel and drain. Between source and channel a Schottky junction is formed. Since graphene is a semi-metal, with low density of states, the junction barrier height is electrostatically modulated by the voltage applied to a control metal gate. A simplified band-diagram is shown in Fig. 2, where $x$ denotes the source to “Silicon channel” direction. Depending on the position of the drain contact, the transport can be entirely in the $x$-direction (back drain contact) or a combination of $x$-direction (flux injection) and transverse ($y$ or $z$) direction (top drain contact). In the latter case after injection into/from the silicon channel, transport continues along the transverse direction till the drain contact. For simplicity, in the rest of this discussion we will neglect (i) the scattering in the silicon channel, so as to treat the case of ballistic transport; (ii) barrier lowering at the Schottky junction due to image charge effects; (iii) tunneling through the Schottky junction; (iv) the possible reflections at the interface due to band-structure mismatch. These assumptions allow us to keep the model simple, and to concentrate on the formalism developed in Sec. II. In support to our assumptions, several works on graphene-silicon Schottky junctions show experimental measurements that agree with the classical equation of the current in a Schottky junction, supporting the thermionic assumption for graphene-silicon junctions. If some assumptions are not fully verified, the only change should be in a pre-factor in the current expression, while the functional dependence of the current with respect to different parameters should not change. Moreover, it has been shown that the interface between chemically inert graphene and a completely saturated semiconductor surface is free of defects so that we can neglect interfacial states. The Schottky barrier $\Phi_B$ is modulated by the graphene surface potential $V_{Gr}$

$$q\Phi_B = q\Phi_B^0 - qV_{Gr}(V_G)$$

where $q\Phi_B^0$ is the equilibrium barrier height when the graphene energy Fermi level is aligned with the silicon energy Fermi level ($q\Phi_B^0 \approx 0.5$eV) and $V_G$ is the control gate voltage. The source is grounded. The barrier height modulation as function of $V_G$ is defined by the device electrostatics which can be expressed in terms of charge balance

$$Q_M + Q_{Gr} + Q_{Si} = 0,$$

$$Q_M = C_{ox}[V_G - V_{Gr}] = \frac{\varepsilon_{ox} \varepsilon_0}{\ell_{ox}} [V_G - V_{Gr}],$$

$$Q_{Gr} = \frac{2q}{\pi} \left( \frac{kT}{\hbar v_F} \right)^2 \left[ \frac{3}{2} \left( 1 - \frac{qV_{Gr}}{kT} \right)^{3/2} \right],$$

$$Q_{Si} = \frac{\Phi_j}{\Phi_D} \sqrt{2\varepsilon_{Si} \ell_0 kT N_D} \left[ \left( \exp \left( \frac{q\Phi_j}{kT} \right) + \exp \left( \frac{-q\Phi_j}{kT} \right) \right) - 1 \right]^{1/2}$$

where $Q_M$, $Q_{Gr}$, and $Q_{Si}$ are the charge (per unit area) in the metal gate, in the 2D graphene layer (source), and in the n-doped silicon layer (channel and drain), respectively. $C_{ox}$, $\varepsilon_{ox}$, and $\ell_{ox}$ are the oxide capacitances per unit area, the relative dielectric oxide constant, and the oxide thickness, respectively. $\ell_0$ is the absolute dielectric constant, $v_F$ is the Fermi velocity of carriers in graphene ($\sim 10^8$ cm/s), $\varepsilon_{Si}$ is the
relative dielectric constant, \( n_i \) is the intrinsic electron concentration, \( N_D \) is the doping concentration of the silicon layer, and \(-\Phi_i \) is the potential drop in the silicon layer calculated as (Fig. 2)

\[
q\Phi_i = q\Phi_B + qV_{DS} + E_F - E_C(\chi_{DC}),
\]

\[
N_D \approx n(\chi_{DC}) = 3D^{1/2}\left(\frac{E_F - E_C(\chi_{DC})}{kT}\right)^{-1/2},
\]

where \( \chi_{DC} \) is the absissa of the drain contact and \( n \) is the electron concentration in the silicon. Let us notice that the expression for \( Q_{Si} \) and \( n \) are approximations valid when charge density associated to current flow is negligible. As expected, the higher is the oxide thickness and dielectric constant values, the stronger is the barrier height modulation with the gate voltage.

As in Eq. (17) we write the current as the sum of two opposite fluxes at the top of the barrier: \( J^- \) is injected from the drain and \( J^+ \) is injected from the source. In this case, it is extremely convenient for us to write the equations at the top of the barrier in silicon, so that we can use a parabolic 3D energy dispersion relation. This is the typical approach followed in the case of metal-semiconductor junctions. From a rigorous point of view, we are neglecting in this case possible reflections at the interface due to bandstructure mismatch; however, since we are dealing with semiclasical electrons, this is a typical and acceptable approximation. For convenience, we first write the expression for \( J^- \)

\[
J^-(\eta) = q\left(\frac{2\pi m_{DOS}kT}{\hbar^2}\right)^{3/2}\sqrt{\frac{2kT}{\pi m_C}}\left(\frac{E_F - E_C}{kT}\right)^{1/2} = A'^2 S_1(\eta),
\]

where \( J^- \) is current density injected from the silicon layer to the graphene layer, \( \eta = (E_F - E_C)_{\chi_{max}}/kT \), \( E_C \) is the conduction band edge, and \( E_F \) is the drain Fermi energy. The normalized electrostatic potential \( \eta \) is related to the drain and to the gate voltage by

\[
\eta = \frac{E_F - E_C}{kT} = \frac{q\Phi_B(V_G)}{kT} - \frac{qV_{DS}}{kT} = \eta_0 - \frac{qV_{DS}}{kT},
\]

\[
\eta_0 = \eta(V_{DS} = 0) = -\frac{q\Phi_B}{kT} + \frac{qV_G}{kT}.
\]

The current flux \( J^+ \) injected from the source to the drain is equal to the flux injected from the drain to the source in the equilibrium case \( (V_{DS} = 0) \) when the total current is zero \( (J_{DS} = 0) \)

\[
J^+ = J^- = A'^2 S_1(\eta_0).
\]

Using this approach we can write the total current without taking into account for the band-structure of the source (graphene) region

\[
J_{DS} = J^+ - J^- = A'^2 [S_1(\eta_0) - 3S_1(\eta)],
\]

In the case in which carriers in silicon do not form a degenerate electron gas, the Fermi-Dirac statistics can be approximated by the Boltzmann statistics so that \( S_1(\eta) \approx e^\eta \)

\[
J_{DS} \approx A'^2 e^{\frac{q\Phi_B}{kT}} e^{\frac{qV_G}{kT}}[1 - e^{\frac{qV_{DS}}{kT}}].
\]

Equation (26) appears very similar but should not be confused with Eq. (17). The latter is used for a homogenous material (e.g., silicon) and \( \eta_S \), \( \eta_D \) are related to Fermi levels of source and drain contacts. Differently, Eq. (26) describes the transport in a non homogenous structure and \( \eta_D \) is related to the Fermi level of the source contact including the modulation effect of the gate potential. At first sight, Eq. (26) seems to lose the information related to the material “graphene.” However, this info is contained in the relation between the graphene charge and the potential (the third in Eq. (21)). This relation is a function of the dispersion relation in the graphene and in particular of the graphene Fermi-velocity \( v_F \).

We stress again the fact that the validity of Eq. (26) is subjected to the assumptions discussed above, although experimental measurements confirm the functional dependence of the current with respect to parameters. Moreover, the actual magnitude of the current is function of material dependent parameters such as the Fermi velocity of graphene and the graphene-silicon conduction band offset which are normally extrapolated by experiments.2,1,22

Fig. 4 shows the calculated current (with parameters \( \varepsilon_{ox} = 21, t_{ox} = 1 \text{ nm}, N_D = 10^{16} \text{ cm}^{-3}, V_{DS} = 1 \text{ V} \) in the cases

\[
\eta = \frac{E_F - E_C}{kT} = \frac{q\Phi_B(V_G)}{kT} - \frac{qV_{DS}}{kT} = \eta_0 - \frac{qV_{DS}}{kT},
\]

\[
\eta_0 = \eta(V_{DS} = 0) = -\frac{q\Phi_B}{kT} + \frac{qV_G}{kT}.
\]
of (i) Boltzmann distribution and (ii) Fermi-Dirac distribution. It is apparent that, especially in the high current regime, it is very important to use the Fermi-Dirac distribution. The same figure shows the current density calculated taking into account the Fermi-Dirac distribution for other values of the parameters $t_{ox}$, $t_{in}$, and $N_D$. Higher channel doping ($N_D = 10^{20}$ cm$^{-3}$) translates in a simple shift of the curve (a change in threshold voltage), while the use of SiO$_2$ as gate oxide ($t_{ox} = 3.9$) results in a strong degradation of the sub-threshold slope even for very thin oxide thickness ($t_{in} = 1$ nm).

IV. CONCLUSION

In this paper we developed analytical expressions for charge transport in ballistic devices with a three-dimensional degenerate electron gas. Such devices have a size in the transport direction that is much smaller than the carrier mean free path, so that transport is ballistic and quasi-ballistic, and larger size in the transversal directions so that quantum confinement is negligible. Solid-state nanoscale devices inspired to vacuum tubes or to bipolar transistors can easily belong to this category. We have shown an application of the proposed physical description to the development of a semi-analytical model for ballistic transport in variable-barrier graphene transistors.